

Ratio and scales: IGCSE Maths worksheet with answers

What you need to know

A ratio compares two or more quantities of the same kind. The ratio 3 : 5 means that for every 3 parts of the first quantity there are 5 parts of the second. A ratio has no units, so both quantities must be written in the same unit before you compare them.

Simplifying ratios

Divide every part of the ratio by the highest common factor (HCF). If the ratio contains decimals or fractions, first multiply every part by the same number to make them whole numbers, then simplify.

- 18 : 24: the HCF is 6, so the ratio simplifies to 3 : 4.
- 0.4 : 1.2: multiply by 10 to get 4 : 12, then divide by 4 to get 1 : 3.
- $\frac{3}{4}$: $\frac{1}{2}$: multiply both parts by 4 (the lowest common denominator) to get 3 : 2.
- 45 cm : 2 m: change to the same unit, 45 : 200, then divide by 5 to get 9 : 40.

The forms 1 : n and n : 1

To write a ratio in the form 1 : n, divide both parts by the first part. For example, 4 : 10 = 1 : 2.5. To write it in the form n : 1, divide both parts by the second part, so 4 : 10 = 0.4 : 1. In these forms n may be a decimal.

Ratios and fractions

If red and blue beads are in the ratio 2 : 5, there are $2 + 5 = 7$ parts in total. The fraction of beads that are red is $\frac{2}{7}$ and the fraction that are blue is $\frac{5}{7}$. The red beads are $\frac{2}{5}$ of the number of blue beads. Read the question carefully: "a fraction of the total" and "a fraction of the other quantity" are different.

Sharing in a given ratio

Method for sharing an amount in a ratio

1. Add the parts of the ratio to find the total number of parts.
2. Divide the amount by the total number of parts to find the value of one part.
3. Multiply the value of one part by each number in the ratio.
4. Check that your answers add up to the original amount.

If the question tells you the size of one share, or the difference between two shares, do not add the parts. Instead, match the information to the correct number of parts. For example, if $A : B = 5 : 2$ and A receives \$30 more than B, then $5 - 2 = 3$ parts are worth \$30, so one part is worth \$10.

Recipe scaling and best buys

To scale a recipe, find the multiplier: new amount $\div$ original amount. Multiply every ingredient by this same number. If an ingredient limits how much you can make, work out how many times the recipe



that ingredient allows.

To find the best buy, compare like with like. Either find the cost per unit (for example cents per gram) and choose the lowest, or find the amount per unit of money (grams per dollar) and choose the highest. Always state which option is better value and show the figures you compared.

Map scales

A map scale of 1 : n means that 1 unit on the map represents n of the same unit on the ground. On a 1 : 50 000 map, 1 cm represents 50 000 cm, which is 500 m or 0.5 km.

Unit facts to learn

1 m = 100 cm, 1 km = 1000 m = 100 000 cm.

1 m² = 100² = 10 000 cm², 1 km² = 1000² = 1 000 000 m² = 10¹⁰ cm².

Length: real length = map length × n. **Area:** real area = map area × n².

Because area is two-dimensional, the area scale factor is the square of the length scale factor. A map with scale 1 : 10 000 has an area scale of 1 : 100 000 000 (that is, 1 : 10⁸). The most common mistake in the exam is to multiply an area by n instead of n².

Worked example

A map has a scale of 1 : 10 000. A park has an area of 5 cm² on the map. Calculate the real area of the park in m².

Area scale factor = 10 000² = 10⁸.

Real area = 5 × 10⁸ cm².

Convert to m² by dividing by 10 000: 5 × 10⁸ ÷ 10⁴ = 5 × 10⁴ m² = 50 000 m².

Check: 1 cm on the map is 100 m on the ground, so 1 cm² is 100 m × 100 m = 10 000 m², and 5 cm² is 50 000 m².

Exam tip

Write the number of parts above each quantity before you start a ratio problem. For map questions, convert to centimetres first, use the scale, and only convert to metres or kilometres at the end.

Questions

Show all your working. Give answers that are not exact to 3 significant figures.

1. Write the ratio 24 : 36 in its simplest form.
2. Write the ratio 1.5 kg : 600 g in its simplest form.
3. Write the ratio 2/3 : 5/6 in its simplest form, using whole numbers.
4. Share \$84 in the ratio 3 : 4.
5. Write the ratio 5 : 7 in the form 1 : n.

6. In a class the ratio of boys to girls is 4 : 5. There are 35 girls.
 - (a) Calculate the number of boys.
 - (b) Write the ratio of boys to the total number of students in its simplest form.
7. A bag contains red and blue counters in the ratio 2 : 7.
 - (a) Write down the fraction of the counters that are red.
 - (b) There are 54 counters in the bag. Calculate the number of blue counters.
8. A recipe for 12 biscuits uses 180 g of flour, 90 g of butter and 60 g of sugar.
 - (a) Calculate the amount of each ingredient needed for 30 biscuits.
 - (b) Imran has 300 g of butter and plenty of the other ingredients. Calculate the greatest number of biscuits he can make.
9. A map has a scale of 1 : 50 000. The distance between two villages on the map is 7.4 cm. Calculate the real distance between the villages in kilometres.
10. A map has a scale of 1 : 25 000. A road is 4.5 km long. Calculate the length of the road on the map in centimetres.
11. Rice is sold in two sizes: a 400 g bag for \$2.60 and a 650 g bag for \$4.10. Showing your working, find which bag is the better value.
12. Three friends share \$360 in the ratio 2 : 3 : 7. Calculate the largest share.
13. Ayesha and Bilal share some money in the ratio 5 : 3. Ayesha receives \$48 more than Bilal. Calculate the total amount of money they share.
14. On a map, 3 cm represents a real distance of 1.5 km. Write the scale of the map in the form 1 : n.
15. Batteries are sold in three packs: Pack A has 6 batteries for \$4.20, Pack B has 10 batteries for \$6.80, and Pack C has 15 batteries for \$10.50. Which pack gives the best value? Show your working.
16. $x : y = 3 : 4$ and $y : z = 6 : 5$. Find the ratio $x : y : z$ in its simplest form, using whole numbers.
17. A map has a scale of 1 : 20 000. A forest has an area of 6 cm^2 on the map. Calculate the real area of the forest in square kilometres.
18. A lake has a real area of 0.72 km^2 . Calculate its area, in cm^2 , on a map of scale 1 : 30 000.
19. A box contains red and green pens in the ratio 5 : 3. Eight red pens are taken out. The ratio of red pens to green pens is now 1 : 1. Calculate the number of pens that were in the box at the start.
20. The ratio of Ali's age to Hamza's age is 3 : 5. In 6 years' time the ratio of their ages will be 5 : 7. Find their ages now.

Answers

1. The HCF of 24 and 36 is 12. $24 \div 12 = 2$ and $36 \div 12 = 3$, so $24 : 36 = 2 : 3$.
2. Change to the same unit: $1.5 \text{ kg} = 1500 \text{ g}$. The ratio is $1500 : 600$. The HCF is 300, so $1500 : 600 = 5 : 2$.
3. The lowest common denominator of 3 and 6 is 6. Multiply both parts by 6: $2/3 \times 6 = 4$ and $5/6 \times 6 = 5$. The ratio is $4 : 5$, which cannot be simplified further.

4. Total parts = $3 + 4 = 7$. One part = $84 \div 7 = \$12$. Shares: $3 \times 12 = \$36$ and $4 \times 12 = \$48$.
Check: $36 + 48 = 84$.

5. Divide both parts by 5: $5 \div 5 = 1$ and $7 \div 5 = 1.4$. So $5 : 7 = 1 : 1.4$.

6. (a) Girls are 5 parts, so 5 parts = 35 and one part = 7. Boys = $4 \times 7 = 28$.

(b) Total students = $28 + 35 = 63$. Boys : total = $28 : 63$. Divide by 7: $4 : 9$.

7. (a) Total parts = $2 + 7 = 9$, so the fraction that are red = $\frac{2}{9}$.

(b) One part = $54 \div 9 = 6$. Blue counters = $7 \times 6 = 42$. Check: red = 12, and $12 + 42 = 54$.

8. (a) Multiplier = $30 \div 12 = 2.5$.

Flour = $180 \times 2.5 = 450$ g, butter = $90 \times 2.5 = 225$ g, sugar = $60 \times 2.5 = 150$ g.

(b) Butter is the limiting ingredient. $300 \div 90 = 10/3$, so he can make $10/3$ times the recipe.

Number of biscuits = $12 \times 10/3 = 40$.

9. Real distance = $7.4 \times 50\,000 = 370\,000$ cm.

$370\,000$ cm $\div 100 = 3700$ m, and 3700 m $\div 1000 = 3.7$ km.

10. Convert to centimetres: 4.5 km = 4500 m = $450\,000$ cm.

Map length = $450\,000 \div 25\,000 = 18$ cm.

11. Cost per gram:

400 g bag: 260 cents $\div 400 = 0.65$ cents per gram.

650 g bag: 410 cents $\div 650 = 0.631$ cents per gram (3 s.f.).

The **650 g bag** is better value because each gram costs less.

(Alternative: grams per dollar are $400 \div 2.60 = 154$ g and $650 \div 4.10 = 159$ g, so the 650 g bag gives more rice for each dollar.)

12. Total parts = $2 + 3 + 7 = 12$. One part = $360 \div 12 = \$30$.

Shares are $2 \times 30 = \$60$, $3 \times 30 = \$90$ and $7 \times 30 = \$210$. The largest share is **\$210**. Check: $60 + 90 + 210 = 360$.

13. The difference between the shares is $5 - 3 = 2$ parts. 2 parts = $\$48$, so one part = $\$24$.

Total parts = $5 + 3 = 8$. Total money = $8 \times 24 = \$192$.

Check: Ayesha gets $5 \times 24 = \$120$, Bilal gets $3 \times 24 = \$72$, and $120 - 72 = 48$.

14. Convert the real distance to centimetres: 1.5 km = 1500 m = $150\,000$ cm.

So 3 cm represents $150\,000$ cm. Divide both by 3: 1 cm represents $50\,000$ cm.

Scale = **1 : 50 000**.

15. Cost per battery:

Pack A: $4.20 \div 6 = \$0.70$.

Pack B: $6.80 \div 10 = \$0.68$.

Pack C: $10.50 \div 15 = \$0.70$.

Pack B gives the best value because each battery costs the least.

16. y appears in both ratios as 4 and 6. Make y the same in both using the LCM of 4 and 6, which is 12.

$x : y = 3 : 4 = 9 : 12$ (multiply by 3).

$y : z = 6 : 5 = 12 : 10$ (multiply by 2).

So $x : y : z = 9 : 12 : 10$. These have no common factor, so this is the simplest form.

17. Area scale factor = $20\,000^2 = 4 \times 10^8$.

Real area = $6 \times 4 \times 10^8 = 2.4 \times 10^9 \text{ cm}^2$.

$1 \text{ km}^2 = 10^{10} \text{ cm}^2$, so real area = $2.4 \times 10^9 \div 10^{10} = \mathbf{0.24 \text{ km}^2}$.

Check: 1 cm on the map is 20 000 cm = 0.2 km, so 1 cm^2 represents $0.2 \times 0.2 = 0.04 \text{ km}^2$. Then $6 \times 0.04 = 0.24 \text{ km}^2$.

18. Convert the real area to cm^2 : $0.72 \text{ km}^2 = 0.72 \times 10^{10} = 7.2 \times 10^9 \text{ cm}^2$.

Area scale factor = $30\,000^2 = 9 \times 10^8$.

Map area = $7.2 \times 10^9 \div (9 \times 10^8) = \mathbf{8 \text{ cm}^2}$.

Check: 1 cm on the map is 0.3 km, so 1 cm^2 represents 0.09 km^2 , and $0.72 \div 0.09 = 8$.

19. Let the numbers at the start be 5k red pens and 3k green pens.

After 8 red pens are removed: 5k – 8 red and 3k green. These are now equal:

$$5k - 8 = 3k$$

$$2k = 8$$

$$k = 4.$$

At the start there were $5 \times 4 = 20$ red and $3 \times 4 = 12$ green, so there were **32 pens**.

Check: $20 - 8 = 12$, and $12 : 12 = 1 : 1$.

20. Let Ali's age be 3k and Hamza's age be 5k. In 6 years they will be 3k + 6 and 5k + 6.

$(3k + 6) : (5k + 6) = 5 : 7$, so $(3k + 6)/(5k + 6) = 5/7$.

Cross-multiply: $7(3k + 6) = 5(5k + 6)$

$$21k + 42 = 25k + 30$$

$$12 = 4k$$

$$k = 3.$$

Ali is $3 \times 3 = \mathbf{9 \text{ years old}}$ and Hamza is $5 \times 3 = \mathbf{15 \text{ years old}}$.

Check: in 6 years they will be 15 and 21, and $15 : 21 = 5 : 7$.