

Kinematics: equations of motion, graphs and projectiles (AS Physics notes)

These notes cover the kinematics part of Cambridge International AS Level Physics 9702: describing motion in a straight line, using motion graphs, deriving and applying the equations of uniformly accelerated motion, free fall and the measurement of the acceleration of free fall, and two-dimensional projectile motion. A short section on air resistance links this topic to dynamics. Learn the boxed definitions word for word. A Quick check of 15 exam-style questions with full worked answers is at the end.

Throughout, the acceleration of free fall is taken as $g = 9.81 \text{ m s}^{-2}$, the value given in the Cambridge data booklet. Answers are given to 2 or 3 significant figures to match the data.

1. Describing motion: scalars, vectors and the key quantities

1.1 Scalars and vectors

A **scalar** quantity has magnitude only, for example distance, speed, time and mass. A **vector** quantity has both magnitude and direction, for example displacement, velocity and acceleration. In one-dimensional motion the direction is shown by a sign: choose one direction as positive, and anything in the opposite direction is negative. Choosing the sign convention at the start of every problem, and sticking to it, prevents most errors in this topic.

1.2 Distance and displacement

Distance: the total length of the path travelled, regardless of direction. It is a scalar.

Displacement: the distance in a straight line from the starting point to the final position, in a specified direction. It is a vector.

If you walk once round a circular track and return to the start, the distance travelled is the circumference of the track but your displacement is zero. Distance can never decrease as time goes on; displacement can increase, decrease or become negative.

1.3 Speed and velocity

Speed: the distance travelled per unit time.

Velocity: the rate of change of displacement, that is the change in displacement per unit time.

Both have the SI unit m s^{-1} . Speed is a scalar; velocity is a vector.

Average speed = total distance $\div$ total time. **Average velocity** = total displacement $\div$ total time.

Instantaneous speed or velocity is the value at one moment, found from the gradient of a tangent to a graph (section 2).

Worked example 1: A student walks 300 m due east then 400 m due north. The whole walk takes 250 s. Calculate (a) the average speed and (b) the average velocity.

(a) Distance = $300 + 400 = 700 \text{ m}$. Average speed = $700 \div 250 = 2.8 \text{ m s}^{-1}$.



(b) Displacement: the two legs are at right angles, so magnitude = $\sqrt{(300^2 + 400^2)} = \sqrt{250\,000} = 500$ m.

Direction: $\tan \theta = 400/300$, so $\theta = 53.1^\circ$ north of east.

Average velocity = $500 \div 250 = 2.0 \text{ m s}^{-1}$ at 53° north of east. Note that the direction must be given for a velocity.

1.4 Acceleration

Acceleration: the rate of change of velocity.

$a = (v - u) \div t$, where u is the initial velocity and v is the final velocity after time t . SI unit: m s^{-2} .

Acceleration is a vector.

Because velocity is a vector, an object accelerates if its speed changes, its direction changes, or both. In straight-line motion, a negative acceleration means the acceleration is in the negative direction. Whether this means the object is slowing down depends on the direction of the velocity: an object moving in the positive direction with negative acceleration slows down, but an object moving in the negative direction with negative acceleration speeds up. For this reason it is clearer to say "the acceleration is 3.0 m s^{-2} backwards" than to use the word deceleration with a sign.

Uniform (constant) acceleration means the velocity changes by equal amounts in equal intervals of time. The equations of motion in section 3 only apply to uniform acceleration.

2. Motion graphs

Two graphs are used most often: displacement against time and velocity against time. The key skill is knowing what the **gradient** and the **area under the line** represent on each one.

2.1 Displacement-time graphs

Rule: the gradient of a displacement-time graph is the velocity.

- A horizontal line: the gradient is zero, so the object is at rest.
- A straight sloping line: constant gradient, so constant velocity. A steeper line means a greater velocity.
- A line sloping downwards: negative gradient, so the velocity is in the negative direction (the object is moving back towards, or past, the start).
- A curve getting steeper: the velocity is increasing, so the object is accelerating.
- A curve getting less steep: the velocity is decreasing.

For a curved graph, the **instantaneous velocity** at a time t is the gradient of the tangent drawn to the curve at t . Draw the tangent with a ruler so that it touches the curve at one point, make the triangle used for the gradient as large as possible (at least half the graph), and read the coordinates carefully.

The area under a displacement-time graph has no useful physical meaning at this level.

2.2 Velocity-time graphs

Rules:

The gradient of a velocity-time graph is the acceleration.

The area between a velocity-time graph and the time axis is the displacement.

- A horizontal line: zero acceleration, constant velocity.
- A straight sloping line: uniform acceleration. The steeper the line, the greater the acceleration.
- A curve: non-uniform acceleration. The instantaneous acceleration is the gradient of the tangent at that time.
- Where the line is below the time axis, the velocity is negative, so the object is moving in the negative direction. Area below the axis counts as negative displacement.

The total **distance** travelled is the sum of all the areas, all counted as positive. The final **displacement** is the area above the axis minus the area below it. For areas under curved lines, count squares: find the displacement represented by one square from the scales, count whole squares and squares more than half inside the area, and multiply.

Worked example 2: A car's velocity increases uniformly from 4.0 m s^{-1} to 16 m s^{-1} in 8.0 s . Find (a) the acceleration and (b) the distance travelled.

(a) Gradient = $(16 - 4.0) \div 8.0 = 1.5 \text{ m s}^{-2}$.

(b) The area under the line is a trapezium: $\frac{1}{2} \times (4.0 + 16) \times 8.0 = 80 \text{ m}$.

Alternatively split it into a rectangle ($4.0 \times 8.0 = 32 \text{ m}$) and a triangle ($\frac{1}{2} \times 8.0 \times 12 = 48 \text{ m}$), total 80 m .

2.3 Linking the graphs: a ball thrown upwards

Take upwards as positive and ignore air resistance. A ball is thrown straight up and caught at the same height.

- **Velocity-time graph:** a single straight line starting at $+u$, falling through zero at the top of the flight and ending at $-u$. The gradient is $-g$ throughout, including at the top, because the acceleration is always 9.81 m s^{-2} downwards. The area above the axis (going up) equals the area below the axis (coming down), so the total displacement is zero.
- **Displacement-time graph:** an upside-down parabola. It starts at zero, is steepest at the start, is horizontal (zero velocity) at the top, and returns to zero.

Exam tip: at the highest point the velocity is zero but the acceleration is not. It is still g downwards. Saying "the acceleration is zero at the top" is one of the most common mistakes in this topic.

3. Equations of uniformly accelerated motion

3.1 The symbols

Symbol	Quantity	Unit
s	displacement	m
u	initial velocity	m s^{-1}
v	final velocity	m s^{-1}

a	acceleration (constant)	m s ⁻²
t	time taken	s

These are often called the SUVAT equations, after the five symbols. Each equation contains four of the five quantities, so if you know any three you can find the other two.

3.2 The equations

Equations of motion (uniform acceleration in a straight line):

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

The Cambridge data and formulae page lists $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$. You must be able to derive and use all of them.

3.3 Deriving the equations

The syllabus requires you to derive the equations from the definitions of velocity and acceleration, and you may be asked to do this in an exam. Use a velocity-time graph that is a straight line from velocity u at time 0 to velocity v at time t .

Equation 1, from the definition of acceleration. Acceleration is the gradient of the graph: $a = (v - u) \div t$. Rearranging gives **$v = u + at$** .

Equation 2, from the area. The displacement is the area under the graph, which is a trapezium with parallel sides u and v and width t . So $s = \frac{1}{2}(u + v)t$. This also follows from average velocity: for uniform acceleration the average velocity is $(u + v)/2$, and displacement = average velocity $\times$ time.

Equation 3. Split the area into a rectangle and a triangle. The rectangle has height u and width t , area ut . The triangle has base t and height $(v - u) = at$, area $\frac{1}{2} \times t \times at = \frac{1}{2}at^2$. Adding: **$s = ut + \frac{1}{2}at^2$** . (Algebraically, substitute $v = u + at$ into $s = \frac{1}{2}(u + v)t$.)

Equation 4, eliminating t . From equation 1, $t = (v - u)/a$. Substitute into $s = \frac{1}{2}(u + v)t$:

$$s = \frac{1}{2}(u + v)(v - u)/a = (v^2 - u^2)/(2a).$$

Rearranging: $2as = v^2 - u^2$, so **$v^2 = u^2 + 2as$** .

3.4 How to use the equations

1. Draw a simple sketch and choose a positive direction.
2. List s , u , v , a and t . Write in the three values you know (with signs) and mark the one you want.
3. Pick the equation that contains the three knowns and the unknown, and does not contain the fifth quantity.
4. Substitute, rearrange and calculate. Check the sign and size of the answer make sense.

Hidden information is common: "starts from rest" means $u = 0$; "comes to rest" or "stops" means $v = 0$; "dropped" means $u = 0$ vertically; "at the highest point" means the vertical velocity is zero.

The equations are only valid when the acceleration is constant. If a journey has several stages with different accelerations, treat each stage separately, using the final velocity of one stage as the initial

velocity of the next.

Worked example 3: A cyclist moving at 3.0 m s^{-1} accelerates uniformly at 0.80 m s^{-2} over a distance of 50 m. Calculate (a) her final speed and (b) the time taken.

Known: $u = 3.0 \text{ m s}^{-1}$, $a = 0.80 \text{ m s}^{-2}$, $s = 50 \text{ m}$.

(a) $v^2 = u^2 + 2as = 3.0^2 + 2 \times 0.80 \times 50 = 9.0 + 80 = 89$, so $v = \sqrt{89} = \mathbf{9.4 \text{ m s}^{-1}}$.

(b) $v = u + at$ gives $t = (v - u)/a = (9.43 - 3.0)/0.80 = \mathbf{8.0 \text{ s}}$.

Check with $s = \frac{1}{2}(u + v)t = \frac{1}{2} \times (3.0 + 9.43) \times 8.04 = 50 \text{ m}$.

Worked example 4 (braking): A train travelling at 32 m s^{-1} brakes with a constant acceleration of 0.40 m s^{-2} opposite to its motion. How far does it travel before stopping?

Take the direction of motion as positive: $u = 32 \text{ m s}^{-1}$, $v = 0$, $a = -0.40 \text{ m s}^{-2}$.

$v^2 = u^2 + 2as$ gives $0 = 32^2 + 2 \times (-0.40) \times s$, so $0.80s = 1024$ and $s = \mathbf{1280 \text{ m}}$ (1.3 km to 2 s.f.).

3.5 Stopping distance

When a driver sees a hazard, the car first travels at constant speed during the driver's **reaction time**; this is the thinking distance = speed $\times$ reaction time. The car then decelerates; this is the braking distance, found from $v^2 = u^2 + 2as$ with $v = 0$. The stopping distance is the sum. Thinking distance is proportional to speed, but braking distance is proportional to speed squared, so doubling the speed roughly quadruples the braking distance.

4. Free fall and the acceleration of free fall

4.1 What free fall means

Acceleration of free fall (g): the acceleration of an object falling under gravity alone, with no air resistance. Near the Earth's surface it is uniform and approximately 9.81 m s^{-2} , directed vertically downwards.

In a uniform gravitational field and with no air resistance, every object has the same acceleration, whatever its mass. A hammer and a feather dropped together in a vacuum land together. In air, the feather falls more slowly only because air resistance on it is large compared with its weight.

Free-fall problems use the equations of motion with $a = g$ (or $a = -g$ if upwards is taken as positive). The value of g varies slightly with location on the Earth; always use the value given in the question or the data booklet.

Worked example 5: A stone is dropped from rest into a well and hits the water 2.4 s later. Calculate the depth of the water surface below the top of the well. (Ignore air resistance and the time for the sound to return.)

Take down as positive: $u = 0$, $a = 9.81 \text{ m s}^{-2}$, $t = 2.4 \text{ s}$.

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 9.81 \times 2.4^2 = 0.5 \times 9.81 \times 5.76 = 28.3 \text{ m}$, so the depth is $\mathbf{28 \text{ m}}$ (2 s.f.).

4.2 Objects thrown upwards

Take upwards as positive. The acceleration is -9.81 m s^{-2} for the whole flight, on the way up, at the top and on the way down. Useful results, with no air resistance:

- At the maximum height $v = 0$, so maximum height $h = u^2/(2g)$.
- Time to reach the top $= u/g$. The time to fall back to the launch height is the same, so the total time $= 2u/g$.
- The object returns to the launch height with the same speed it was thrown with, moving downwards.

When an object is thrown upwards from a height (for example from a cliff), you do not need to split the motion into "up" and "down" parts. Using $s = ut + \frac{1}{2}at^2$ with the displacement measured to the final point (negative if it ends below the start) handles the whole flight in one step, giving a quadratic in t . Choose the positive root.

4.3 Measuring the acceleration of free fall

The syllabus asks you to describe an experiment to determine g using a falling object. A common method uses an electromagnet and a trapdoor (or two light gates).

Method (electromagnet and trapdoor):

1. A small steel ball is held by an electromagnet above a hinged trapdoor. The electromagnet and trapdoor are connected to an electronic timer.
2. Switching off the electromagnet releases the ball and starts the timer. The ball hits the trapdoor, which opens and stops the timer. The timer reads the fall time t .
3. Measure the height h from the bottom of the ball to the trapdoor with a metre rule, using a set square to avoid parallax.
4. Repeat each height several times and find the mean time. Repeat for at least five or six different heights, for example from 0.30 m to 1.20 m.

Analysis: the ball starts from rest, so $h = \frac{1}{2}gt^2$. Plot h on the y-axis against t^2 on the x-axis. The graph should be a straight line through the origin with **gradient** $= g/2$, so $g = 2 \times \text{gradient}$.

Sources of error and improvements:

- The electromagnet may not release the ball the instant the current is switched off (residual magnetism), so measured times are slightly too long. This is a systematic error. On the graph of h against t^2 it shows up as a line that does not pass through the origin, but the gradient (and so g) is less affected, which is one advantage of the graph method.
- Air resistance is negligible for a small dense steel ball over these heights, which is why a steel ball is used rather than, say, a table-tennis ball.
- Measuring h : it is hard to measure exactly from the bottom of the ball, giving an uncertainty of a few millimetres.

Alternative methods: (a) Drop a card or ball through two light gates a known distance apart connected to a data logger. Each gate gives a speed (card length $\div$ time to pass), and $g = (v_2^2 - v_1^2)/(2 \times \text{separation})$, or $g = (v_2 - v_1)/(\text{time between gates})$. (b) Video a falling ball in front of a metre rule and step through the frames, using the known frame rate to find time.

Worked example 6: In a trapdoor experiment the gradient of a graph of h (in m) against t^2 (in s^2) is 4.90 m s^{-2} . Determine g .
 $h = \frac{1}{2}gt^2$, so gradient $= g/2$. $g = 2 \times 4.90 = \mathbf{9.80 \text{ m s}^{-2}}$.

5. Projectile motion

5.1 The key idea

A projectile is any object that, once launched, moves only under the influence of gravity (when air resistance is ignored). Its motion is two-dimensional, but it can be analysed as two separate, independent one-dimensional motions at right angles:

Projectile motion (no air resistance):

Horizontally: no force acts, so the horizontal component of velocity is constant (uniform velocity).

Horizontal displacement $x = u_x t$.

Vertically: the weight acts, so there is a uniform acceleration g downwards. Use the equations of motion with $a = g$.

The two motions are independent. The only quantity they share is the time t .

The path of a projectile is a parabola. A well-known demonstration: if one ball is dropped from rest and another is projected horizontally from the same height at the same instant, both hit level ground at the same time, because both have zero initial vertical velocity and the same vertical acceleration. The horizontal motion does not affect the vertical motion.

5.2 Resolving the initial velocity

If a projectile is launched with speed u at an angle θ above the horizontal:

- horizontal component $u_x = u \cos \theta$
- vertical component $u_y = u \sin \theta$

At any later time the velocity has components $v_x = u_x$ (unchanged) and $v_y = u_y - gt$ (taking up as positive). The speed is $\sqrt{v_x^2 + v_y^2}$, and the angle to the horizontal is given by $\tan \phi = v_y/v_x$.

5.3 Horizontal launch

For an object projected horizontally from a height, $u_y = 0$. The time of flight depends only on the height: from $h = \frac{1}{2}gt^2$, $t = \sqrt{2h/g}$. The horizontal distance travelled is then $u_x \times t$.

Worked example 7: An aircraft flying horizontally at 50 m s^{-1} at a height of 80 m releases a package. Ignoring air resistance, calculate (a) the time for the package to reach the ground and (b) the horizontal distance it travels in this time.

(a) Vertically: $u_y = 0$, $s = 80 \text{ m}$, $a = 9.81 \text{ m s}^{-2}$ (down positive).

$80 = \frac{1}{2} \times 9.81 \times t^2$, so $t^2 = 160/9.81 = 16.3$, $t = \mathbf{4.04 \text{ s}}$.

(b) Horizontally the package keeps the aircraft's velocity: $x = 50 \times 4.04 = 202 \text{ m}$, so about **200 m**.

Note: with no air resistance the package stays directly below the aircraft all the way down, because they have the same horizontal velocity.

5.4 Launch at an angle

For a projectile launched from level ground and landing at the same level (no air resistance):

- At the top, $v_y = 0$. Time to the top $= u_y/g$; maximum height $H = u_y^2/(2g)$.
- Time of flight $T = 2u_y/g$, by symmetry.
- Range $R = u_x \times T$.

- At the top the velocity is not zero: it is horizontal and equal to u_x . The speed is a minimum there.
- The projectile lands with the same speed it was launched with, at the same angle below the horizontal.

For a given launch speed on level ground, the range is greatest at a launch angle of 45° , and angles that add up to 90° (for example 30° and 60°) give the same range. You do not need to prove this for AS, but it is a useful check.

Worked example 8: A ball is kicked from level ground at 25 m s^{-1} at 40° above the horizontal. Ignoring air resistance, calculate (a) the time of flight, (b) the range and (c) the maximum height.

$u_x = 25 \cos 40^\circ = 19.15 \text{ m s}^{-1}$; $u_y = 25 \sin 40^\circ = 16.07 \text{ m s}^{-1}$.

(a) Taking up as positive, the ball lands when the vertical displacement is zero: $0 = u_y t - \frac{1}{2}gt^2$, so $t = 2u_y/g = 2 \times 16.07/9.81 = \mathbf{3.28 \text{ s}}$.

(b) $R = u_x \times t = 19.15 \times 3.276 = 62.7 \text{ m}$, so **63 m** (2 s.f.).

(c) At the top $v_y = 0$: $0 = u_y^2 - 2gH$, so $H = 16.07^2/(2 \times 9.81) = 258.2/19.62 = \mathbf{13 \text{ m}}$.

6. Air resistance (qualitative)

6.1 Drag forces

Air resistance is a drag force: a frictional force that opposes the motion of an object through a fluid. It acts in the opposite direction to the velocity. Its size increases as the speed increases (at the speeds met in everyday life, roughly in proportion to the speed squared), and it also depends on the cross-sectional area and shape of the object. You are only expected to describe its effects qualitatively.

6.2 Falling with air resistance: terminal velocity

Consider a skydiver jumping from a hovering helicopter, taking down as positive.

1. At the start the speed is zero, so there is no air resistance. The only force is the weight, and the acceleration is g .
2. As the speed increases, the air resistance increases. The resultant force (weight minus air resistance) decreases, so the acceleration decreases. The velocity is still increasing, but more slowly.
3. Eventually the air resistance equals the weight. The resultant force is zero, the acceleration is zero, and the skydiver falls at a constant speed called the **terminal velocity**.
4. When the parachute opens, the cross-sectional area increases sharply, so the air resistance becomes much larger than the weight. The resultant force is upwards, so the skydiver decelerates. As the speed falls, air resistance decreases, until it again equals the weight at a new, much lower terminal velocity.

Terminal velocity: the constant maximum velocity reached by an object falling through a fluid, when the drag force (plus any upthrust) is equal and opposite to its weight, so the resultant force and the acceleration are zero.

Velocity-time graph in words: the line starts at the origin with gradient g , then curves, becoming less and less steep, and levels off at the terminal velocity. When the parachute opens, the velocity falls quickly along a curve that becomes less steep, levelling off at the lower terminal velocity. The gradient

at any point is the acceleration, which is never greater than g for a falling object starting from rest.

6.3 Effect of air resistance on a projectile

Air resistance acts opposite to the velocity at every instant, so it has a horizontal component (opposing the horizontal motion) and a vertical component. Compared with the ideal parabola:

- the horizontal velocity decreases during the flight instead of staying constant;
- the maximum height is lower;
- the range is shorter;
- the path is no longer symmetrical: the descent is steeper than the ascent;
- the landing speed is less than the launch speed.

Exam tip: when asked to show the effect of air resistance on a sketched trajectory, draw a path that is lower and shorter, and steeper on the way down. Then state each change in words, because a sketch alone may not earn all the marks.

Quick check

- (a) Define acceleration.
(b) State the difference between distance and displacement.
- An athlete runs exactly one lap of a 400 m track in 80 s, finishing where she started. Calculate (a) her average speed and (b) the magnitude of her average velocity. Explain your answer to (b).
- A car accelerates uniformly from 12 m s^{-1} to 30 m s^{-1} in 6.0 s. Calculate (a) its acceleration and (b) the distance it travels in this time.
- A driver travelling at 25 m s^{-1} sees a hazard. Her reaction time is 0.60 s, after which the car decelerates uniformly at 5.0 m s^{-2} until it stops. Calculate the total stopping distance.
- A trolley starts from rest and accelerates uniformly to 8.0 m s^{-1} in 4.0 s. It then moves at constant velocity for 6.0 s and finally decelerates uniformly to rest in 2.0 s.
(a) Describe the velocity-time graph for the whole motion.
(b) Use the graph to calculate the total distance travelled.
(c) Calculate the average speed for the whole motion.
- Describe the motion of an object whose displacement-time graph is (a) a horizontal straight line, (b) a straight line with a negative gradient, (c) a curve that becomes steeper as time increases. State in each case what the gradient tells you.
- Starting from the definitions of acceleration and of displacement as the area under a velocity-time graph, derive the equation $s = ut + \frac{1}{2}at^2$ for uniform acceleration.
- A ball is dropped from rest from a height of 20.0 m. Ignoring air resistance, calculate (a) the time it takes to reach the ground and (b) its speed just before it lands.
- A ball is thrown vertically upwards at 15.0 m s^{-1} and caught at the same height. Ignoring air resistance, calculate (a) the maximum height reached and (b) the time the ball is in the air. (c) State the acceleration of the ball at its highest point.
- A stone is thrown vertically upwards at 12 m s^{-1} from the edge of a cliff 30 m above the sea. It

misses the cliff on the way down. Ignoring air resistance, calculate (a) the speed with which it hits the sea and (b) the time from throwing to hitting the sea.

11. In an experiment to measure g , a steel ball released from an electromagnet falls 0.800 m on to a trapdoor in a time of 0.405 s.

(a) Calculate a value for g from this single reading.

(b) Explain why plotting a graph of h against t^2 for several heights gives a more reliable value, and how g is found from the graph.

(c) Suggest one source of systematic error in this experiment.

12. A ball rolls off a horizontal table top 1.25 m high with a speed of 3.0 m s^{-1} . Ignoring air resistance, calculate (a) the time it takes to reach the floor, (b) how far from the table it lands, measured horizontally, and (c) its velocity just before it lands (magnitude and direction).

13. A football is kicked from level ground with a velocity of 20 m s^{-1} at 30° above the horizontal. Ignoring air resistance, calculate (a) the time of flight, (b) the range and (c) the maximum height. (d) State the speed of the ball at its highest point.

14. At the same instant, ball P is dropped from rest and ball Q is projected horizontally at 10 m s^{-1} , both from the same height above level ground. Ignoring air resistance, state and explain which ball hits the ground first.

15. A skydiver jumps from a hovering helicopter. Describe and explain how her velocity and acceleration change from the moment she jumps until she reaches terminal velocity, before her parachute opens.

1. (a) Acceleration is the rate of change of velocity.

(b) Distance is the total length of the path travelled and is a scalar. Displacement is the straight-line distance from the start point to the end point in a specified direction, and is a vector.

2. (a) Average speed = total distance $\div$ time = $400 \div 80 = 5.0 \text{ m s}^{-1}$.

(b) She finishes at her starting point, so her displacement is zero. Average velocity = displacement $\div$ time = $0 \div 80 = 0$.

3. (a) $a = (v - u)/t = (30 - 12)/6.0 = 3.0 \text{ m s}^{-2}$.

(b) $s = \frac{1}{2}(u + v)t = \frac{1}{2} \times (12 + 30) \times 6.0 = 126 \text{ m}$ (130 m to 2 s.f.).

4. Thinking distance (constant speed) = $25 \times 0.60 = 15 \text{ m}$.

Braking distance: $u = 25 \text{ m s}^{-1}$, $v = 0$, $a = -5.0 \text{ m s}^{-2}$.

$v^2 = u^2 + 2as$: $0 = 625 + 2 \times (-5.0) \times s$, so $10s = 625$ and $s = 62.5 \text{ m}$.

Stopping distance = $15 + 62.5 = 77.5 \text{ m}$, about **78 m**.

5. (a) A straight line from the origin rising to 8.0 m s^{-1} at 4.0 s; a horizontal line at 8.0 m s^{-1} from 4.0 s to 10.0 s; a straight line falling to zero at 12.0 s. The first line has gradient 2.0 m s^{-2} and the last -4.0 m s^{-2} .

(b) Distance = area under the graph.

Stage 1 (triangle): $\frac{1}{2} \times 4.0 \times 8.0 = 16 \text{ m}$.

Stage 2 (rectangle): $6.0 \times 8.0 = 48 \text{ m}$.

Stage 3 (triangle): $\frac{1}{2} \times 2.0 \times 8.0 = 8.0 \text{ m}$.

Total = $16 + 48 + 8.0 = 72 \text{ m}$.

(c) Average speed = $72 \div 12.0 = 6.0 \text{ m s}^{-1}$.

6. The gradient of a displacement-time graph is the velocity.

- (a) Gradient zero: the object is at rest (stationary).
 (b) Constant negative gradient: the object moves with constant velocity in the negative direction (back towards the start).
 (c) Gradient increasing: the velocity is increasing, so the object is accelerating.

7. Draw a velocity-time graph: a straight line from (0, u) to (t, v).

From the definition of acceleration, $a = (v - u)/t$, so the rise of the line is $v - u = at$.

The displacement is the area under the line. Split it into a rectangle of height u and width t, area ut, and a triangle on top with base t and height at, area $\frac{1}{2} \times t \times at = \frac{1}{2}at^2$.

Total area: $s = ut + \frac{1}{2}at^2$.

8. Take down as positive: $u = 0$, $s = 20.0$ m, $a = 9.81$ m s⁻².

(a) $s = \frac{1}{2}at^2$: $t^2 = 2 \times 20.0/9.81 = 4.08$, so $t = \mathbf{2.02$ s.

(b) $v^2 = u^2 + 2as = 0 + 2 \times 9.81 \times 20.0 = 392.4$, so $v = \mathbf{19.8$ m s⁻¹.

9. Take up as positive: $u = 15.0$ m s⁻¹, $a = -9.81$ m s⁻².

(a) At the top $v = 0$: $0 = 15.0^2 - 2 \times 9.81 \times h$, so $h = 225/19.62 = \mathbf{11.5}$ m.

(b) The ball returns to its starting height, so $s = 0$: $0 = 15.0t - 4.905t^2$, giving $t = 0$ (the throw) or $t = 15.0/4.905 = \mathbf{3.06}$ s.

(c) **9.81 m s⁻² vertically downwards**. The velocity is zero at the top, but the acceleration is not.

10. Take up as positive: $u = +12$ m s⁻¹, $a = -9.81$ m s⁻², $s = -30$ m (the sea is below the start).

(a) $v^2 = u^2 + 2as = 144 + 2 \times (-9.81) \times (-30) = 144 + 588.6 = 732.6$, so speed = $\sqrt{732.6} = \mathbf{27}$ m s⁻¹ (27.1 m s⁻¹), moving downwards.

(b) $v = -27.07$ m s⁻¹ (downwards). From $v = u + at$: $-27.07 = 12 - 9.81t$, so $t = 39.07/9.81 = \mathbf{3.98}$ s (4.0 s to 2 s.f.).

11. (a) The ball starts from rest, so $h = \frac{1}{2}gt^2$ and $g = 2h/t^2 = 2 \times 0.800/0.405^2 = 1.600/0.1640 = \mathbf{9.75}$ m s⁻².

(b) Using several heights, each repeated and averaged, reduces the effect of random errors, and the line of best fit averages the readings. Plotting h against t^2 gives a straight line through the origin because $h = (g/2)t^2$. The gradient equals g/2, so **g = 2 × gradient**. A constant systematic error in t or h shifts the line without much effect on its gradient.

(c) Any one of: delay in the electromagnet releasing the ball because of residual magnetism (times too long); the trapdoor taking time to open and stop the timer; h measured to the wrong point on the ball (zero error).

12. Vertically (down positive): $u_y = 0$, $s = 1.25$ m, $a = 9.81$ m s⁻².

(a) $1.25 = \frac{1}{2} \times 9.81 \times t^2$, so $t^2 = 0.2548$ and $t = \mathbf{0.505}$ s.

(b) Horizontally the velocity stays at 3.0 m s⁻¹: $x = 3.0 \times 0.505 = \mathbf{1.5}$ m.

(c) $v_y = u_y + at = 9.81 \times 0.505 = 4.95$ m s⁻¹ downwards; $v_x = 3.0$ m s⁻¹.

Speed = $\sqrt{(3.0^2 + 4.95^2)} = \sqrt{33.5} = \mathbf{5.8}$ m s⁻¹.

Angle below horizontal: $\tan \phi = 4.95/3.0 = 1.65$, so $\phi = \mathbf{59^\circ}$ below the horizontal.

13. $u_x = 20 \cos 30^\circ = 17.3$ m s⁻¹; $u_y = 20 \sin 30^\circ = 10.0$ m s⁻¹.

(a) Taking up as positive, it lands when the vertical displacement is zero: $t = 2u_y/g = 2 \times 10.0/9.81 = \mathbf{2.04}$ s.

(b) Range = $u_x \times t = 17.32 \times 2.039 = 35.3$ m, so **35** m.

(c) At the top $v_y = 0$: $H = u_y^2/(2g) = 100/19.62 = \mathbf{5.1}$ m.

(d) At the top the vertical component is zero and the horizontal component is unchanged, so the speed is **17 m s⁻¹** (17.3 m s⁻¹), horizontally.

14. They hit the ground at the same time. Both start with zero vertical velocity, both have the same vertical acceleration g , and both fall the same vertical distance, so from $s = \frac{1}{2}gt^2$ they take the same time. The horizontal and vertical motions are independent, so Q's horizontal velocity has no effect on its vertical motion; it only makes Q land further away.

15. At the instant she jumps her velocity is zero, so there is no air resistance. The only force is her weight and her acceleration is g (9.81 m s^{-2}) downwards.

As her velocity increases, the air resistance increases. The resultant downward force (weight – air resistance) decreases, so by $F = ma$ her acceleration decreases. Her velocity keeps increasing, but at a decreasing rate.

Eventually the air resistance becomes equal to her weight. The resultant force is zero, so her acceleration is zero and she falls at a constant velocity, the terminal velocity.

On a velocity-time graph this appears as a curve starting with gradient g that becomes less steep and levels off at the terminal velocity.