SAT Math: Geometry and Trigonometry, study guide and worked solutions

What the SAT asks

Geometry and Trigonometry covers four skills: area and volume, lines, angles and triangles, right triangles and trigonometry, and circles. Together they make up about 15% of the Math section, so expect about 6 or 7 of the 44 Math questions, shared roughly equally among the four skills. Each question tests whether you pick the right fact or formula and feed it the right numbers.

Area and volume

Find perimeter, area, surface area and volume, including the effect of scaling

The idea. Perimeter is the distance around a flat shape. Area is the flat space inside it (square units). Volume is the space inside a solid (cubic units). Surface area is the total area of all the faces of a solid (square units).

The reference sheet in the Bluebook app gives the area of a circle, a rectangle and a triangle, and the volume of a rectangular box, a cylinder, a sphere, a cone and a pyramid. It does not give the area of a trapezoid or any surface area formula, so learn those by heart. Most mistakes come from the numbers, not the formula: a diameter used where the formula needs a radius, or a slanted side used where it needs the perpendicular height.

Unusual shapes are built from simple ones: split them into rectangles, triangles and parts of circles and add, or subtract a missing piece from a bigger shape.

The rules.

- Rectangle: area = length × width; perimeter = 2(length + width).
- Triangle: area = ½ × base × height, where the height is perpendicular to the base.
- Trapezoid: area = ½ × (sum of the two parallel sides) × height.
- Circle: area = πr^2; circumference = $2\pi r$. The radius r is half the diameter.
- Rectangular box: volume = lwh; surface area = 2(lw + lh + wh).
- Cylinder: volume = $\pi r^2 h$; surface area of a closed cylinder = $2\pi r^2 + 2\pi rh$ (two circles plus the curved side).
- Cone: volume = $(1/3)\pi r^2 h$. Pyramid: volume = (1/3) × base area × height.
- Sphere: volume = $(4/3)\pi r^3$; surface area = $4\pi r^2$.
- Scale factor k: if every length is multiplied by k, every area is multiplied by k^2 and every volume by k^3.

> **Worked example 1: a cut-out shape**
>
> A rectangle is 10 cm long and 8 cm wide. A smaller rectangle, 4 cm by 3 cm, is cut out of one corner, leaving an L-shaped piece. Find the area and the perimeter of the L-shaped piece.

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1. Area of the full rectangle: 10 × 8 = 80 square centimeters.
2. Area of the corner piece: 4 × 3 = 12 square centimeters.
3. Area of the L shape: 80 − 12 = 68 square centimeters.
4. Perimeter: walk around the six sides: 10, 8 − 3 = 5, 4, 3, 10 − 4 = 6 and 8. Total: 10 + 5 + 4 + 3 + 6 + 8 = 36 cm.
5. Check: this equals the perimeter of the full rectangle, 2(10 + 8) = 36, because the two new inside edges replace the two outer pieces that were removed.

Answer: area 68 cm^2, perimeter 36 cm.

Fastest route: subtract areas; for the perimeter of a corner cut, use the full rectangle's perimeter.

Worked example 2: scaling a sphere

A ball has a radius of 3 cm. A larger ball has a radius 50% greater. How many times larger is the volume of the larger ball, and what is its volume?

1. A 50% increase means the scale factor is k = 1.5.
2. Volumes scale by $k^3 = 1.5^3 = 1.5 \times 1.5 \times 1.5 = 3.375$.
3. Small ball: $(4/3)\pi \times 3^3 = (4/3)\pi \times 27 = 36\pi$.
4. Large ball: $36\pi \times 3.375 = 121.5\pi$ cubic centimeters.
5. Check directly: the new radius is 4.5, and $4.5^3 = 91.125$. Then $(4/3)\pi \times 91.125 = 121.5\pi$. The results agree.

Answer: 3.375 times as large; 121.5π cm^3 (about 382).

Desmos route: type (4/3)π(4.5)^3 to get about 381.7, then compare with the decimal value of each choice.

SAT trap: scaling by the wrong power

If a question says every length is doubled, the area is not doubled. It is multiplied by $2^2 = 4$, and the volume by $2^3 = 8$. Expect wrong choices built from these slips. Ask: am I scaling a length (power 1), an area (power 2) or a volume (power 3)? And a diameter of 10 means r = 5.

Lines, angles and triangles

Use angle facts, triangle angle sums and similar triangles

The idea. These questions give a figure, or describe one in words, and ask for a missing angle or side. If there is no figure, draw one and mark everything you are told. Trust the numbers, not the look of the picture.

Nearly every angle question uses a few facts: angles on a straight line, angles at parallel lines, and the triangle angle sum. Write one equation from one fact and solve it.

Similar triangles have the same three angles, so one is an enlarged (or shrunk) copy of the other. Their matching sides are all in the same ratio. A common SAT setup is a line inside a triangle parallel to one side: it cuts off a smaller triangle similar to the whole one, because the parallel line makes equal

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corresponding angles and the triangles share the top angle.

The rules.

- Angles on a straight line add to 180°. Angles around a point add to 360°. Vertical (opposite) angles are equal.
- Two parallel lines cut by a transversal: corresponding angles are equal, alternate interior angles are equal, and same-side interior angles add to 180°.
- The three angles of a triangle add to 180°.
- An exterior angle of a triangle equals the sum of the two interior angles that are not next to it.
- Isosceles triangle: the angles opposite the two equal sides are equal. Equilateral triangle: all angles are 60°.
- Similar triangles: if matching sides are in the ratio k, the areas are in the ratio k^2.
- A polygon with n sides: interior angles add to (n – 2) × 180°; the exterior angles (one at each vertex) add to 360°. For a regular polygon, each exterior angle is 360 ÷ n, so a regular hexagon has exterior angles of 60° and interior angles of 120°.

Worked example 1: exterior angle with algebra

In triangle ABC, the exterior angle at C measures 130°. Angle A measures (x + 20)° and angle B measures (2x + 5)°. What is the measure of angle B?

1. The exterior angle at C equals the sum of the two interior angles not next to it, A and B: (x + 20) + (2x + 5) = 130.
2. Simplify: 3x + 25 = 130, so 3x = 105 and x = 35.
3. Angle B = 2(35) + 5 = 75°. Angle A = 35 + 20 = 55°.
4. Check: the interior angle at C is 180 – 130 = 50°, and 55 + 75 + 50 = 180. Correct.

Answer: 75°.

Desmos route: type 3x+25=130 and Desmos draws the vertical line x = 35. By hand is just as fast here.

Worked example 2: a parallel line inside a triangle

In triangle PQR, point S is on side PQ and point T is on side PR, and segment ST is parallel to side QR. PS = 8, SQ = 12 and QR = 25. What is the length of ST?

1. Because ST is parallel to QR, triangle PST is similar to triangle PQR (they share angle P, and the parallel lines make the other angles equal).
2. Find the whole side of the large triangle: PQ = PS + SQ = 8 + 12 = 20.
3. Ratio of small to large: PS/PQ = 8/20 = 2/5.
4. So ST/QR = 2/5, which gives ST = (2/5) × 25 = 10.
5. Check: 10/25 = 8/20 = 0.4.

Answer: ST = 10.

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SAT trap: using part of a side in a similar triangle

In worked example 2, a common wrong setup is PS/SQ = ST/QR, which gives ST = (8/12) × 25 ≈ 16.7. SQ is not a side of either triangle. Compare whole sides: PS with PQ, not PS with SQ.

Right triangles and trigonometry

Use Pythagoras, special right triangles and sine, cosine and tangent, including radians

The idea. A right triangle has one 90° angle. The side across from the right angle is the **hypotenuse**, the longest side; the other two are the legs. Know two sides, and the Pythagorean theorem gives the third.

Two **special right triangles** come up again and again. The 45°-45°-90° triangle is half of a square, cut along the diagonal. The 30°-60°-90° triangle is half of an equilateral triangle. Their side ratios are on the reference sheet and give exact sides without a calculator.

Sine, cosine and tangent are ratios of two sides of a right triangle, named from the acute angle you are looking at. "Opposite" is the side across from that angle and "adjacent" is the leg touching it. The ratios depend only on the angle, so similar right triangles have the same sine, cosine and tangent.

Angles can be measured in degrees or in **radians**. A full turn is 360°, which is 2π radians, so a half turn is 180° = π radians.

The rules.

- Pythagorean theorem: $a^2 + b^2 = c^2$, where c is the hypotenuse.
- Whole-number right triangles to recognize: 3-4-5, 5-12-13, 8-15-17, 7-24-25, and their multiples such as 6-8-10 and 10-24-26.
- 45°-45°-90°: sides x, x and x√2 (hypotenuse = leg × √2).
- 30°-60°-90°: sides x, x√3 and 2x. The shortest side x is opposite 30°, x√3 is opposite 60°, and 2x is the hypotenuse.
- sin = opposite ÷ hypotenuse; cos = adjacent ÷ hypotenuse; tan = opposite ÷ adjacent. (Memory aid: SOH CAH TOA.)
- For the two acute angles of a right triangle, sin x° = cos(90° − x°). The sine of one acute angle equals the cosine of the other.
- π radians = 180°. Degrees to radians: multiply by π/180. Radians to degrees: multiply by 180/π.

Worked example 1: a missing leg

A 13-foot ladder leans against a vertical wall. Its foot is 5 feet from the base of the wall on level ground. How high up the wall does the ladder reach?

1. The wall and the ground make a right angle, so the ladder is the hypotenuse: c = 13, one leg a = 5, and the height h is the other leg.
2. $5^2 + h^2 = 13^2$, so $25 + h^2 = 169$.
3. $h^2 = 169 - 25 = 144$, so h = 12.
4. Check: 5-12-13 is in the list of whole-number triangles, so you could skip the algebra.

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Answer: 12 feet.

Worked example 2: special right triangles

An equilateral triangle has sides of length 8. What is its height?

1. The height splits the equilateral triangle into two 30°-60°-90° triangles. The hypotenuse is 8 = 2x, so x = 4 (half the base).
2. The height is opposite the 60° angle, so it is $x\sqrt{3} = 4\sqrt{3}$ (about 6.93).
3. Check with Pythagoras: $4^2 + (4\sqrt{3})^2 = 16 + 48 = 64 = 8^2$. Correct.

Answer: $4\sqrt{3}$. In the same way, a square with diagonal 10 has side $10/\sqrt{2} = 5\sqrt{2}$ (a 45°-45°-90° triangle).

Worked example 3: sine, cosine and tangent

Right triangle DEF has its right angle at E. DE = 8, EF = 15 and DF = 17. Find sin F, cos F and tan F, and then cos D.

1. DF = 17 is the hypotenuse (it is opposite the right angle at E). Check: $8^2 + 15^2 = 64 + 225 = 289 = 17^2$.
2. From angle F: the opposite side is DE = 8, the adjacent leg is EF = 15.
3. sin F = 8/17, cos F = 15/17, tan F = 8/15.
4. From angle D: the adjacent leg is DE = 8, so cos D = 8/17.
5. Check: cos D = sin F, as the rule says, because D and F add to 90°.

Answer: sin F = 8/17, cos F = 15/17, tan F = 8/15, cos D = 8/17.

Worked example 4: using a trig ratio to find a side

From a point 40 meters from the base of a tower, on level ground, the angle up to the top of the tower is 35°. To the nearest meter, how tall is the tower?

1. The tower, the ground and the line of sight form a right triangle. The height h is opposite the 35° angle and the 40 m is adjacent to it.
2. Opposite and adjacent means tangent: $\tan 35° = h/40$.
3. $h = 40 \times \tan 35° \approx 40 \times 0.7002 \approx 28.0$.
4. Check: 45° would give a height of 40 ($\tan 45° = 1$), so a smaller angle should give less than 40.

Answer: about 28 meters.

Desmos route: Desmos works in radians unless you change it. Open the settings (the wrench icon) and choose Degrees, then type 40tan(35). If you stay in radians, type 40tan(35π/180) instead. Forgetting this gives a completely wrong number.

Worked example 5: degrees and radians

Convert $3\pi/4$ radians to degrees, and 210° to radians.

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1. $3\pi/4 \times 180/\pi = (3 \times 180)/4 = 540/4 = 135°$.
2. $210 \times \pi/180 = 210\pi/180 = 7\pi/6$ radians (divide 210 and 180 by 30).
3. Check: $\pi/4$ is 45°, so $3\pi/4$ is $3 \times 45 = 135°$. And $\pi/6$ is 30°, so $7\pi/6$ is $7 \times 30 = 210°$.

Answer: 135°; $7\pi/6$ radians.

Fastest route: remember $\pi/6 = 30°$, $\pi/4 = 45°$, $\pi/3 = 60°$, $\pi/2 = 90°$, and count in those steps.

SAT trap: the hypotenuse in the wrong place

If you are given the hypotenuse and one leg, do not add their squares. The hypotenuse goes alone on one side: leg^2 = hypotenuse^2 – other leg^2. The missing leg must be shorter than the hypotenuse. Also, opposite and adjacent change with the angle: name the angle first, then label the sides.

Circles

Use arc length, sector area, central angles and the equation of a circle

The idea. Circle questions come in two kinds. The first is about parts of a circle. A **central angle** has its vertex at the center. The **arc** it cuts off is a fraction of the circumference, and the **sector** (the pizza-slice region) is the same fraction of the area. The fraction is the central angle over a full turn: 360° or 2π radians.

The second is the equation of a circle in the xy-plane. In standard form you read the center and radius directly; if the equation is multiplied out, complete the square in x and in y first. Remember that the number on the right of the standard form is the radius squared, not the radius.

The rules.

- Circumference = $2\pi r$; area = πr^2.
- Arc length = $(\theta/360) \times 2\pi r$ when θ is in degrees, or $r\theta$ when θ is in radians.
- Sector area = $(\theta/360) \times \pi r^2$ when θ is in degrees, or $\frac{1}{2}r^2\theta$ when θ is in radians.
- A circle with center (h, k) and radius r has equation $(x - h)^2 + (y - k)^2 = r^2$.
- Completing the square: $x^2 + bx = (x + b/2)^2 - (b/2)^2$.
- A tangent line is perpendicular to the radius at the point where it touches the circle.
- An inscribed angle (vertex on the circle) is half the central angle that cuts off the same arc. An angle inscribed in a semicircle is 90°.

Worked example 1: arc and sector, in degrees and in radians

A circle has a radius of 12. A central angle measures 60°. Find the length of the arc and the area of the sector.

1. Fraction of the circle: $60/360 = 1/6$.
2. Circumference = $2\pi \times 12 = 24\pi$, so the arc = $(1/6) \times 24\pi = 4\pi$.
3. Area = $\pi \times 12^2 = 144\pi$, so the sector = $(1/6) \times 144\pi = 24\pi$.
4. Check in radians: $60° = \pi/3$. Arc = $r\theta = 12 \times \pi/3 = 4\pi$. Sector = $\frac{1}{2} \times 144 \times \pi/3 = 24\pi$. Both

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agree.

Answer: arc 4π (about 12.6), sector 24π (about 75.4).

Worked example 2: from an inscribed angle to an arc

Points A, B and C lie on a circle with radius 18. Inscribed angle ABC measures 35°. What is the length of arc AC (the arc that does not contain B)?

1. The central angle on the same arc is twice the inscribed angle: 2 × 35° = 70°.
2. Circumference = 2π × 18 = 36π.
3. Arc AC = (70/360) × 36π = (70 × 36/360)π = 7π.
4. Check: 70° is just under a fifth of 360°, and 7π is just under a fifth of 36π (7.2π).

Answer: 7π (about 22.0).

Worked example 3: completing the square

The graph of $x^2 + y^2 - 10x + 4y + 13 = 0$ in the xy-plane is a circle. Find its center and radius.

1. Group the x terms and the y terms, and move the number across: $(x^2 - 10x) + (y^2 + 4y) = -13$.
2. Half of −10 is −5, and $(-5)^2 = 25$. Half of 4 is 2, and $2^2 = 4$. Add 25 and 4 to both sides.
3. $(x - 5)^2 + (y + 2)^2 = -13 + 25 + 4 = 16$.
4. Center (5, −2). Radius = $\sqrt{16} = 4$.
5. Check: expanding gives back $x^2 + y^2 - 10x + 4y + 25 + 4 - 16$, and 25 + 4 − 16 = 13. It matches.

Answer: center (5, −2), radius 4.

Desmos route: type the equation as given. The circle runs from x = 1 to x = 9 and from y = −6 to y = 2, so the center is the midpoint, (5, −2), and the radius is half the width, 4.

SAT trap: signs of the center and the squared radius

The equation $(x + 2)^2 + (y - 7)^2 = 49$ has center (−2, 7), not (2, −7), because x + 2 is the same as x − (−2). Its radius is $\sqrt{49} = 7$, not 49. Answer choices are built from these two slips. When you complete the square, add the same numbers to the right-hand side too.

Practice set

Questions 1 to 11 are multiple choice. Questions 12 to 14 are student-produced response: type your own answer. Aim for 60 to 90 seconds each.

1. (Easy) A rectangle has a length of 12 centimeters and a perimeter of 40 centimeters. What is the area of the rectangle, in square centimeters?

A) 96
B) 240

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C) 336
D) 480

2. (Medium) A cylinder has a volume of 20 cubic inches. The radius and the height of the cylinder are both multiplied by 3 to make a new cylinder. What is the volume of the new cylinder, in cubic inches?

A) 60
B) 180
C) 540
D) 1,620

3. (Hard) An isosceles trapezoid has parallel sides of lengths 10 and 22. Each of its two nonparallel sides has length 10. What is the area of the trapezoid?

A) 96
B) 128
C) 160
D) 256

4. (Easy) Line ℓ is parallel to line m, and line t crosses both of them. Between the two parallel lines and on the same side of t, the angle at line ℓ measures $(3x + 10)°$ and the angle at line m measures $(2x + 20)°$. What is the value of x?

A) 10
B) 30
C) 34
D) 36

5. (Medium) In triangle ABC, point D is on side AB and point E is on side AC, and segment DE is parallel to side BC. If AD = 6, DB = 4 and DE = 9, what is the length of BC?

A) 6
B) 13
C) 13.5
D) 15

6. (Easy) In a right triangle, the hypotenuse has length 26 and one leg has length 10. What is the length of the other leg?

A) 16
B) 24
C) $2\sqrt{194}$
D) 36

7. (Medium) In triangle JKL, angle K is a right angle, angle J measures 30° and KL = 7. What is the length of JK?

A) $7\sqrt{3}$
B) 14
C) $7\sqrt{2}$
D) $7\sqrt{3}/3$

8. (Hard) In a right triangle, the two acute angles measure x° and y°. If $\cos x° = 12/13$, what is the value of $\tan y°$?

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A) 5/13
B) 5/12
C) 12/13
D) 12/5

9. (Easy) An angle measures 5π/6 radians. What is its measure in degrees?

A) 30
B) 75
C) 150
D) 300

10. (Easy) In the xy-plane, a circle has the equation $(x - 3)^2 + (y + 5)^2 = 36$. Which of the following gives the center and the radius of the circle?

A) center (3, –5), radius 6
B) center (–3, 5), radius 6
C) center (3, –5), radius 36
D) center (–3, 5), radius 36

11. (Hard) In the xy-plane, the graph of $x^2 + y^2 + 8x - 6y - 11 = 0$ is a circle. What is the radius of the circle?

A) √11
B) 5
C) 6
D) 36

12. (Medium, student-produced response) A solid cone has a radius of 6 centimeters and a volume of 96π cubic centimeters. A cylinder has the same radius and the same volume as the cone. What is the height of the cylinder, in centimeters?

13. (Hard, student-produced response) Triangle PQR is similar to triangle STU, and side PQ corresponds to side ST. PQ = 4, ST = 10 and the area of triangle PQR is 12 square units. What is the area of triangle STU, in square units?

14. (Medium, student-produced response) A circle has a radius of 9. A sector of the circle has a central angle of 2π/3 radians. The area of the sector is kπ square units. What is the value of k?

Solutions

1. Answer: A (96)

1. Perimeter: 2(12 + w) = 40, so 12 + w = 20 and w = 8.
2. Area = 12 × 8 = 96 square centimeters.

Wrong options: B (240) uses 20, the half-perimeter, as the width. C (336) uses 40 – 12 = 28 as the width. D (480) multiplies the length by the whole perimeter.

Skill: Area and volume.

2. Answer: C (540)

1. Volume = $\pi r^2 h$. Replace r with 3r and h with 3h: $\pi(3r)^2(3h) = \pi \times 9r^2 \times 3h = 27\pi r^2 h$.

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2. So the new volume is 27 times the old one: 27 × 20 = 540 cubic inches.

Wrong options: A (60) multiplies by 3, as if volume scaled like length. B (180) multiplies by 9, the area factor, or triples only the radius. D (1,620) multiplies by 81, raising 3 to the fourth power.

Skill: Area and volume.

3. Answer: B (128)

1. Put the 22 side at the bottom and drop a height from each end of the top side. The bottom splits into 10 in the middle and two end pieces of (22 – 10) ÷ 2 = 6.
2. Each end makes a right triangle with hypotenuse 10 (the slanted side) and one leg 6. Height: $h^2 = 10^2 - 6^2 = 100 - 36 = 64$, so h = 8 (a 6-8-10 triangle).
3. Area = ½ × (10 + 22) × 8 = ½ × 32 × 8 = 128.

Wrong options: A (96) uses 6, the end piece, as the height. C (160) uses the slanted side 10 as the height. D (256) forgets the ½.

Skill: Area and volume.

4. Answer: B (30)

1. The two angles are same-side interior angles (between the parallel lines, on the same side of t), so they add to 180°.
2. (3x + 10) + (2x + 20) = 180, so 5x + 30 = 180.
3. 5x = 150, so x = 30.
4. Check: 100° + 80° = 180°.

Wrong options: A (10) sets the angles equal, which is only true for corresponding or alternate interior angles. C (34) drops the 20 and solves 5x + 10 = 180. D (36) drops both constants and solves 5x = 180.

Skill: Lines, angles and triangles.

5. Answer: D (15)

1. DE is parallel to BC, so triangle ADE is similar to triangle ABC.
2. Whole side: AB = AD + DB = 6 + 4 = 10.
3. Ratio of small to large: AD/AB = 6/10 = 3/5. So DE/BC = 3/5.
4. 9/BC = 3/5, so 3 × BC = 45 and BC = 15.
5. Check: 9/15 = 0.6 = 6/10.

Wrong options: A (6) uses AD/DB = DE/BC with the parts of the side, then solves BC = 9 × 4/6. B (13) adds DB to DE. C (13.5) uses the parts of the side the other way round, 9 × 6/4.

Skill: Lines, angles and triangles.

6. Answer: B (24)

1. The hypotenuse is 26, so the missing leg b satisfies $10^2 + b^2 = 26^2$.
2. $b^2 = 676 - 100 = 576$, so b = 24.
3. Check: this is the 5-12-13 triangle doubled (10-24-26).

Wrong options: A (16) subtracts the sides instead of their squares. C (2√194) adds the squares,

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100 + 676 = 776, treating 26 as a leg. D (36) adds the sides.

Skill: Right triangles and trigonometry.

7. Answer: A ($7\sqrt{3}$)

1. The angles are J = 30°, K = 90°, so L = 60°. This is a 30°-60°-90° triangle.
2. KL is opposite angle J (30°), so it is the shortest side: x = 7.
3. JK is opposite angle L (60°), so JK = $x\sqrt{3} = 7\sqrt{3}$.
4. Check: JL = 2x = 14, and $49 + 147 = 196 = 14^2$.

Wrong options: B (14) is the hypotenuse JL. C ($7\sqrt{2}$) uses the 45°-45°-90° ratio. D ($7\sqrt{3}/3$, which is $7/\sqrt{3}$) treats 7 as the side opposite 60° and divides by $\sqrt{3}$.

Skill: Right triangles and trigonometry.

8. Answer: D (12/5)

1. cos x° = adjacent/hypotenuse = 12/13. Draw a right triangle with the leg next to angle x equal to 12 and the hypotenuse 13.
2. The third side: $13^2 - 12^2 = 169 - 144 = 25$, so it is 5. This leg is opposite angle x.
3. Angle y is the other acute angle. The leg of length 12 is opposite y and the leg of length 5 is adjacent to y.
4. tan y° = opposite/adjacent = 12/5.

Wrong options: A (5/13) is sin x°, which is also cos y°. B (5/12) is tan x°, the tangent of the wrong angle. C (12/13) is sin y°, the sine instead of the tangent.

Skill: Right triangles and trigonometry.

9. Answer: C (150)

1. Multiply by 180/π: $(5\pi/6) \times (180/\pi) = (5 \times 180)/6$.
2. $5 \times 180 = 900$, and $900 \div 6 = 150°$.

Wrong options: A (30) converts π/6 and forgets the 5. B (75) uses 90° instead of 180° for π. D (300) uses 360° for π.

Skill: Right triangles and trigonometry.

10. Answer: A (center (3, –5), radius 6)

1. Compare with $(x - h)^2 + (y - k)^2 = r^2$.
2. x – 3 gives h = 3. y + 5 = y – (–5) gives k = –5. So the center is (3, –5).
3. $r^2 = 36$, so r = 6.

Wrong options: B flips both signs of the center. C gives r^2 as the radius. D makes both mistakes.

Skill: Circles.

11. Answer: C (6)

1. Group and move the constant: $(x^2 + 8x) + (y^2 - 6y) = 11$.
2. Half of 8 is 4, and $4^2 = 16$. Half of –6 is –3, and $(-3)^2 = 9$. Add 16 and 9 to both sides.
3. $(x + 4)^2 + (y - 3)^2 = 11 + 16 + 9 = 36$.

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4. Radius = $\sqrt{36} = 6$. (The center is (–4, 3).)

Desmos route: type the equation as given. The circle reaches from x = –10 to x = 2, a width of 12, so the radius is 6.

Wrong options: A ($\sqrt{11}$) takes the square root of the constant without completing the square. B (5) adds 8 and 6 instead of 16 and 9: $\sqrt{(11 + 8 + 6)} = \sqrt{25} = 5$. D (36) is r^2, not r.

Skill: Circles.

12. Answer: 8/3 (also accepted: 2.666 or 2.667)

1. Cone: $(1/3)\pi \times 6^2 \times h = 96\pi$. Since $(1/3) \times 36 = 12$, this is $12\pi h = 96\pi$, so the cone's height is 8.
2. Cylinder with the same volume: $\pi \times 6^2 \times H = 96\pi$, so $36\pi H = 96\pi$.
3. $H = 96/36 = 8/3$.
4. Check: a cylinder holds 3 times a cone of the same radius and height, so for equal volume it is one third as tall: 8 ÷ 3.

Skill: Area and volume.

13. Answer: 75

1. Scale factor from PQR to STU: ST/PQ = 10/4 = 2.5.
2. Areas scale by the square of the factor: $2.5^2 = 6.25$.
3. Area of STU = 12 × 6.25 = 75.

Common wrong answer: 30, from multiplying the area by 2.5 instead of 2.5^2.

Skill: Lines, angles and triangles.

14. Answer: 27

1. Sector area in radians: $\frac{1}{2}r^2\theta = \frac{1}{2} \times 81 \times (2\pi/3)$.
2. $\frac{1}{2} \times 2\pi/3 = \pi/3$, and $81 \times \pi/3 = 27\pi$. So k = 27.
3. Check in degrees: $2\pi/3 = 120°$, which is one third of a circle. Area of the circle = 81π, and one third of it is 27π.

Common wrong answers: 6, which is the arc length coefficient ($r\theta = 9 \times 2\pi/3 = 6\pi$), and 54, from forgetting the ½.

Skill: Circles.

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