

Physical quantities, units and measurement: AS Physics notes with questions

These notes cover the first topic of Cambridge International AS Level Physics 9702, together with the measurement skills needed for the practical papers: physical quantities and SI units, estimation, scalars and vectors, measurement techniques, and errors and uncertainties. Ideas from here are tested in every paper, and especially in Paper 3 (practical) and Paper 5 (planning, analysis and evaluation), so learn the definitions in the boxes word for word.

SI base quantities and units

A physical quantity is a property that can be measured. Every measurement is written as a number multiplied by a unit, for example 2.5 kg. The SI system is built on a small set of base quantities, each with its own base unit. Every other unit is derived from these.

Base quantity	Base unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity (not needed for 9702)	candela	cd

Note that the kilogram, not the gram, is the base unit of mass. Units named after people are written with a capital letter as a symbol (N, J, W, Pa) but with a small letter as a word (newton, joule, watt, pascal).

Derived units and homogeneity of equations

A derived unit is a product or quotient of base units. Many derived units have special names. You should be able to express each of these in base units by starting from a defining equation.

Quantity	Derived unit	Defining equation	In base units
Force	newton, N	$F = ma$	kg m s^{-2}
Energy, work	joule, J	$W = Fs$	$\text{kg m}^2 \text{s}^{-2}$
Power	watt, W	$P = W/t$	$\text{kg m}^2 \text{s}^{-3}$
Pressure	pascal, Pa	$p = F/A$	$\text{kg m}^{-1} \text{s}^{-2}$



Frequency	hertz, Hz	$f = 1/T$	s^{-1}
Charge	coulomb, C	$Q = It$	A s
Potential difference	volt, V	$V = W/Q$	$kg\ m^2\ s^{-3}\ A^{-1}$
Resistance	ohm, Ω	$R = V/I$	$kg\ m^2\ s^{-3}\ A^{-2}$

Definition. An equation is **homogeneous** when every term in it has the same base units.

A correct equation must be homogeneous, so checking units is a quick test for errors. The test is not perfect: a homogeneous equation can still be wrong, because pure numbers such as $\frac{1}{2}$ or 2π have no units and a missing term with the right units is not detected.

Worked example 1. Show that $v^2 = u^2 + 2as$ is homogeneous.

Left-hand side: v^2 has units $(m\ s^{-1})^2 = m^2\ s^{-2}$.

u^2 has the same units, $m^2\ s^{-2}$.

$2as$: the 2 has no units; $a \times s$ has units $m\ s^{-2} \times m = m^2\ s^{-2}$.

All three terms are $m^2\ s^{-2}$, so the equation is homogeneous.

Worked example 2. The drag force on a car moving at speed v is given by $F = kv^2$. Find the base units of k .

$k = F / v^2$, so the units of k are $kg\ m\ s^{-2}$ divided by $m^2\ s^{-2}$.

$kg\ m\ s^{-2} / (m^2\ s^{-2}) = kg\ m^{-1}$.

So k has base units $kg\ m^{-1}$.

Prefixes

Prefixes stand for powers of ten and are attached to a unit. A prefix on a squared or cubed unit is also squared or cubed: $1\ mm^2 = (10^{-3}\ m)^2 = 10^{-6}\ m^2$, and $1\ cm^3 = 10^{-6}\ m^3$.

Prefix	Symbol	Factor
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
kilo	k	10^3
mega	M	10^6
giga	G	10^9

tera	T	10 ¹²
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Always convert to base units before substituting into an equation: 250 nm = 250×10^{-9} m = 2.5×10^{-7} m, and 3.0 GHz = 3.0×10^9 Hz.

Estimating magnitudes

Exam questions ask you to make reasonable estimates of physical quantities. A sensible estimate is within a factor of about 3 of the true value and is given to one significant figure. Learn the values below as anchors and build other estimates from them, stating any assumptions you make.

Quantity	Typical value
Mass of an adult	70 kg
Mass of a car	1×10^3 kg
Mass of a proton	1.7×10^{-27} kg
Diameter of an atom	1×10^{-10} m
Wavelength of visible light	5×10^{-7} m
Height of a door	2 m
Radius of the Earth	6.4×10^6 m
Walking speed	1.5 m s ⁻¹
Speed of a car on a motorway	30 m s ⁻¹
Speed of sound in air	340 m s ⁻¹
Density of water	1000 kg m ⁻³
Density of air	1.2 kg m ⁻³
Atmospheric pressure	1×10^5 Pa
Power of an electric kettle	2 kW
Energy stored in an AA cell	1×10^4 J
Young modulus of steel	2×10^{11} Pa
Frequency of mains electricity	50 Hz
Gravitational field strength at the Earth's surface	9.81 N kg ⁻¹

Exam tip. In an estimate question, write the equation, state each value you assume with its unit, then calculate. Marks are given for a reasonable method and a sensible order of magnitude, not for a precise number.

Scalars and vectors

Definitions. A **scalar** is a quantity that has magnitude only.

A **vector** is a quantity that has both magnitude and direction.

Scalars	Vectors
distance, speed, mass, time, energy, work, power, temperature, density, pressure, volume, charge	displacement, velocity, acceleration, force, weight, momentum, electric field strength, gravitational field strength

Adding vectors by drawing

Draw the first vector to scale, then draw the second vector starting from the tip of the first. The resultant runs from the start of the first vector to the tip of the second (the tip-to-tail rule). Measure its length and convert with the scale, and measure its direction with a protractor. For two vectors you can also complete the parallelogram: the resultant is the diagonal from the common starting point.

Adding vectors by calculation

- Vectors at right angles: use Pythagoras for the magnitude and a tangent for the direction.
- Vectors at any other angle: either use the cosine rule and sine rule on the triangle, or resolve each vector into two perpendicular components, add the components, and recombine.

Resolving a vector

A vector of magnitude V at angle θ to a chosen axis has a component $V \cos \theta$ along the axis and a component $V \sin \theta$ perpendicular to it. For a weight W on a slope inclined at θ to the horizontal, the component down the slope is $W \sin \theta$ and the component perpendicular to the slope is $W \cos \theta$.

Worked example 3. Two forces of 6.0 N and 8.0 N act on a point at right angles to each other. Find the resultant.

Magnitude: $R = \sqrt{(6.0^2 + 8.0^2)} = \sqrt{(36 + 64)} = \sqrt{100} = 10.0 \text{ N}$.

Direction: $\tan \theta = 6.0 / 8.0 = 0.75$, so $\theta = 36.9^\circ$ measured from the 8.0 N force.

Worked example 4. Forces of 7.0 N and 5.0 N act on an object with an angle of 60° between them. Find the resultant by resolving.

Take the x-axis along the 7.0 N force.

Sum of x-components: $7.0 + 5.0 \cos 60^\circ = 7.0 + 2.50 = 9.50 \text{ N}$.

Sum of y-components: $0 + 5.0 \sin 60^\circ = 4.33 \text{ N}$.

Magnitude: $R = \sqrt{(9.50^2 + 4.33^2)} = \sqrt{(90.25 + 18.75)} = \sqrt{109.0} = 10.4 \text{ N}$.

Direction: $\tan \theta = 4.33 / 9.50 = 0.456$, so $\theta = 24.5^\circ$ from the 7.0 N force.

Check with the cosine rule: $R^2 = 7.0^2 + 5.0^2 + 2 \times 7.0 \times 5.0 \times \cos 60^\circ = 49 + 25 + 35 = 109$, so $R = 10.4 \text{ N}$. Agreed.

Measurement techniques and instruments

Choose an instrument whose range covers the quantity and whose precision is small compared with the value being measured. The precision of an instrument is usually taken as its smallest scale division (for an analogue scale, half a division is also acceptable).

Instrument	Measures	Typical range	Precision
Metre rule	length	up to 1 m	1 mm
Tape measure	length	up to several metres	1 mm
Vernier calipers	length, internal and external diameters, depth	up to about 150 mm	0.1 mm (0.02 mm on some models)
Micrometer screw gauge	small lengths, wire diameters, sheet thickness	up to 25 mm	0.01 mm
Digital balance	mass	up to a few kilograms	0.01 g or 0.1 g
Stopwatch	time	hours	0.01 s on the display, but human reaction time of 0.1 to 0.3 s dominates
Measuring cylinder	volume	10 cm ³ to 1000 cm ³	1 cm ³ (varies with size)
Liquid-in-glass thermometer	temperature	-10 °C to 110 °C	0.5 °C or 1 °C
Thermocouple	temperature, rapid changes and small objects	very wide	needs a calibration curve
Analogue ammeter or voltmeter	current, p.d.	depends on the range selected	typically 0.1 A or 0.1 V per division
Digital multimeter	current, p.d., resistance	switchable ranges	last digit of the display
Cathode-ray oscilloscope	p.d. against time, period, frequency	set by the Y-gain and time-base	a fraction of a division

Points to remember

- Check for a zero error before using a micrometer or calipers, and subtract it from every reading. A micrometer that reads 0.03 mm with its jaws closed has a zero error of +0.03 mm, so a reading of 1.27 mm means a true diameter of 1.24 mm.
- Avoid parallax error by viewing a scale at right angles, and by reading the bottom of the meniscus of water in a measuring cylinder at eye level.
- Measure a small quantity many times over: time 20 oscillations rather than one, or measure the thickness of 50 sheets rather than one sheet. The reaction time or scale precision is then divided by 20 or 50.
- Use a fiducial marker (a fixed reference point, usually at the centre of an oscillation where the

object moves fastest) when timing oscillations.

- An oscilloscope gives the period from the time-base setting multiplied by the number of horizontal divisions for one cycle; the frequency is 1 over the period. The peak voltage is the Y-gain setting multiplied by the vertical height in divisions.
- Some sensors (thermistor, thermocouple, light-dependent resistor) do not give a straight-line relationship with the quantity measured, so a calibration curve is plotted against a known instrument and used to convert readings.

Errors and uncertainties

Systematic and random errors

Definitions. A **systematic error** is an error that shifts every reading away from the true value by the same amount or in the same direction. It cannot be reduced by repeating readings and taking an average. Examples: a zero error on an instrument, an instrument that has been wrongly calibrated, heat loss to the surroundings in a heating experiment.

A **random error** is an error that causes readings to be scattered on either side of the true value. It can be reduced by repeating readings and taking an average. Examples: human reaction time when starting a stopwatch, parallax when reading a scale from slightly different angles, fluctuations in a reading from a changing room temperature.

Precision and accuracy

Definitions. **Precision** is the degree to which repeated readings of the same quantity agree with one another. A precise set of readings has a small spread, which means small random error.

Accuracy is the degree to which a measured value is close to the true value. An accurate result has small systematic error and small random error.

A set of readings can be precise but not accurate (all close together but all shifted by a zero error), or accurate but not precise (widely scattered but with a mean close to the true value). Think of arrows on a target: a tight cluster away from the centre is precise but not accurate.

Absolute, fractional and percentage uncertainty

Definitions. The **absolute uncertainty** Δx is the range on either side of a measured value x within which the true value is expected to lie. It has the same unit as x , and a result is written $x \pm \Delta x$.

The **fractional uncertainty** is $\Delta x / x$. It has no unit.

The **percentage uncertainty** is $(\Delta x / x) \times 100\%$.

- For a single reading, the absolute uncertainty is usually the precision of the instrument, for example 25.0 ± 0.1 cm with a metre rule (two readings, one at each end, each ± 0.05 cm, combine to ± 0.1 cm).
- For a set of repeated readings, take the mean as the value and half the range (largest minus

smallest, divided by 2) as the absolute uncertainty.

- Quote an uncertainty to one significant figure, and round the value to the same decimal place as its uncertainty: $9.8 \pm 0.2 \text{ m s}^{-2}$, not $9.802 \pm 0.17 \text{ m s}^{-2}$.

Combining uncertainties

Operation	Rule
Adding or subtracting: $y = a + b$ or $y = a - b$	Add the absolute uncertainties: $\Delta y = \Delta a + \Delta b$
Multiplying or dividing: $y = ab$ or $y = a/b$	Add the fractional (or percentage) uncertainties: $\Delta y/y = \Delta a/a + \Delta b/b$
Raising to a power: $y = a^n$	Multiply the fractional uncertainty by the power: $\Delta y/y = n \times \Delta a/a$ (n is taken as positive, so a square root gives $\frac{1}{2}$ and $1/a$ gives 1)
Multiplying by a constant: $y = ka$	The fractional uncertainty is unchanged: $\Delta y/y = \Delta a/a$

Worked example 5 (multiplying). A rectangular sheet has length $25.0 \pm 0.1 \text{ cm}$ and width $12.0 \pm 0.1 \text{ cm}$. Find its area with its uncertainty.

$$\text{Area } A = 25.0 \times 12.0 = 300 \text{ cm}^2.$$

$$\text{Percentage uncertainty in length} = (0.1 / 25.0) \times 100\% = 0.40\%.$$

$$\text{Percentage uncertainty in width} = (0.1 / 12.0) \times 100\% = 0.83\%.$$

$$\text{Percentage uncertainty in area} = 0.40\% + 0.83\% = 1.23\%.$$

$$\text{Absolute uncertainty } \Delta A = 0.0123 \times 300 = 3.7 \text{ cm}^2, \text{ rounded to } 4 \text{ cm}^2.$$

$$A = 300 \pm 4 \text{ cm}^2.$$

Worked example 6 (subtracting). Water is heated from $15.0 \pm 0.5 \text{ }^\circ\text{C}$ to $42.0 \pm 0.5 \text{ }^\circ\text{C}$. Find the temperature rise.

$$\text{Rise} = 42.0 - 15.0 = 27.0 \text{ }^\circ\text{C}. \text{ Uncertainty} = 0.5 + 0.5 = 1.0 \text{ }^\circ\text{C}.$$

Temperature rise = $27.0 \pm 1.0 \text{ }^\circ\text{C}$. Notice that the percentage uncertainty (3.7%) is much larger than the percentage uncertainty in either reading, which is why subtracting two similar values gives a poor result.

Worked example 7 (powers). The diameter of a wire is measured five times with a micrometer: 0.52, 0.54, 0.53, 0.55 and 0.51 mm. Find the cross-sectional area with its uncertainty.

$$\text{Mean diameter } d = (0.52 + 0.54 + 0.53 + 0.55 + 0.51) / 5 = 2.65 / 5 = 0.530 \text{ mm}.$$

$$\text{Absolute uncertainty in } d = \text{half the range} = (0.55 - 0.51) / 2 = 0.02 \text{ mm}.$$

$$\text{Area } A = \pi d^2 / 4 = \pi \times 0.530^2 / 4 = \pi \times 0.2809 / 4 = 0.2206 \text{ mm}^2.$$

$$\text{Percentage uncertainty in } d = (0.02 / 0.530) \times 100\% = 3.8\%.$$

$$\text{Percentage uncertainty in } A = 2 \times 3.8\% = 7.6\% \text{ (the power is 2; } \pi \text{ and 4 are exact).}$$

$$\Delta A = 0.076 \times 0.2206 = 0.017 \text{ mm}^2, \text{ rounded to } 0.02 \text{ mm}^2.$$

$$A = 0.22 \pm 0.02 \text{ mm}^2.$$

Worked example 8 (a full calculation). A pendulum of length $L = 0.800 \pm 0.005 \text{ m}$ has period $T = 1.795 \pm 0.010 \text{ s}$. Use $g = 4\pi^2 L / T^2$ to find g with its uncertainty.

$$g = 4\pi^2 \times 0.800 / 1.795^2 = 31.58 / 3.222 = 9.80 \text{ m s}^{-2}.$$

$$\text{Percentage uncertainty in } L = (0.005 / 0.800) \times 100\% = 0.63\%.$$

Percentage uncertainty in $T = (0.010 / 1.795) \times 100\% = 0.56\%$, and T is squared, so its contribution is $2 \times 0.56\% = 1.12\%$.

$$\text{Percentage uncertainty in } g = 0.63\% + 1.12\% = 1.75\%.$$

$$\Delta g = 0.0175 \times 9.80 = 0.17 \text{ m s}^{-2}, \text{ rounded to } 0.2 \text{ m s}^{-2}.$$

$g = 9.8 \pm 0.2 \text{ m s}^{-2}$. The accepted value 9.81 m s^{-2} lies inside this range, so the result is consistent with it.

Exam tip. When a question asks which measurement contributes most to the uncertainty, compare the percentage uncertainties after they have been multiplied by their powers. In Worked example 8 it is the period, because its 0.56% is doubled. Suggest reducing it by timing more oscillations, not by using a stopwatch with more decimal places.

Key terms

Term	Meaning
Physical quantity	A property that can be measured, written as a number and a unit.
Base unit	One of the seven SI units from which all others are derived (kg, m, s, A, K, mol, cd).
Derived unit	A unit formed by multiplying or dividing base units, for example $\text{N} = \text{kg m s}^{-2}$.
Homogeneous equation	An equation in which every term has the same base units.
Prefix	A multiplier that is a power of ten attached to a unit, for example $\text{k} = 10^3$.
Order of magnitude	The power of ten nearest to the size of a quantity.
Scalar	A quantity with magnitude only.
Vector	A quantity with magnitude and direction.
Resultant	The single vector that has the same effect as two or more vectors added together.
Component	One of two perpendicular vectors into which a vector is resolved.
Precision (of an instrument)	The smallest change the instrument can show, usually one scale division.
Range	The smallest and largest values an instrument can measure.

Zero error	A non-zero reading when the true value is zero; a systematic error.
Parallax error	An error from viewing a scale and pointer at an angle rather than square on.
Calibration curve	A graph of the reading of an instrument against the true value, used to convert readings.
Systematic error	An error that shifts all readings in the same direction; not reduced by averaging.
Random error	An error that scatters readings around the true value; reduced by averaging.
Precision (of readings)	How closely repeated readings agree with each other.
Accuracy	How close a measured value is to the true value.
Absolute uncertainty	The margin Δx on either side of a value x ; same unit as x .
Fractional uncertainty	$\Delta x / x$, with no unit.
Percentage uncertainty	$(\Delta x / x) \times 100\%$.

Practice

Marks are shown in square brackets. Full answers are given at the end.

1. State, in SI base units, the units of (a) force, (b) pressure, (c) energy. [3]
2. The pressure p at a depth h in a liquid of density ρ is $p = \rho gh$, where g is the acceleration of free fall. Show that this equation is homogeneous. [3]
3. The viscous drag force F on a small sphere of radius r moving at speed v through a liquid is $F = 6\pi\eta rv$, where η is a constant for the liquid. Determine the base units of η . [3]
4. Convert each of the following to the SI unit without a prefix, giving your answer in standard form: (a) 3.5 GHz, (b) 0.25 mm², (c) 1.2 g cm⁻³. [3]
5. Estimate the kinetic energy of a family car travelling on a motorway. State the values you assume. [3]
6. (a) State the difference between a scalar quantity and a vector quantity. (b) Sort the following into scalars and vectors: distance, displacement, speed, velocity, acceleration, force, energy, momentum, temperature, power. [4]
7. A boat is steered due north at 4.0 m s⁻¹ relative to the water of a river that flows due east at 3.0 m s⁻¹. Calculate the magnitude and direction of the resultant velocity of the boat. [4]
8. Two forces of 12 N and 9.0 N act at a point. The angle between them is 50°. Calculate the magnitude of the resultant force and the angle it makes with the 12 N force. [4]
9. A box of weight 45 N rests on a slope inclined at 25° to the horizontal. Calculate the components of the weight (a) parallel to the slope, (b) perpendicular to the slope. [3]
10. (a) With the jaws closed, a micrometer reads 0.03 mm. With a wire between the jaws it reads 1.27 mm. State the diameter of the wire. (b) Name the most suitable instrument for measuring: (i) the diameter of a wire of about 0.5 mm, (ii) the internal diameter of a pipe of about 2 cm, (iii) the length of a laboratory bench of about 2 m. [4]
11. A student times 20 complete oscillations of a pendulum with a stopwatch and records 28.4 s. The student's reaction time is estimated as 0.2 s. (a) Calculate the period of the pendulum. (b)

Calculate the absolute uncertainty in the period. (c) Explain why the student timed 20 oscillations rather than one. (d) The stopwatch runs slow, so that every reading is 1% too small. State the type of error this introduces and whether repeating the readings would reduce it. [6]

12. In an experiment to find the resistivity of a wire, the potential difference across it is 4.80 ± 0.05 V and the current in it is 0.240 ± 0.005 A. The wire has length 1.500 ± 0.002 m and diameter 0.32 ± 0.01 mm. (a) Calculate the resistance R of the wire with its absolute uncertainty. (b) Calculate the resistivity $\rho = RA / L$ of the wire, where A is the cross-sectional area, with its percentage and absolute uncertainty. (c) State which measurement should be improved first to reduce the uncertainty in ρ , with a reason. [6]

1. (a) Force: $F = ma$, so $\text{kg} \times \text{m s}^{-2} = \text{kg m s}^{-2}$. (b) Pressure: $p = F/A$, so $\text{kg m s}^{-2} / \text{m}^2 = \text{kg m}^{-1} \text{s}^{-2}$. (c) Energy: $W = Fs$, so $\text{kg m s}^{-2} \times \text{m} = \text{kg m}^2 \text{s}^{-2}$. [3]

2. Left-hand side: pressure has base units $\text{kg m}^{-1} \text{s}^{-2}$. Right-hand side: ρ is kg m^{-3} , g is m s^{-2} , h is m. Multiplying: $\text{kg m}^{-3} \times \text{m s}^{-2} \times \text{m} = \text{kg m}^{-3+1+1} \text{s}^{-2} = \text{kg m}^{-1} \text{s}^{-2}$. Both sides have the same base units, so the equation is homogeneous. [3]

3. Rearranging, $\eta = F / (6\pi r v)$. The 6π has no units. Units of $\eta = \text{kg m s}^{-2} / (\text{m} \times \text{m s}^{-1}) = \text{kg m s}^{-2} / (\text{m}^2 \text{s}^{-1}) = \text{kg m}^{-1} \text{s}^{-1}$. [3]

4. (a) $3.5 \text{ GHz} = 3.5 \times 10^9 \text{ Hz}$. (b) $1 \text{ mm}^2 = (10^{-3} \text{ m})^2 = 10^{-6} \text{ m}^2$, so $0.25 \text{ mm}^2 = 0.25 \times 10^{-6} \text{ m}^2 = 2.5 \times 10^{-7} \text{ m}^2$. (c) $1 \text{ g} = 10^{-3} \text{ kg}$ and $1 \text{ cm}^3 = 10^{-6} \text{ m}^3$, so $1 \text{ g cm}^{-3} = 10^{-3} / 10^{-6} = 10^3 \text{ kg m}^{-3}$, and $1.2 \text{ g cm}^{-3} = 1.2 \times 10^3 \text{ kg m}^{-3}$. [3]

5. Assume the mass of the car is 1200 kg and its speed is 30 m s^{-1} (about 110 km h^{-1}). Kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2} \times 1200 \times 30^2 = \frac{1}{2} \times 1200 \times 900 = 5.4 \times 10^5 \text{ J}$, so about $5 \times 10^5 \text{ J}$. Any answer between about $2 \times 10^5 \text{ J}$ and $1 \times 10^6 \text{ J}$ with stated assumptions is acceptable. [3]

6. (a) A scalar has magnitude only; a vector has both magnitude and direction. (b) Scalars: distance, speed, energy, temperature, power. Vectors: displacement, velocity, acceleration, force, momentum. [4]

7. The two velocities are at right angles. Magnitude: $v = \sqrt{4.0^2 + 3.0^2} = \sqrt{16 + 9} = \sqrt{25} = 5.0 \text{ m s}^{-1}$. Direction: $\tan \theta = 3.0 / 4.0 = 0.75$, so $\theta = 36.9^\circ$, measured from north towards east. The boat moves at 5.0 m s^{-1} on a bearing of 037° (36.9° east of north). [4]

8. Resolve along and perpendicular to the 12 N force. Along: $12 + 9.0 \cos 50^\circ = 12 + 9.0 \times 0.6428 = 12 + 5.79 = 17.79 \text{ N}$. Perpendicular: $9.0 \sin 50^\circ = 9.0 \times 0.7660 = 6.89 \text{ N}$. Magnitude: $R = \sqrt{17.79^2 + 6.89^2} = \sqrt{316.5 + 47.5} = \sqrt{364.0} = 19.1 \text{ N}$. Angle with the 12 N force: $\tan \theta = 6.89 / 17.79 = 0.387$, so $\theta = 21.2^\circ$. Check by the cosine rule: $R^2 = 12^2 + 9.0^2 + 2 \times 12 \times 9.0 \times \cos 50^\circ = 144 + 81 + 138.8 = 363.8$, $R = 19.1 \text{ N}$. [4]

9. (a) Parallel to the slope: $45 \sin 25^\circ = 45 \times 0.4226 = 19.0 \text{ N}$ (acting down the slope). (b) Perpendicular to the slope: $45 \cos 25^\circ = 45 \times 0.9063 = 40.8 \text{ N}$ (acting into the slope). Check: $19.0^2 + 40.8^2 = 361 + 1665 = 2026$, and $\sqrt{2026} = 45.0 \text{ N}$. [3]

10. (a) The zero error is $+0.03 \text{ mm}$, so it must be subtracted: diameter $= 1.27 - 0.03 = 1.24 \text{ mm}$. (b) (i) Micrometer screw gauge, precision 0.01 mm , because the diameter is only about 50 times its precision and a vernier (0.1 mm) would give a 20% uncertainty. (ii) Vernier calipers, using the internal jaws. (iii) A tape measure, or a metre rule used twice end to end. [4]

11. (a) Period $T = 28.4 / 20 = 1.42 \text{ s}$. (b) The uncertainty in the total time is the reaction time, 0.2 s ,

so the uncertainty in the period is $0.2 / 20 = 0.01$ s, giving $T = 1.42 \pm 0.01$ s. (c) The reaction time of about 0.2 s is the same whether one oscillation or twenty is timed. Timing one oscillation would give 1.42 ± 0.2 s, a percentage uncertainty of 14%; timing twenty divides the absolute uncertainty by 20 and reduces the percentage uncertainty to about 0.7%. (d) A systematic error: every reading is shifted in the same direction by the same fraction. Repeating and averaging would not reduce it, because every repeat is 1% too small as well; it can only be removed by calibrating the stopwatch against a reliable clock and correcting the readings. [6]

12. (a) $R = V / I = 4.80 / 0.240 = 20.0 \Omega$. Percentage uncertainty in $V = (0.05 / 4.80) \times 100\% = 1.04\%$. Percentage uncertainty in $I = (0.005 / 0.240) \times 100\% = 2.08\%$. Percentage uncertainty in $R = 1.04\% + 2.08\% = 3.1\%$. $\Delta R = 0.031 \times 20.0 = 0.62 \Omega$, rounded to 0.6Ω . $R = 20.0 \pm 0.6 \Omega$.

(b) Convert the diameter: $d = 0.32 \text{ mm} = 3.2 \times 10^{-4} \text{ m}$. Area $A = \pi d^2 / 4 = \pi \times (3.2 \times 10^{-4})^2 / 4 = \pi \times 1.024 \times 10^{-7} / 4 = 8.04 \times 10^{-8} \text{ m}^2$. Resistivity $\rho = RA / L = 20.0 \times 8.04 \times 10^{-8} / 1.500 = 1.07 \times 10^{-6} \Omega \text{ m}$.

Percentage uncertainties: R is 3.1% (from part a); d is $(0.01 / 0.32) \times 100\% = 3.1\%$, and d is squared, so its contribution is $2 \times 3.1\% = 6.2\%$; L is $(0.002 / 1.500) \times 100\% = 0.13\%$. Total percentage uncertainty in $\rho = 3.1\% + 6.2\% + 0.13\% = 9.4\%$, which is about 10%. Absolute uncertainty $\Delta\rho = 0.094 \times 1.07 \times 10^{-6} = 1.0 \times 10^{-7} \Omega \text{ m}$. So $\rho = (1.07 \pm 0.10) \times 10^{-6} \Omega \text{ m}$, which could be written $(1.1 \pm 0.1) \times 10^{-6} \Omega \text{ m}$.

(c) The diameter. Its 3.1% uncertainty is doubled to 6.2% because the area depends on d^2 , making it the largest contribution by far. It should be measured at several places along the wire and in two directions at right angles at each place, with a micrometer that has been checked for zero error, and the mean taken. [6]