

SAT Math: domain and range of radical functions (practice with answers)

Radical functions appear in the Advanced Math domain of the digital SAT. You are asked for the domain (allowed x-values), the range (possible outputs) or the equation that matches a described graph. Below are the rules, one worked example and 20 graded practice questions with worked answers.

What you need to know

The two parent functions

Function	Domain	Range	Shape of graph
$y = \sqrt{x}$ (square root)	$x \geq 0$	$y \geq 0$	Starts at (0, 0) and rises to the right, getting flatter. Passes through (1, 1), (4, 2), (9, 3).
$y = \sqrt[3]{x}$ (cube root)	all real numbers	all real numbers	An S-shaped curve through (-8, -2), (-1, -1), (0, 0), (1, 1), (8, 2). No starting point.

A negative number has no real square root, but it does have a cube root: $\sqrt[3]{-8} = -2$.

Rule 1 (domain of a square root): the expression under the square root sign (the radicand) must be greater than or equal to zero. Write radicand ≥ 0 and solve for x.

Rule 2 (square root in a denominator): if the square root is on the bottom of a fraction, the radicand must be strictly greater than zero, because you cannot divide by zero.

Rule 3 (cube root): a cube root on its own never restricts the domain or the range.

Transformations of $y = \sqrt{x}$

Most SAT radical functions can be written in the form

$f(x) = a\sqrt{x - h} + k$

- h moves the graph right by h (left if h is negative). The radicand is zero when $x = h$, so the domain is $x \geq h$.
- k moves the graph up by k (down if k is negative). The graph starts at the point (h, k), called the endpoint.
- a stretches the graph. If a is positive the curve rises from the endpoint, so the range is $y \geq k$. If a is negative the curve is reflected in a horizontal line and falls from the endpoint, so the range is $y \leq k$.
- A minus sign inside, as in $\sqrt{h - x}$, reflects the graph left to right. Now the radicand is zero at $x = h$ and positive for $x < h$, so the domain is $x \leq h$ and the curve runs to the left of the endpoint.

Quick method for $f(x) = a\sqrt{x - h} + k$

Domain: $x \geq h$. Range: $y \geq k$ if $a > 0$, $y \leq k$ if $a < 0$. The endpoint (h, k) is the minimum point when a



> 0 and the maximum point when $a < 0$.

Worked example

Find the domain and range of $f(x) = -2\sqrt{x-3} + 5$.

Domain: the radicand must not be negative, so $x - 3 \geq 0$, giving $x \geq 3$.

Range: $\sqrt{x-3}$ takes every value from 0 upwards. Multiplying by -2 gives every value from 0 downwards. Adding 5 gives every value from 5 downwards. So the range is $y \leq 5$.

Check: $f(3) = 5$, $f(7) = 1$, $f(12) = -1$. The graph starts at the endpoint $(3, 5)$ and falls to the right.

Things that trip students up

- When you solve the inequality, remember to reverse the sign if you divide by a negative number. For $\sqrt{6-2x}$: $6-2x \geq 0$ gives $-2x \geq -6$, so $x \leq 3$.
- If the radicand is a quadratic, such as $x^2 - 16$, the domain can split into two pieces. Solve $x^2 \geq 16$ to get $x \leq -4$ or $x \geq 4$.
- If a function has two square roots added together, both radicands must be non-negative at the same time, so the domain is the overlap of the two conditions.
- On graph questions, find the endpoint first. Its coordinates give h and k directly. Then use one other point on the curve to find a .

Questions

Questions 1 to 7 are warm-up, 8 to 14 medium, 15 to 20 hard. Grid-in questions need a single number.

- What is the domain of $f(x) = \sqrt{x-4}$?
A) $x \geq -4$; B) $x \geq 0$; C) $x \geq 4$; D) all real numbers
- What is the range of $g(x) = \sqrt{x} + 6$?
A) $y \geq 0$; B) $y \geq 6$; C) $y \geq -6$; D) $y \leq 6$
- What is the domain of $h(x) = \sqrt[3]{x-2} + 1$?
A) $x \geq 2$; B) $x \geq 1$; C) $x \geq -1$; D) all real numbers
- Find the smallest value of x in the domain of $f(x) = \sqrt{2x+10}$. (Grid-in)
- What is the range of $f(x) = -\sqrt{x} + 3$?
A) $y \geq 3$; B) $y \leq 3$; C) $y \geq -3$; D) $y \leq 0$
- The function f is defined by $f(x) = \sqrt{9-x}$.
(a) State the domain of f .
(b) State the range of f .
(c) Describe how the graph of f is related to the graph of $y = \sqrt{x}$.
- Which of the following is NOT in the domain of $f(x) = \sqrt{3x-12}$?
A) 3; B) 4; C) 5; D) 10
- The graph of $y = \sqrt{x}$ is translated 2 units to the left and 7 units down to give the graph of $y = g(x)$.
(a) Write an equation for $g(x)$.
(b) State the coordinates of the endpoint of the graph.
(c) State the range of g .

9. The function f is defined by $f(x) = 3\sqrt{x+1} - 4$. What is the minimum value of $f(x)$? (Grid-in)
10. What is the domain of $f(x) = 1/\sqrt{x-5}$?
A) $x \geq 5$; B) $x > 5$; C) $x \neq 5$; D) $x \leq 5$
11. Which of the following describes the domain of $f(x) = \sqrt{6-2x}$?
A) $x \geq 3$; B) $x \leq 3$; C) $x \geq -3$; D) $x \leq -3$
12. The graph of $y = f(x)$ starts at the point $(4, -1)$ and rises to the right, passing through the point $(8, 1)$. The equation of the graph has the form $f(x) = a\sqrt{x-h} + k$.
(a) Find the values of a , h and k .
(b) State the domain and range of f .
13. The graph of a function in the xy -plane starts at the point $(-3, 5)$ and falls to the right. Which of the following could be the equation of the graph?
A) $y = \sqrt{x+3} + 5$; B) $y = -\sqrt{x+3} + 5$; C) $y = -\sqrt{x-3} + 5$; D) $y = \sqrt{x-3} - 5$
14. The function g is defined by $g(x) = -2\sqrt{4-x} + 10$.
(a) State the domain of g .
(b) State the range of g .
(c) Calculate $g(0)$.
15. What is the domain of $f(x) = \sqrt{x^2 - 16}$?
A) $x \geq 4$; B) $-4 \leq x \leq 4$; C) $x \leq -4$ or $x \geq 4$; D) all real numbers
16. The function f is defined by $f(x) = \sqrt[3]{x+8} - 2$.
(a) State the range of f .
(b) Calculate $f(0)$.
(c) Find the value of x for which $f(x) = 1$.
17. How many integers are in the domain of $f(x) = \sqrt{x+3} + \sqrt{7-x}$? (Grid-in)
18. The function f is defined by $f(x) = \sqrt{25-x^2}$.
(a) Find the domain of f .
(b) Find the range of f .
(c) Describe the shape of the graph.
19. The function $f(x) = \sqrt{x-c}$, where c is a constant, has domain $x \geq -6$. What is the value of $f(10)$? (Grid-in)
20. The function $f(x) = a\sqrt{x-h} + k$, where a , h and k are constants, has range $y \geq -3$. Its graph passes through the points $(2, -3)$ and $(11, 3)$. What is the value of $a + h + k$? (Grid-in)

Answers

1. **C.** The radicand must be non-negative: $x - 4 \geq 0$, so $x \geq 4$.
2. **B.** $\sqrt{x} \geq 0$ for every x in its domain, so $\sqrt{x} + 6 \geq 6$. The range is $y \geq 6$.
3. **D.** A cube root can take any real input, including negatives, so there is no restriction. The domain is all real numbers.
4. **-5.** $2x + 10 \geq 0$ gives $2x \geq -10$, so $x \geq -5$. The smallest value in the domain is -5 .
5. **B.** $\sqrt{x} \geq 0$, so $-\sqrt{x} \leq 0$, so $-\sqrt{x} + 3 \leq 3$. The range is $y \leq 3$.
6. (a) $9 - x \geq 0$ gives $x \leq 9$. Domain: $x \leq 9$.

(b) A square root on its own is never negative, and it takes the value 0 at $x = 9$ and grows without limit as x decreases. Range: $y \geq 0$.

(c) $\sqrt{9 - x} = \sqrt{-(x - 9)}$: the graph of $y = \sqrt{x}$ is reflected in the y -axis, then translated 9 units right. It starts at $(9, 0)$ and rises to the left.

7. A. We need $3x - 12 \geq 0$, so $x \geq 4$. Checking $x = 3$: $3(3) - 12 = -3$, which is negative, so 3 is not in the domain.

8. (a) A shift of 2 to the left replaces x with $x + 2$, and a shift of 7 down subtracts 7: $g(x) = \sqrt{x + 2} - 7$.

(b) The endpoint of $y = \sqrt{x}$ is $(0, 0)$. Moving it 2 left and 7 down gives $(-2, -7)$.

(c) The graph rises from the endpoint, so the range is $y \geq -7$.

9. -4. $\sqrt{x + 1} \geq 0$, so $3\sqrt{x + 1} \geq 0$ and $3\sqrt{x + 1} - 4 \geq -4$. The minimum value -4 occurs at $x = -1$, where the radicand is zero.

10. B. The radicand must be non-negative, but $x = 5$ gives $\sqrt{0} = 0$ in the denominator, which is undefined. So $x - 5 > 0$, giving $x > 5$. The inequality is strict.

11. B. $6 - 2x \geq 0$ gives $-2x \geq -6$. Dividing both sides by -2 reverses the inequality: $x \leq 3$.

12. (a) The endpoint is (h, k) , so $h = 4$ and $k = -1$. Substitute the point $(8, 1)$: $1 = a\sqrt{(8 - 4) - 1}$, so $2 = a \times 2$ and $a = 1$. The function is $f(x) = \sqrt{(x - 4) - 1}$.

(b) Domain: $x \geq 4$. Since $a = 1$ is positive the graph rises from the endpoint, so the range is $y \geq -1$.

13. B. The endpoint $(-3, 5)$ means $h = -3$ and $k = 5$, so the radicand is $x - (-3) = x + 3$ and the constant added is $+5$. Falling to the right needs a negative coefficient in front of the root, so B. Option A starts at the same point but rises.

14. (a) $4 - x \geq 0$ gives $x \leq 4$. Domain: $x \leq 4$.

(b) $\sqrt{4 - x} \geq 0$, so $-2\sqrt{4 - x} \leq 0$, so $g(x) \leq 10$. Range: $y \leq 10$.

(c) $g(0) = -2\sqrt{4} + 10 = -2 \times 2 + 10 = 6$.

15. C. We need $x^2 - 16 \geq 0$, so $x^2 \geq 16$. This is true when $x \geq 4$ or when $x \leq -4$. Values between -4 and 4 , such as $x = 0$, give a negative radicand.

16. (a) A cube root can output any real number, and subtracting 2 does not change that. Range: all real numbers.

(b) $f(0) = \sqrt[3]{8} - 2 = 2 - 2 = 0$.

(c) $\sqrt[3]{x + 8} - 2 = 1$ gives $\sqrt[3]{x + 8} = 3$. Cube both sides: $x + 8 = 27$, so $x = 19$.

17. 11. Both radicands must be non-negative at the same time. $x + 3 \geq 0$ gives $x \geq -3$, and $7 - x \geq 0$ gives $x \leq 7$. The overlap is $-3 \leq x \leq 7$. The integers are $-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7$, which is $7 - (-3) + 1 = 11$ integers.

18. (a) $25 - x^2 \geq 0$ gives $x^2 \leq 25$, so $-5 \leq x \leq 5$.

(b) The largest value of $25 - x^2$ is 25, at $x = 0$, giving $\sqrt{25} = 5$. The smallest value of the radicand in the domain is 0, at $x = \pm 5$, giving $\sqrt{0} = 0$. Range: $0 \leq y \leq 5$.

(c) If $y = \sqrt{25 - x^2}$ then $y^2 = 25 - x^2$, so $x^2 + y^2 = 25$ with $y \geq 0$. The graph is the upper half of the circle with centre $(0, 0)$ and radius 5.

19. 4. The domain of $\sqrt{x - c}$ is $x \geq c$, and we are told it is $x \geq -6$, so $c = -6$. Then $f(x) = \sqrt{x + 6}$ and $f(10) = \sqrt{16} = 4$.

20. 1. The range $y \geq -3$ tells us that the graph rises from an endpoint with y -coordinate -3 , so $k =$

-3 and a is positive. The point $(2, -3)$ lies on the graph and has the minimum y -value, so it is the endpoint, giving $h = 2$. Substitute $(11, 3)$: $3 = a\sqrt{(11 - 2) - 3}$, so $6 = a \times 3$ and $a = 2$. Then $a + h + k = 2 + 2 + (-3) = 1$. Check: $f(x) = 2\sqrt{(x - 2) - 3}$ gives $f(11) = 2 \times 3 - 3 = 3$.