

Sets and Venn diagrams: IGCSE Maths worksheet with answers

Cambridge IGCSE Mathematics 0580 (Extended). Read the summary, then attempt all 20 questions before checking the answers at the end.

What you need to know

Set notation

Symbol	Meaning	Example
{ }	a set, listed inside curly brackets	$A = \{1, 2, 3, 4\}$
$\in$	is an element of	$3 \in \{1, 2, 3, 4\}$
$\notin$	is not an element of	$7 \notin \{1, 2, 3, 4\}$
$\subseteq$	is a subset of (every element of the first set is in the second)	$\{1, 2\} \subseteq \{1, 2, 3, 4\}$
$\mathcal{E}$	the universal set, everything being considered	$\mathcal{E} = \{\text{integers from 1 to 10}\}$
A'	the complement of A, everything in $\mathcal{E}$ not in A	$\mathcal{E} = \{1, 2, 3, 4\}$, $A = \{1, 2\}$, so $A' = \{3, 4\}$
$A \cup B$	union, elements in A or B or both	$\{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}$
$A \cap B$	intersection, elements in both A and B	$\{1, 2\} \cap \{2, 3\} = \{2\}$
$\emptyset$ or { }	the empty set, with no elements	$\{1, 2\} \cap \{3, 4\} = \emptyset$
$n(A)$	the number of elements in A	$A = \{1, 2, 3, 4\}$, so $n(A) = 4$

Each element is listed once: the set of letters in BOOK is {B, O, K}, so $n = 3$. The empty set is a subset of every set, and when two sets share nothing their intersection is $\emptyset$, never the number 0.

Regions of a two-set Venn diagram

Two overlapping circles A and B inside a rectangle (the universal set $\mathcal{E}$) make four regions:

- the overlap, inside both circles: $A \cap B$
- inside A but outside B: $A \cap B'$
- inside B but outside A: $A' \cap B$
- outside both circles: $(A \cup B)'$, the same region as $A' \cap B'$

Build other regions from these. $A \cup B$ is all three parts of the circles. A' is the B-only crescent together with the outside. $(A \cap B)'$ is everything except the overlap, the same as $A' \cup B'$.

Regions of a three-set Venn diagram

Three overlapping circles A, B and C give eight regions: the centre ($A \cap B \cap C$), three regions where exactly two circles overlap (for example $A \cap B \cap C'$, inside A and B but outside C), three "only" regions



(for example $A \cap B' \cap C'$, A only), and the outside, $(A \cup B \cup C)'$. Fill in numbers from the centre outwards: the number given for "A and B" includes the centre, so the "A and B only" region is $n(A \cap B)$ minus $n(A \cap B \cap C)$.

Worked example (two sets). In a class of 30 students, 18 play hockey (H) and 14 play tennis (T). 5 play neither. Find how many play both, and write the number in each region.

At least one sport: $n(H \cup T) = 30 - 5 = 25$.

Use $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, so $n(H \cap T) = 18 + 14 - 25 = 7$.

Regions: hockey only $18 - 7 = 11$, both 7, tennis only $14 - 7 = 7$, neither 5. Check: $11 + 7 + 7 + 5 = 30$.

The formula works because adding $n(H)$ and $n(T)$ counts the overlap twice.

Exam tip. Never write a set's total inside its circle. Fill the overlap first, subtract to get the "only" regions, then check that the regions add up to $n(\mathcal{E})$. Call an unknown region x and form an equation from $n(\mathcal{E})$.

Worksheet

Marks are in square brackets; total 92. Venn diagrams are described in words, so sketch each one yourself.

1. $\mathcal{E} = \{\text{integers from 1 to 12}\}$, $A = \{\text{factors of 12}\}$, $B = \{\text{multiples of 3}\}$.

- List the elements of A.
- List the elements of B.
- List $A \cap B$.
- Find $n(A \cup B)$. [4]

2. $\mathcal{E} = \{x : x \text{ is an integer and } 1 \leq x \leq 10\}$, $P = \{\text{prime numbers}\}$, $Q = \{\text{odd numbers}\}$.

- List P' .
- List $P \cap Q$.
- Is $P \subseteq Q$? Give a reason.
- Find $n(P \cup Q)$. [5]

3. State whether each statement is true or false, giving a reason.

- $7 \in \{\text{multiples of 7}\}$
- $\{2, 4\} \subseteq \{\text{even numbers}\}$
- $\emptyset \subseteq \{1, 2\}$
- $\{1, 2, 3\} \cap \{4, 5\} = \emptyset$ [4]

4. A Venn diagram shows sets A and B inside $\mathcal{E}$. Write in set notation the region that is

- inside both circles,
- inside A but outside B,
- outside both circles,
- everywhere except the overlap. [4]

5. $\mathcal{E} = \{\text{letters of the word MATHEMATICS}\}$, $V = \{\text{vowels in } \mathcal{E}\}$, $T = \{\text{letters of the word TEAM}\}$.

- List the elements of $\mathcal{E}$.

(b) Write down $n(\mathcal{E})$.

(c) List $V \cap T$.

(d) List T' . [4]

6. In a class of 32 students, 20 study Physics (P), 15 study Chemistry (C) and 9 study both.

(a) Write down the number in each of the four regions of the Venn diagram.

(b) How many study neither subject?

(c) How many study exactly one subject? [4]

7. $n(\mathcal{E}) = 50$, $n(A) = 28$, $n(B) = 23$ and $n((A \cup B)') = 7$.

(a) Find $n(A \cup B)$.

(b) Find $n(A \cap B)$.

(c) Find $n(A' \cap B)$. [4]

8. A Venn diagram shows sets A, B and C inside $\mathcal{E}$. Write in set notation the region that is

(a) inside all three circles,

(b) inside A and B but outside C,

(c) inside C only,

(d) inside at least one circle.

(e) Describe in words the region $(A \cup B) \cap C'$. [5]

9. $\mathcal{E} = \{x : x \text{ is an integer and } 1 \leq x \leq 20\}$, $A = \{\text{multiples of 4}\}$, $B = \{\text{factors of 20}\}$.

(a) List $A' \cap B$.

(b) List $(A \cup B)'$.

(c) Find $n(A' \cup B')$. [5]

10. In a two-set Venn diagram, $n(A) = 3x$, $n(B) = 2x + 4$ and $n(A \cap B) = x$. Also $n(\mathcal{E}) = 40$ and $n((A \cup B)') = 4$.

(a) Form an equation in x and solve it.

(b) Find the number of elements in A only.

(c) Find $n(B)$. [4]

11. 60 people are asked whether they drink tea (T) or coffee (C). 12 drink neither, 18 drink both, and the number who drink only tea is twice the number who drink only coffee.

(a) Find the number who drink only tea.

(b) Find $n(T)$.

(c) Find $n(C \cap T')$. [4]

12. $\mathcal{E} = \{\text{integers from 1 to 12}\}$, $A = \{\text{even numbers}\}$, $B = \{\text{multiples of 3}\}$, $C = \{\text{factors of 12}\}$.

(a) List $A \cap B \cap C$.

(b) List $A \cap C \cap B'$.

(c) List $(A \cup B \cup C)'$.

(d) Find $n(A \cup B \cup C)$. [4]

13. 40 students were asked which sports they like. 22 like football (F), 18 like cricket (C) and 15 like hockey (H). 8 like football and cricket, 7 like cricket and hockey, 6 like football and hockey, and 3 like all three.

(a) Write down the number in each of the eight regions of the Venn diagram.

(b) How many like none of the three sports?

(c) How many like exactly two of the sports? [5]

14. In a college of 50 students, 26 study French (F), 21 study Spanish (S) and 17 study German (G).

12 study French and Spanish, 9 study Spanish and German, and 10 study French and German. 12 students study none of these languages.

- Let x be the number who study all three. Form an equation in x and solve it.
- Write down the number in each region of the Venn diagram.
- How many students study exactly one language? [6]

15. A three-set Venn diagram has these region counts: $A \cap B \cap C = 4$, $A \cap B \cap C' = x$, $A \cap C \cap B' = 6$, $B \cap C \cap A' = 5$, A only $= 2x$, B only $= 9$, C only $= 7$, outside all three $= 3$. Also $n(\mathcal{C}) = 52$.

- Find x .
- Find $n(A)$.
- Find $n((B \cup C)')$. [4]

16. In a school of 80 students, 45 own a bicycle (B), 30 own a skateboard (S) and 12 own both. A student is chosen at random.

- Find the probability that the student owns neither.
- Find the probability that the student owns exactly one of the two.
- A student who owns a bicycle is chosen at random. Find the probability that this student also owns a skateboard. [5]

17. 60 people were asked which music they listen to. 32 listen to pop (P), 24 to rock (R) and 14 to jazz (J). 12 listen to pop and rock, 6 to rock and jazz, 5 to pop and jazz, and 3 listen to all three.

- Write down the number in each region of the Venn diagram.
- One person is chosen at random. Find the probability that they listen to exactly one kind of music.
- Find the probability that a person chosen at random listens to jazz but not pop.
- Two different people are chosen at random. Find the probability that neither listens to any of the three kinds. [6]

18. $n(\mathcal{C}) = 20$, $n(A) = 15$ and $n(B) = 11$.

- Write down the greatest possible value of $n(A \cap B)$.
- Find the least possible value of $n(A \cap B)$.
- Find the greatest possible value of $n((A \cup B)')$. [4]

19. 80 shoppers were asked whether they bought bread (B), milk (M) or eggs (E). The Venn diagram regions are: all three 5, bread and milk only x , milk and eggs only 6, bread and eggs only y , bread only 20, milk only 14, eggs only 9, none 7. Also $n(M) = 36$.

- Find x and y .
- Find $n(E)$.
- Find $n(B \cap M')$.
- A shopper is chosen at random. Find the probability that they bought exactly one of the three items. [6]

20. $n(\mathcal{C}) = 60$, $n(A)$ is twice $n(B)$, $n(A \cap B) = 6$ and $n((A \cup B)') = 12$.

- Find $n(A)$ and $n(B)$.
- An element of A is chosen at random. Find the probability that it is also in B .
- An element of $\mathcal{C}$ is chosen at random. Find the probability that it is in exactly one of the two sets. [5]

Answers

1. (a) $A = \{1, 2, 3, 4, 6, 12\}$. (b) $B = \{3, 6, 9, 12\}$. (c) $A \cap B = \{3, 6, 12\}$. (d) $A \cup B = \{1, 2, 3, 4, 6, 9, 12\}$, so $n(A \cup B) = 7$. Check: $6 + 4 - 3 = 7$.

2. $P = \{2, 3, 5, 7\}$, $Q = \{1, 3, 5, 7, 9\}$. (a) $P' = \{1, 4, 6, 8, 9, 10\}$. (b) $P \cap Q = \{3, 5, 7\}$. (c) No: $2 \in P$ but $2 \notin Q$. (d) $P \cup Q = \{1, 2, 3, 5, 7, 9\}$, so $n(P \cup Q) = 6$. Check: $4 + 5 - 3 = 6$.
3. (a) True, $7 = 7 \times 1$. (b) True, 2 and 4 are both even. (c) True, the empty set is a subset of every set. (d) False, the sets share no elements so the intersection is the empty set $\emptyset$, not the number 0.
4. (a) $A \cap B$. (b) $A \cap B'$. (c) $(A \cup B)'$ or $A' \cap B'$. (d) $(A \cap B)'$ or $A' \cup B'$.
5. (a) $\mathcal{E} = \{M, A, T, H, E, I, C, S\}$, each letter once. (b) $n(\mathcal{E}) = 8$. (c) $V = \{A, E, I\}$ and $T = \{T, E, A, M\}$, so $V \cap T = \{A, E\}$. (d) $T' = \{H, I, C, S\}$.
6. (a) Both 9; Physics only $20 - 9 = 11$; Chemistry only $15 - 9 = 6$; neither $32 - (11 + 9 + 6) = 6$. (b) 6. (c) $11 + 6 = 17$.
7. (a) $n(A \cup B) = 50 - 7 = 43$. (b) $n(A \cap B) = 28 + 23 - 43 = 8$. (c) $n(A' \cap B)$ is B only: $23 - 8 = 15$.
8. (a) $A \cap B \cap C$. (b) $A \cap B \cap C'$. (c) $C \cap A' \cap B'$, also written $C \cap (A \cup B)'$. (d) $A \cup B \cup C$. (e) Inside A or B (or both) but outside C.
9. $A = \{4, 8, 12, 16, 20\}$, $B = \{1, 2, 4, 5, 10, 20\}$, $A \cap B = \{4, 20\}$. (a) $A' \cap B$ is B without A: $\{1, 2, 5, 10\}$. (b) $A \cup B = \{1, 2, 4, 5, 8, 10, 12, 16, 20\}$, 9 elements, so $(A \cup B)' = \{3, 6, 7, 9, 11, 13, 14, 15, 17, 18, 19\}$. (c) $A' \cup B'$ is the same region as $(A \cap B)'$, so $n = 20 - 2 = 18$.
10. (a) $n(A \cup B) = 40 - 4 = 36$ and $n(A \cup B) = 3x + (2x + 4) - x = 4x + 4$. So $4x + 4 = 36$, $x = 8$. (b) A only $= 3x - x = 24 - 8 = 16$. (c) $n(B) = 2 \times 8 + 4 = 20$. Check: $16 + 8 + 12 + 4 = 40$.
11. Only tea + only coffee $= 60 - 12 - 18 = 30$. Let only coffee $= y$, so only tea $= 2y$ and $3y = 30$, $y = 10$. (a) 20. (b) $n(T) = 20 + 18 = 38$. (c) Coffee only: 10.
12. $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{3, 6, 9, 12\}$, $C = \{1, 2, 3, 4, 6, 12\}$. (a) $A \cap B = \{6, 12\}$ and both are in C, so $A \cap B \cap C = \{6, 12\}$. (b) $A \cap C = \{2, 4, 6, 12\}$; remove multiples of 3: $\{2, 4\}$. (c) $A \cup B \cup C = \{1, 2, 3, 4, 6, 8, 9, 10, 12\}$, so $(A \cup B \cup C)' = \{5, 7, 11\}$. (d) $12 - 3 = 9$.
13. (a) Centre 3. Football and cricket only $8 - 3 = 5$; cricket and hockey only $7 - 3 = 4$; football and hockey only $6 - 3 = 3$. Football only $22 - 5 - 3 - 3 = 11$; cricket only $18 - 5 - 4 - 3 = 6$; hockey only $15 - 4 - 3 - 3 = 5$. Inside the circles $3 + 5 + 4 + 3 + 11 + 6 + 5 = 37$, so none $= 40 - 37 = 3$. (b) 3. (c) $5 + 4 + 3 = 12$.
14. (a) At least one language: $50 - 12 = 38$. Adding the three totals counts each "two language" overlap twice and the centre three times, so $n(F \cup S \cup G) = 26 + 21 + 17 - 12 - 9 - 10 + x = 33 + x$. So $33 + x = 38$, $x = 5$. (b) Centre 5; French and Spanish only $12 - 5 = 7$; Spanish and German only $9 - 5 = 4$; French and German only $10 - 5 = 5$; French only $26 - 7 - 5 - 5 = 9$; Spanish only $21 - 7 - 4 - 5 = 5$; German only $17 - 4 - 5 - 5 = 3$; none 12. Check: $5 + 7 + 4 + 5 + 9 + 5 + 3 + 12 = 50$. (c) $9 + 5 + 3 = 17$.
15. (a) $4 + x + 6 + 5 + 2x + 9 + 7 + 3 = 34 + 3x = 52$, so $x = 6$. (b) $n(A) = 4 + 6 + 6 + 12 = 28$. (c) $(B \cup C)'$ is A only plus the outside: $12 + 3 = 15$.
16. Both 12; bicycle only $45 - 12 = 33$; skateboard only $30 - 12 = 18$; neither $80 - (33 + 12 + 18) = 17$. (a) $17/80$. (b) $(33 + 18)/80 = 51/80$. (c) 12 of the 45 bicycle owners own a skateboard: $12/45 = 4/15$.
17. (a) Centre 3; pop and rock only $12 - 3 = 9$; rock and jazz only $6 - 3 = 3$; pop and jazz only $5 - 3 = 2$; pop only $32 - 9 - 2 - 3 = 18$; rock only $24 - 9 - 3 - 3 = 9$; jazz only $14 - 3 - 2 - 3 = 6$; inside the

circles 50, so none = 10. (b) Exactly one: $18 + 9 + 6 = 33$, so $33/60 = 11/20$. (c) Jazz but not pop: $3 + 6 = 9$, so $9/60 = 3/20$. (d) $10/60 \times 9/59 = 90/3540 = 3/118$.

18. (a) 11, when $B \subseteq A$. (b) $n(A \cup B)$ cannot exceed 20, so $n(A \cap B) = 15 + 11 - n(A \cup B) \geq 26 - 20 = 6$. (c) $n((A \cup B)') = 20 - n(A \cup B)$, and $n(A \cup B)$ is smallest, 15, when $B \subseteq A$, giving $20 - 15 = 5$.

19. (a) $n(M) = 5 + x + 6 + 14 = 25 + x = 36$, so $x = 11$. All regions add to 80: $5 + 11 + 6 + y + 20 + 14 + 9 + 7 = 72 + y = 80$, so $y = 8$. (b) $n(E) = 5 + 6 + 8 + 9 = 28$. (c) Bread without milk: bread and eggs only 8, plus bread only 20, so 28. (d) Exactly one: $20 + 14 + 9 = 43$, so $43/80$.

20. (a) Let $n(B) = b$, so $n(A) = 2b$. $n(A \cup B) = 60 - 12 = 48 = 2b + b - 6$, so $3b = 54$, $b = 18$. $n(B) = 18$ and $n(A) = 36$. (b) $6/36 = 1/6$. (c) A only = 30, B only = 12, so exactly one = 42 and the probability is $42/60 = 7/10$. Check: $30 + 6 + 12 + 12 = 60$.