

GCSE Maths instant revision: key facts and methods for every topic

This is a fast revision guide for GCSE (9-1) Mathematics, also suitable for Cambridge IGCSE Mathematics 0580. Each topic gives you the key facts, one worked example and a "watch out" line on the mistake that costs most marks. Topics marked **(Higher)** are for the Higher tier (GCSE) or Extended tier (IGCSE). Use the key terms table near the end to check your vocabulary, then try the Quick check.

1. Number: place value, rounding and estimation

- To round, look at the digit after the last one you keep: 5 or more rounds up, 4 or less stays the same.
- Decimal places (d.p.) count digits after the decimal point. Significant figures (s.f.) count from the first non-zero digit.
- 0.004 07 to 2 s.f. is 0.0041. 38 250 to 2 s.f. is 38 000 (keep the zeros as place holders).
- To estimate, round every number to 1 significant figure, then calculate.
- Multiplying by a number between 0 and 1 makes a value smaller; dividing by a number between 0 and 1 makes it bigger.

Worked example: Estimate the value of $(49.7 \times 3.1) / 0.52$.

Round to 1 s.f.: $(50 \times 3) / 0.5 = 150 / 0.5 = 300$.

Dividing by 0.5 is the same as multiplying by 2, so the estimate is 300.

Watch out: when asked to estimate, you must show the rounded values. Writing only the final answer from a calculator gets no marks.

2. Number: bounds and error intervals

- A value rounded to the nearest unit could be up to half a unit above or below.
- Length = 12 cm to the nearest cm gives the error interval $11.5 \leq L < 12.5$.
- Truncated values: 4.7 truncated to 1 d.p. means $4.7 \leq x < 4.8$.
- **(Higher)** To make a sum or product as large as possible, use the upper bounds. To make a difference or a quotient as large as possible, use upper bound minus lower bound, and upper bound divided by lower bound.

Worked example: A rectangle measures 8 cm by 5 cm, both to the nearest cm. Find the upper bound of its area.

Upper bounds: 8.5 cm and 5.5 cm.

Upper bound of area = $8.5 \times 5.5 = 46.75 \text{ cm}^2$.

Watch out: the upper bound uses "<", not " $\leq$ ", because 12.5 itself would round to 13. In the exam you still use 12.5 in calculations.



3. Number: factors, multiples and primes

- A prime number has exactly two factors, 1 and itself. 1 is not prime. 2 is the only even prime.
- Every whole number above 1 can be written as a product of prime factors, for example $60 = 2^2 \times 3 \times 5$.
- Highest common factor (HCF): multiply the prime factors the numbers share.
- Lowest common multiple (LCM): multiply every prime factor that appears, using the highest power of each.
- Use HCF for "largest equal groups" problems and LCM for "when will they next coincide" problems.

Worked example: Find the HCF and LCM of 60 and 84.

$$60 = 2^2 \times 3 \times 5 \text{ and } 84 = 2^2 \times 3 \times 7$$

$$\text{HCF} = 2^2 \times 3 = 12$$

$$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$$

Watch out: give a product of primes in index form when asked, and check by multiplying back to the original number.

4. Number: fractions and decimals

- To add or subtract fractions, use a common denominator.
- To multiply, multiply the tops and multiply the bottoms. To divide, flip the second fraction and multiply.
- Change mixed numbers to improper fractions before multiplying or dividing: $2 \frac{1}{3} = \frac{7}{3}$.
- A fraction in its simplest form gives a terminating decimal only if the denominator has no prime factors other than 2 and 5.
- **(Higher)** Recurring decimal to fraction: let $x = 0.272727\dots$, then $100x = 27.2727\dots$, subtract to get $99x = 27$, so $x = \frac{27}{99} = \frac{3}{11}$.

Worked example: Work out $2 \frac{1}{3} \div 1 \frac{3}{4}$.

$$= \frac{7}{3} \div \frac{7}{4} = \frac{7}{3} \times \frac{4}{7} = \frac{28}{21} = \frac{4}{3} = 1 \frac{1}{3}$$

Watch out: never add the denominators. $\frac{3}{4} + \frac{2}{5}$ is $\frac{15}{20} + \frac{8}{20} = \frac{23}{20}$, not $\frac{5}{9}$.

5. Number: percentages

- Use multipliers. An increase of 15% is $\times 1.15$; a decrease of 15% is $\times 0.85$.
- Percentage change = $(\text{change} / \text{original}) \times 100$.
- Reverse percentage: divide the new amount by the multiplier to find the original.
- Compound interest or growth for n years: final amount = original $\times (\text{multiplier})^n$.
- Simple interest adds the same amount each year; compound interest adds interest on the interest.

Worked example 1 (reverse percentage): After a 20% increase a price is £84. Find the original price.

Original $\times 1.20 = 84$, so original $= 84 \div 1.20 = £70$.

Worked example 2 (compound interest): £5000 is invested at 3% per year compound interest. Find the value after 4 years.

$5000 \times 1.03^4 = 5000 \times 1.12550881 = £5627.54$

Watch out: in a reverse percentage, do not take 20% off £84. 20% of the new price is not the same as 20% of the original.

6. Number: indices and standard form

- $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$
- $a^0 = 1$ and $a^{-n} = 1/a^n$
- **(Higher)** $a^{1/n}$ is the n th root of a ; $a^{m/n} = (\text{nth root of } a)^m$. Take the root first, it keeps the numbers small.
- Standard form is $A \times 10^n$ where $1 \leq A < 10$ and n is an integer.
- Large numbers have positive n ; small numbers (less than 1) have negative n .

Worked example: (a) Evaluate $8^{2/3}$ and $27^{-1/3}$. (b) Work out $(3 \times 10^4) \times (5 \times 10^6)$ in standard form.

(a) $8^{2/3} = (\text{cube root of } 8)^2 = 2^2 = 4$. $27^{-1/3} = 1 / (\text{cube root of } 27) = 1/3$.

(b) $3 \times 5 = 15$ and $10^4 \times 10^6 = 10^{10}$, so $15 \times 10^{10} = 1.5 \times 10^{11}$

Watch out: 15×10^{10} is not in standard form because 15 is not between 1 and 10. Always adjust the final answer.

7. Number: surds (Higher)

- A surd is a root that cannot be simplified to a whole number or fraction, such as $\sqrt{2}$.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and $\sqrt{a/b} = \sqrt{a} / \sqrt{b}$.
- Simplify by finding the largest square factor: $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$.
- Rationalise a denominator $\sqrt{a}$ by multiplying top and bottom by $\sqrt{a}$.
- For a denominator such as $3 + \sqrt{2}$, multiply top and bottom by $3 - \sqrt{2}$.

Worked example: (a) Rationalise $6/\sqrt{3}$. (b) Expand $(3 + \sqrt{2})(3 - \sqrt{2})$.

(a) $6/\sqrt{3} \times \sqrt{3}/\sqrt{3} = 6\sqrt{3}/3 = 2\sqrt{3}$

(b) $9 - 3\sqrt{2} + 3\sqrt{2} - 2 = 7$

Watch out: $\sqrt{a} + \sqrt{b}$ is not $\sqrt{a+b}$. $\sqrt{9} + \sqrt{16} = 7$, but $\sqrt{25} = 5$.

8. Algebra: expanding and factorising

- Expand single brackets by multiplying every term inside by the term outside.
- Expand double brackets by multiplying each term in the first by each term in the second (a grid helps).
- Factorise by taking out the highest common factor: $6x^2 + 9x = 3x(2x + 3)$.
- Quadratics $x^2 + bx + c$: find two numbers that multiply to c and add to b .
- Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$.
- **(Higher)** For $ax^2 + bx + c$, find two numbers that multiply to ac and add to b , then split the middle term.

Worked example: (a) Expand $(x + 3)(x - 5)$. (b) Factorise $2x^2 + 7x + 3$.

(a) $x^2 - 5x + 3x - 15 = x^2 - 2x - 15$

(b) $ac = 6$; 6 and 1 multiply to 6 and add to 7. $2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$

Watch out: $(x + 3)^2$ is $x^2 + 6x + 9$, not $x^2 + 9$. Write it as $(x + 3)(x + 3)$ and expand.

9. Algebra: linear equations and changing the subject

- Do the same to both sides. Expand brackets first, collect the unknown on the side where it is larger, then divide.
- With fractions, multiply every term by the common denominator.
- Changing the subject uses the same steps as solving an equation.
- **(Higher)** If the new subject appears twice, collect those terms on one side and factorise it out.

Worked example 1: Solve $5(x - 2) = 3x + 8$.

$5x - 10 = 3x + 8$, so $2x = 18$ and $x = 9$.

Worked example 2 (Higher): Make x the subject of $y = (x + 3)/(x - 2)$.

$$y(x - 2) = x + 3$$

$$yx - 2y = x + 3$$

$$yx - x = 3 + 2y$$

$$x(y - 1) = 3 + 2y$$

$$x = (3 + 2y)/(y - 1)$$

Watch out: when you multiply out a bracket with a minus in front, every sign inside changes: $-2(x - 4) = -2x + 8$.

10. Algebra: quadratic equations

- Rearrange to $= 0$ first.
- Method 1, factorise: if $(x - 2)(x - 3) = 0$ then $x = 2$ or $x = 3$.
- Method 2, the quadratic formula for $ax^2 + bx + c = 0$: $x = (-b \pm \sqrt{b^2 - 4ac}) / 2a$. Use it when the question asks for decimal answers.
- **(Higher)** Method 3, completing the square: $x^2 + bx + c = (x + b/2)^2 - (b/2)^2 + c$.

- The completed square form $(x + p)^2 + q$ shows the turning point $(-p, q)$.

Worked example 1: Solve $2x^2 + 3x - 4 = 0$, giving answers to 2 d.p.

$$a = 2, b = 3, c = -4. b^2 - 4ac = 9 + 32 = 41$$

$$x = (-3 \pm \sqrt{41}) / 4, \text{ so } x = 0.85 \text{ or } x = -2.35$$

Worked example 2: Write $x^2 + 6x + 2$ in the form $(x + p)^2 + q$ and state the turning point.

$$(x + 3)^2 - 9 + 2 = (x + 3)^2 - 7. \text{ Turning point } (-3, -7), \text{ a minimum.}$$

Watch out: with a negative c , $-4ac$ becomes positive. Write $b^2 - 4ac$ out in full before you press the calculator.

11. Algebra: simultaneous equations

- Elimination: make the coefficients of one letter equal, then add (different signs) or subtract (same signs).
- Substitution: rearrange one equation for one letter and substitute into the other.
- Always substitute back to find the second letter, and check in the other equation.
- **(Higher)** For one linear and one quadratic equation, substitute the linear one into the quadratic. You usually get two pairs of answers.
- Graphically, the solutions are the points where the two graphs cross.

Worked example 1: Solve $3x + 2y = 16$ and $x - 2y = 0$.

Add: $4x = 16$, so $x = 4$. Then $4 - 2y = 0$, so $y = 2$. Check: $12 + 4 = 16$.

Worked example 2 (Higher): Solve $y = x + 1$ and $x^2 + y^2 = 13$.

$$x^2 + (x + 1)^2 = 13, \text{ so } 2x^2 + 2x - 12 = 0, \text{ so } x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0, \text{ so } x = 2, y = 3 \text{ or } x = -3, y = -2$$

Watch out: give the answers as matched pairs. " $x = 2, y = 3$ " and " $x = -3, y = -2$ " are not interchangeable.

12. Algebra: inequalities

- Solve like an equation, but if you multiply or divide by a negative number, reverse the inequality sign.
- On a number line, an open circle means not included ($<$ or $>$); a filled circle means included ($\leq$ or $\geq$).
- Integer solutions: list the whole numbers that fit. For $-2 < n \leq 3$ they are $-1, 0, 1, 2, 3$.
- Regions on graphs: draw the boundary line (dashed for $<$ or $>$, solid for $\leq$ or $\geq$) and test a point such as $(0, 0)$.
- **(Higher)** Quadratic inequalities: find the roots, sketch the curve, and read off where it is above or below the x -axis.

Worked example (Higher): Solve $x^2 - x - 12 < 0$.

$(x - 4)(x + 3) = 0$ gives roots -3 and 4 . The curve is U-shaped, so it is below the axis between the roots.

Solution: $-3 < x < 4$

Watch out: $-2x \leq 6$ gives $x \geq -3$, not $x \leq -3$. Dividing by -2 flips the sign.

13. Algebra: sequences

- Arithmetic (linear) sequence: same difference each time. n th term = $dn + (\text{first term} - d)$, where d is the common difference.
- Geometric sequence: multiply by the same ratio r each time, for example 3, 6, 12, 24.
- Fibonacci-type: each term is the sum of the previous two.
- To test whether a number is in a sequence, set the n th term equal to it and check whether n is a positive whole number.
- **(Higher)** Quadratic sequence: the second differences are constant. Half the second difference is the coefficient of n^2 .

Worked example 1: Find the n th term of 5, 8, 11, 14, ...

$d = 3$, so $3n$. First term $5 = 3 + 2$, so n th term = $3n + 2$.

Worked example 2 (Higher): Find the n th term of 3, 8, 15, 24, ...

First differences 5, 7, 9; second difference 2, so the sequence starts with n^2 .

Subtract n^2 (1, 4, 9, 16): left with 2, 4, 6, 8, which is $2n$.

n th term = $n^2 + 2n$. Check $n = 4$: $16 + 8 = 24$.

Watch out: the n th term of 5, 8, 11 is not $n + 3$. The number in front of n is always the common difference.

14. Algebra: straight-line graphs

- The equation $y = mx + c$ has gradient m and y -intercept c .
- Gradient = change in y / change in x between two points.
- Parallel lines have the same gradient.
- **(Higher)** Perpendicular lines have gradients that multiply to -1 : if one gradient is m , the other is $-1/m$.
- Midpoint of (x_1, y_1) and (x_2, y_2) is $((x_1 + x_2)/2, (y_1 + y_2)/2)$. Length uses Pythagoras.

Worked example: Find the equation of the line through (1, 3) and (4, 9).

$m = (9 - 3)/(4 - 1) = 6/3 = 2$

Substitute (1, 3) into $y = 2x + c$: $3 = 2 + c$, so $c = 1$.

$y = 2x + 1$

Watch out: rearrange to $y = \dots$ before reading off the gradient. For $2y = 6x - 4$, the gradient is 3, not 6.

15. Algebra: other graphs

- Quadratic $y = ax^2 + bx + c$: a U shape if $a > 0$, an upside-down U if $a < 0$. Roots are where $y = 0$.
- Cubic $y = x^3$: an S shape through the origin.
- Reciprocal $y = 1/x$: two curves, never touching the axes (the axes are asymptotes).
- **(Higher)** Exponential $y = k^x$ ($k > 1$): passes through (0, 1) and rises more and more steeply; never goes below the x-axis.
- **(Higher)** The circle $x^2 + y^2 = r^2$ has centre (0, 0) and radius r . A tangent is perpendicular to the radius at that point.
- On a distance-time graph, gradient = speed. On a velocity-time graph, gradient = acceleration and area under the graph = distance.

Worked example: For $y = x^2 - 4x - 5$, find the roots, the y-intercept and the turning point.

Roots: $(x - 5)(x + 1) = 0$, so $x = 5$ and $x = -1$.

y-intercept: $x = 0$ gives $y = -5$.

The turning point is halfway between the roots, at $x = 2$: $y = 4 - 8 - 5 = -9$. Turning point (2, -9).

Watch out: when you plot a curve from a table, join the points with a smooth curve, not straight line segments.

16. Algebra: functions (Higher)

- $f(x)$ is a rule. $f(3)$ means substitute $x = 3$.
- Composite function $fg(x)$ means do g first, then f .
- Inverse function $f^{-1}(x)$ undoes f . Write $y = f(x)$, swap x and y , and rearrange for y .
- $f^{-1}f(x) = x$.

Worked example: $f(x) = 2x + 1$ and $g(x) = x^2$. Find $fg(3)$, $gf(3)$ and $f^{-1}(x)$.

$fg(3) = f(9) = 19$. $gf(3) = g(7) = 49$.

$y = 2x + 1$; swap: $x = 2y + 1$, so $y = (x - 1)/2$. $f^{-1}(x) = (x - 1)/2$.

Watch out: $fg(x)$ is not $f(x) \times g(x)$. Work from the inside (right-hand letter) outwards.

17. Ratio and proportion: ratio

- Simplify a ratio by dividing all parts by their HCF. Use the same units first.
- To share in a ratio: add the parts, find one part, then multiply.
- A ratio $a : b$ means the first quantity is $a/(a + b)$ of the total.
- To write a ratio in the form $1 : n$, divide both parts by the first.
- Best buy: compare the cost per unit or the amount per £1.

Worked example: Share £120 in the ratio 3 : 5.

Total parts = 8. One part = $120 \div 8 = £15$.

$3 \times 15 = £45$ and $5 \times 15 = £75$. Check: $45 + 75 = 120$.

Watch out: if the question gives the difference between the shares (for example "B gets £30 more than A"), the difference matches the difference in parts, not the total.

18. Ratio and proportion: direct and inverse proportion

- y is directly proportional to x : $y = kx$. The graph is a straight line through the origin.
- y is inversely proportional to x : $y = k/x$. When x doubles, y halves.
- **(Higher)** Other forms: $y = kx^2$, $y = k\sqrt{x}$, $y = k/x^2$.
- Method: write the equation with k , substitute the given pair to find k , then use the equation.

Worked example: y is directly proportional to x^2 . When $x = 2$, $y = 12$. Find y when $x = 5$.

$$y = kx^2: 12 = k \times 4, \text{ so } k = 3.$$

$$y = 3 \times 5^2 = 75$$

Watch out: read the wording carefully: "proportional to the square of x " means x^2 , and "inversely" means divide.

19. Ratio and proportion: compound measures

- Speed = distance $\div$ time
- Density = mass $\div$ volume
- Pressure = force $\div$ area
- Rates such as flow (litres per minute) and unit pricing work the same way.
- Average speed for a whole journey = total distance $\div$ total time.

Worked example: A car travels 150 km in 2 hours 30 minutes. Find its average speed.

$$2 \text{ hours } 30 \text{ minutes} = 2.5 \text{ hours.}$$

$$\text{Speed} = 150 \div 2.5 = 60 \text{ km/h}$$

Watch out: 2 hours 30 minutes is 2.5 hours, not 2.3 hours. Convert minutes by dividing by 60.

20. Geometry: angles and polygons

- Angles on a straight line add to 180° . Angles around a point add to 360° . Vertically opposite angles are equal.
- Angles in a triangle add to 180° ; in a quadrilateral, 360° .
- Parallel lines: alternate angles are equal, corresponding angles are equal, co-interior angles add to 180° .
- Sum of interior angles of an n -sided polygon = $(n - 2) \times 180^\circ$.
- Exterior angles of any polygon add to 360° . For a regular polygon, each exterior angle = $360^\circ \div n$, and interior + exterior = 180° .

Worked example: Each exterior angle of a regular polygon is 24° . Find the number of sides and the size of each interior angle.

$$n = 360 \div 24 = 15 \text{ sides.}$$

$$\text{Interior angle} = 180 - 24 = 156^\circ.$$

Watch out: give a reason for every angle you find, using the correct words ("alternate angles are equal", not "Z angles").

21. Geometry: perimeter, area and circles

- Rectangle: length $\times$ width. Triangle: $\frac{1}{2} \times \text{base} \times \text{perpendicular height}$.
- Parallelogram: base $\times$ perpendicular height. Trapezium: $\frac{1}{2} (a + b) h$.
- Circle: circumference = $\pi d = 2\pi r$; area = πr^2 .
- Sector with angle θ : arc length = $(\theta/360) \times 2\pi r$; area = $(\theta/360) \times \pi r^2$.
- Perimeter of a sector = arc length + 2 radii.
- Area units: $1 \text{ m}^2 = 10\,000 \text{ cm}^2$ (because 100×100).

Worked example: A sector has radius 6 cm and angle 120° . Find its area and arc length.

$$\text{Area} = (120/360) \times \pi \times 6^2 = \frac{1}{3} \times 36\pi = 12\pi = 37.7 \text{ cm}^2 \text{ (3 s.f.)}$$

$$\text{Arc} = (120/360) \times 2 \times \pi \times 6 = 4\pi = 12.6 \text{ cm (3 s.f.)}$$

Watch out: if a question says "leave your answer in terms of π ", write 12π , not 37.7.

22. Geometry: volume and surface area

- Prism (including a cylinder): volume = area of cross-section $\times$ length. Cylinder: $V = \pi r^2 h$.
- Pyramid and cone: volume = $\frac{1}{3} \times \text{base area} \times \text{height}$. Cone: $V = \frac{1}{3} \pi r^2 h$.
- Sphere: $V = \frac{4}{3} \pi r^3$, surface area = $4\pi r^2$.
- Cone curved surface area = $\pi r l$, where l is the slant height. Cylinder curved surface area = $2\pi r h$.
- Surface area = total area of all faces; sketch the net if it helps.
- Volume units: $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$; 1 litre = 1000 cm^3 .

Worked example: Find the volume of a cylinder with radius 3 cm and height 10 cm.

$$V = \pi \times 3^2 \times 10 = 90\pi = 283 \text{ cm}^3 \text{ (3 s.f.)}$$

Watch out: the cone and sphere formulae are usually given in the question or on the formula list, but you must still know which length is which: l is the slant height, h is the vertical height.

23. Geometry: Pythagoras and trigonometry

Pythagoras' theorem: in a right-angled triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse (the longest side, opposite the right angle).

Trigonometric ratios: $\sin \theta = \text{opposite/hypotenuse}$, $\cos \theta = \text{adjacent/hypotenuse}$, $\tan \theta = \text{opposite/adjacent}$ (SOH CAH TOA).

- Finding the hypotenuse: add the squares. Finding a shorter side: subtract.

- To find an angle, use the inverse function: $\theta = \sin^{-1}(\text{opp/hyp})$, and so on.
- Exact values: $\sin 30^\circ = 1/2$, $\cos 60^\circ = 1/2$, $\tan 45^\circ = 1$, $\sin 60^\circ = \cos 30^\circ = \sqrt{3}/2$, $\sin 45^\circ = \cos 45^\circ = \sqrt{2}/2$, $\tan 60^\circ = \sqrt{3}$, $\tan 30^\circ = 1/\sqrt{3}$, $\sin 0^\circ = 0$, $\cos 0^\circ = 1$.
- Angle of elevation is measured up from the horizontal; angle of depression is measured down from the horizontal.
- **(Higher)** In 3D, find the right-angled triangle you need, often using a diagonal of the base first.

Worked example: (a) A right-angled triangle has shorter sides 5 cm and 12 cm. Find the hypotenuse. (b) In another right-angled triangle, the side opposite angle θ is 7 cm and the hypotenuse is 10 cm. Find θ .

(a) $c^2 = 25 + 144 = 169$, so $c = 13$ cm.

(b) $\sin \theta = 7/10 = 0.7$, so $\theta = \sin^{-1}(0.7) = 44.4^\circ$ (1 d.p.)

Watch out: check your calculator is in degrees (a small D on the screen), not radians.

24. Geometry: sine rule, cosine rule and area of a triangle (Higher)

Sine rule: $a/\sin A = b/\sin B = c/\sin C$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of a triangle: $1/2 ab \sin C$

Small letters are sides; capital letters are the angles opposite them.

- Use the sine rule when you know a side and its opposite angle.
- Use the cosine rule when you know two sides and the angle between them (to find the third side), or all three sides (to find an angle).
- To find an angle with the cosine rule: $\cos A = (b^2 + c^2 - a^2) / 2bc$.

Worked example: In triangle ABC, $b = 8$ cm, $c = 11$ cm and $A = 40^\circ$. Find side a and the area.

$$a^2 = 64 + 121 - 2 \times 8 \times 11 \times \cos 40^\circ = 185 - 176 \times 0.7660 = 185 - 134.82 = 50.18$$

$$a = 7.08 \text{ cm (3 s.f.)}$$

$$\text{Area} = 1/2 \times 8 \times 11 \times \sin 40^\circ = 44 \times 0.6428 = 28.3 \text{ cm}^2 \text{ (3 s.f.)}$$

Watch out: work out $2bc \cos A$ as one term and subtract it at the end. A common error is to calculate $(185 - 176) \times \cos 40^\circ$.

25. Geometry: circle theorems (Higher)

- The angle at the centre is twice the angle at the circumference subtended by the same arc.
- The angle in a semicircle is 90° .
- Angles in the same segment are equal.
- Opposite angles of a cyclic quadrilateral add to 180° .
- A tangent meets a radius at 90° .
- Two tangents from the same external point are equal in length.

- The perpendicular from the centre to a chord bisects the chord.
- Alternate segment theorem: the angle between a tangent and a chord equals the angle in the alternate segment.

Worked example: The angle at the centre of a circle subtended by arc AB is 130° . Find the angle at the circumference subtended by the same arc, and the opposite angle of a cyclic quadrilateral that contains that angle.

Angle at circumference = $130 \div 2 = 65^\circ$.

Opposite angle of the cyclic quadrilateral = $180 - 65 = 115^\circ$.

Watch out: state the theorem in full as your reason. "Angle at centre is twice the angle at the circumference" earns the mark; "circle theorem" does not.

26. Geometry: transformations

- Translation: described by a column vector (x to the right, y up).
- Reflection: described by the equation of the mirror line, for example $y = x$ or $x = 2$.
- Rotation: give the angle, the direction (clockwise or anticlockwise) and the centre.
- Enlargement: give the scale factor and the centre. A fractional scale factor makes the shape smaller. **(Higher)** A negative scale factor puts the image on the opposite side of the centre, upside down.
- Rotation of 90° clockwise about the origin sends (x, y) to (y, -x).
- Translations, reflections and rotations give congruent images; enlargements give similar images.

Worked example: Enlarge the point (3, 2) by scale factor 2, centre (1, 1).

Vector from centre to point: $(3 - 1, 2 - 1) = (2, 1)$.

Multiply by 2: (4, 2). Add to the centre: $(1 + 4, 1 + 2) = (5, 3)$.

Watch out: "describe fully a single transformation" means one transformation with all its details. Two transformations, or a rotation with no centre, scores zero.

27. Geometry: vectors

- A vector has size (magnitude) and direction. Write it as a column vector or a bold letter (underlined when handwritten).
- Add column vectors by adding the top numbers and the bottom numbers.
- Magnitude of the vector with components (x, y) is $\sqrt{x^2 + y^2}$.
- Vector $\mathbf{AB} = -(\text{vector BA})$. To go from A to B, follow any route of known vectors.
- **(Higher)** Vectors are parallel if one is a multiple of the other. Points are collinear if two vectors joining them are parallel and share a point.

Worked example: O is the origin, vector $\mathbf{OA} = \mathbf{a}$ and vector $\mathbf{OB} = \mathbf{b}$. M is the midpoint of AB. Find vector OM.

$\mathbf{AB} = \mathbf{AO} + \mathbf{OB} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$

$$OM = OA + \frac{1}{2} AB = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{a} + \mathbf{b})$$

Watch out: the direction matters. Going against an arrow changes the sign of the vector.

28. Geometry: congruence and similarity

- Congruent shapes are identical in shape and size.
- Conditions for congruent triangles: SSS, SAS, ASA (or AAS), RHS.
- Similar shapes have equal angles and sides in the same ratio.
- **(Higher)** If the length scale factor is k , the area scale factor is k^2 and the volume scale factor is k^3 .

Worked example (Higher): Two similar cylinders have heights 4 cm and 6 cm. The smaller has volume 32 cm^3 . Find the volume of the larger.

Length scale factor $k = 6 \div 4 = 1.5$. Volume scale factor $= 1.5^3 = 3.375$.

Volume $= 32 \times 3.375 = 108 \text{ cm}^3$

Watch out: AAA does not prove congruence; it only proves similarity.

29. Geometry: bearings, scale drawings, constructions and loci

- A bearing is measured clockwise from north and written with three figures, for example 070° .
- Back bearing: add 180° (if the bearing is less than 180°) or subtract 180° (if it is more).
- Scale 1 : 50 000 means 1 cm on the map is 50 000 cm = 500 m in real life.
- Constructions with ruler and compasses: perpendicular bisector of a line, angle bisector, perpendicular from a point. Leave the construction arcs.
- Locus of points a fixed distance from a point: a circle. Equidistant from two points: the perpendicular bisector. Equidistant from two lines: the angle bisector.

Worked example: The bearing of B from A is 070° . Find the bearing of A from B.

$070 + 180 = 250^\circ$.

Watch out: "the bearing of B from A" means stand at A, face north and turn clockwise to B. Draw the north line at the point after "from".

30. Probability

- Probability is between 0 (impossible) and 1 (certain). Write it as a fraction, decimal or percentage, never as "1 in 4" or a ratio.
- $P(\text{not } A) = 1 - P(A)$. The probabilities of all possible outcomes add to 1.
- Mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$.
- Independent events: $P(A \text{ and } B) = P(A) \times P(B)$.
- Expected frequency = probability $\times$ number of trials. Relative frequency = number of successes $\div$ number of trials; more trials give a better estimate.

- Tree diagrams: multiply along the branches, add between the final outcomes. Without replacement, the numbers on the second set of branches change.
- Venn diagrams: $A \cap B$ is "A and B", $A \cup B$ is "A or B (or both)", A' is "not A".
- **(Higher)** Conditional probability: $P(A \text{ given } B) = P(A \text{ and } B) \div P(B)$.

Worked example: A bag holds 5 red and 3 blue counters. Two are taken without replacement. Find the probability that (a) both are red, (b) one of each colour.

(a) $5/8 \times 4/7 = 20/56 = 5/14$

(b) Red then blue + blue then red = $5/8 \times 3/7 + 3/8 \times 5/7 = 15/56 + 15/56 = 30/56 = 15/28$

Watch out: "one of each" has two routes through the tree. Missing the second route is the most common error.

31. Statistics: averages and spread

- Mean = total of values $\div$ number of values. Median = middle value when in order. Mode = most common value.
- Range = largest - smallest. Interquartile range (IQR) = upper quartile - lower quartile; it ignores extreme values.
- From a frequency table: mean = $\Sigma fx \div \Sigma f$.
- For grouped data, use the midpoint of each class to find an estimate of the mean. The modal class is the class with the highest frequency.
- To compare two sets of data, compare an average and a measure of spread, in context.

Worked example: Scores 1, 2, 3, 4 have frequencies 3, 5, 8, 4. Find the mean, median and mode.

$\Sigma f = 20$; $\Sigma fx = 3 + 10 + 24 + 16 = 53$. Mean = $53 \div 20 = 2.65$.

The median is between the 10th and 11th values. Running totals 3, 8, 16: both are 3, so median = 3.

Mode = 3 (highest frequency).

Watch out: for grouped data the mean is only an estimate, and the question will expect the word "estimate" in your method.

32. Statistics: charts and diagrams

- Pie chart angle = $(\text{frequency} \div \text{total}) \times 360^\circ$.
- Scatter graphs show correlation (positive, negative or none). Draw a line of best fit through the middle of the points. Correlation does not prove one thing causes the other.
- Using the line of best fit outside the range of the data (extrapolation) is unreliable.
- **(Higher)** Cumulative frequency: plot running totals at the upper class boundary and join with a smooth curve. Read the median at half the total, and the quartiles at a quarter and three quarters.
- **(Higher)** Box plot: shows minimum, lower quartile, median, upper quartile and maximum.

- **(Higher)** Histogram with unequal classes: frequency density = frequency $\div$ class width. The area of each bar is the frequency.
- Sampling: a random sample gives every member an equal chance. A larger sample is more reliable.

Worked example (Higher): The class $10 < x \leq 30$ has a frequency of 12. Find the height of its histogram bar.

Class width = $30 - 10 = 20$. Frequency density = $12 \div 20 = 0.6$.

Watch out: on a histogram the vertical axis is frequency density, not frequency. Find a frequency by multiplying frequency density by class width.

Key terms

Term	Meaning
Integer	A whole number, positive, negative or zero.
Prime number	A number with exactly two factors, 1 and itself.
Reciprocal	1 divided by the number. The reciprocal of 4 is $\frac{1}{4}$; of $\frac{2}{3}$ is $\frac{3}{2}$.
Significant figure	A digit counted from the first non-zero digit of a number.
Standard form	$A \times 10^n$ with $1 \leq A < 10$ and n an integer.
Surd	An irrational root, such as $\sqrt{3}$, left in root form.
Expression	A collection of terms with no equals sign, such as $3x + 2$.
Equation	A statement that two expressions are equal, true for particular values.
Identity	A statement true for all values, written with $\equiv$.
Formula	A rule linking two or more variables, such as $A = \pi r^2$.
Coefficient	The number in front of a letter. In $5x^2$, the coefficient is 5.
Gradient	The steepness of a line: change in $y \div$ change in x .
Root	A solution of an equation; on a graph, a point where it crosses the x -axis.
Hypotenuse	The longest side of a right-angled triangle, opposite the right angle.
Congruent	Exactly the same shape and size.
Similar	The same shape, with corresponding sides in the same ratio.
Subtend	The angle an arc or chord makes at a point.
Locus	The set of all points that obey a rule.
Mutually exclusive	Events that cannot happen at the same time.
Independent	Events where the outcome of one does not affect the other.
Interquartile range	Upper quartile minus lower quartile; the spread of the middle half of the data.
Frequency density	Frequency $\div$ class width; the height of a histogram bar.

Quick check

A mixed set covering all five strands. Show your working. Questions 12 to 15 include Higher tier content.

1. Write 0.000 405 in standard form.
2. Increase £240 by 15%.
3. Simplify $(2x^3y)^4$.
4. Solve $4(2x - 1) = 3x + 11$.
5. Find the n th term of the sequence 7, 11, 15, 19, ...
6. Share 84 sweets between Ali and Ben in the ratio 2 : 5.
7. Find the equation of the straight line through (0, -2) and (3, 4).
8. Calculate the size of each interior angle of a regular octagon.
9. A right-angled triangle has shorter sides 8 cm and 15 cm. Calculate the length of the hypotenuse.
10. Find the mean, median and range of 4, 7, 7, 9, 13.
11. The probability that it rains on any day is 0.3, independently of other days. Calculate the probability that it rains on exactly one of two days.
12. In a right-angled triangle, the side adjacent to angle θ is 6 cm and the hypotenuse is 9 cm. Calculate θ to 1 decimal place.
13. Solve $x^2 + 2x - 15 = 0$.
14. Calculate the volume of a sphere of radius 3 cm. Give your answer to 3 significant figures.
15. y is inversely proportional to x . When $x = 3$, $y = 10$. Find y when $x = 6$.

1. Move the decimal point 4 places to the right to get 4.05, which is between 1 and 10. The number is small, so the power is negative: 4.05×10^{-4} .
2. Multiplier for a 15% increase is 1.15. $240 \times 1.15 = \text{£}276$.
3. Raise each part to the power 4: $2^4 = 16$, $(x^3)^4 = x^{12}$, y^4 . Answer: $16x^{12}y^4$.
4. $8x - 4 = 3x + 11$, so $5x = 15$, so $x = 3$. Check: $4(6 - 1) = 20$ and $9 + 11 = 20$.
5. Common difference 4, so $4n$. First term $7 = 4 + 3$, so n th term $= 4n + 3$.
6. Total parts $2 + 5 = 7$. One part $= 84 \div 7 = 12$. Ali gets $2 \times 12 = 24$; Ben gets $5 \times 12 = 60$. Check: $24 + 60 = 84$.
7. Gradient $= (4 - (-2)) \div (3 - 0) = 6 \div 3 = 2$. The y -intercept is -2 because the line passes through (0, -2). Equation: $y = 2x - 2$.
8. Exterior angle $= 360 \div 8 = 45^\circ$. Interior angle $= 180 - 45 = 135^\circ$. (Or: sum $= (8 - 2) \times 180 = 1080^\circ$, and $1080 \div 8 = 135^\circ$.)
9. $c^2 = 8^2 + 15^2 = 64 + 225 = 289$, so $c = \sqrt{289} = 17$ cm.
10. Mean $= (4 + 7 + 7 + 9 + 13) \div 5 = 40 \div 5 = 8$. Median = middle (3rd) value = 7. Range $= 13 - 4 = 9$.
11. Two routes: rain then dry, or dry then rain. $P(\text{dry}) = 1 - 0.3 = 0.7$.

$$0.3 \times 0.7 + 0.7 \times 0.3 = 0.21 + 0.21 = 0.42$$

12. Adjacent and hypotenuse, so use cosine: $\cos \theta = 6/9 = 0.6667$. $\theta = \cos^{-1}(0.6667) = 48.2^\circ$.

13. Find two numbers that multiply to -15 and add to 2: 5 and -3. $(x + 5)(x - 3) = 0$, so $x = -5$ or $x = 3$.

14. $V = \frac{4}{3} \times \pi \times 3^3 = \frac{4}{3} \times \pi \times 27 = 36\pi = 113.097... = 113 \text{ cm}^3$ (3 s.f.)

15. $y = k/x$. $10 = k/3$, so $k = 30$. When $x = 6$, $y = 30/6 = 5$.