

Fractions and decimals: the four operations

(IGCSE Maths worksheet with answers)

This worksheet is for Cambridge IGCSE Mathematics 0580 (Extended). Paper 2 and Paper 4 both test fractions and decimals, and on the non-calculator paper you must show every step. Read the rules, study the worked example, then try the 25 questions. They start easy and get harder. Full worked answers are at the end.

What you need to know

Simplifying and equivalent fractions

Multiplying or dividing the numerator and the denominator by the same number gives an equivalent fraction. A fraction is in its simplest form when the numerator and denominator have no common factor other than 1. Always give final answers in their simplest form, and write improper fractions as mixed numbers unless the question says otherwise.

Rule 1: adding and subtracting fractions

Write both fractions over a common denominator (the lowest common multiple of the denominators is best), then add or subtract the numerators only. The denominator does not change.

For mixed numbers, change them to improper fractions first. This avoids mistakes when the fraction part of the second number is larger than the first.

Rule 2: multiplying fractions

Multiply the numerators together and the denominators together. Cancel common factors before multiplying to keep the numbers small.

Mixed numbers must be changed to improper fractions first. Never multiply the whole numbers and the fraction parts separately.

Rule 3: dividing fractions

Dividing by a fraction is the same as multiplying by its reciprocal. Keep the first fraction, change $\div$ to $\times$, and turn the second fraction upside down.

The reciprocal of a/b is b/a . The reciprocal of a whole number n is $1/n$.

Decimal operations without a calculator

- **Adding and subtracting:** line up the decimal points. Fill empty places with zeros, for example $12.48 - 5.90$.
- **Multiplying:** ignore the decimal points and multiply the whole numbers. Then count the total number of decimal places in the two numbers. The answer has that many decimal places. For example, 0.3×0.02 : $3 \times 2 = 6$, and there are $1 + 2 = 3$ decimal places, so the answer is 0.006 .
- **Dividing:** multiply both numbers by the same power of 10 until the number you are dividing by is a whole number, then divide. For example, $1.56 \div 0.4 = 15.6 \div 4 = 3.9$.



Converting between fractions, decimals and percentages

- Fraction to decimal: divide the numerator by the denominator (short division), or find an equivalent fraction with denominator 10, 100 or 1000. For example, $\frac{3}{25} = \frac{12}{100} = 0.12$.
- Decimal to fraction: write it over 10, 100 or 1000 and simplify. For example, $0.125 = \frac{125}{1000} = \frac{1}{8}$.
- Decimal to percentage: multiply by 100. Percentage to decimal: divide by 100.
- Percentage to fraction: write it over 100 and simplify. For example, $35\% = \frac{35}{100} = \frac{7}{20}$.

Learn these common equivalents: $\frac{1}{2} = 0.5 = 50\%$, $\frac{1}{4} = 0.25 = 25\%$, $\frac{3}{4} = 0.75 = 75\%$, $\frac{1}{5} = 0.2 = 20\%$, $\frac{1}{8} = 0.125 = 12.5\%$, $\frac{1}{3} = 0.333... = 33 \frac{1}{3} \%$, $\frac{2}{3} = 0.666... = 66 \frac{2}{3} \%$.

Terminating and recurring decimals

A fraction in its simplest form gives a terminating decimal only if the prime factors of its denominator are 2 and 5 only. Any other prime factor (3, 7, 11 and so on) gives a recurring decimal. In this worksheet a recurring decimal is written with its repeating block shown at least twice followed by dots, for example 0.272727... In the exam, dots are placed over the first and last digits of the repeating block.

Rule 4: converting a recurring decimal to a fraction

1. Let x equal the recurring decimal.
2. Multiply x by 10, 100 or 1000 (one zero for each digit in the repeating block) so that the repeating parts line up.
3. If the decimal has non-repeating digits before the repeating block, first multiply by a power of 10 to move them in front of the decimal point, so you subtract two equations whose decimal parts are identical.
4. Subtract to remove the recurring part, solve for x , and simplify.

Worked example

(a) Work out $2 \frac{2}{5} \div 1 \frac{1}{5} - \frac{3}{4}$.

Change to improper fractions: $2 \frac{2}{5} = \frac{12}{5}$ and $1 \frac{1}{5} = \frac{6}{5}$.

Divide first (order of operations): $\frac{12}{5} \div \frac{6}{5} = \frac{12}{5} \times \frac{5}{6} = \frac{60}{30} = 2$.

Then subtract: $2 - \frac{3}{4} = \frac{8}{4} - \frac{3}{4} = \frac{5}{4} = 1 \frac{1}{4}$.

(b) Convert 0.3181818... to a fraction in its simplest form.

Let $x = 0.3181818...$

The repeating block is 18 (two digits), and the 3 does not repeat.

$10x = 3.181818...$

$1000x = 318.181818...$

Subtract: $1000x - 10x = 318.181818... - 3.181818...$, so $990x = 315$.

$x = \frac{315}{990}$. Divide top and bottom by 45: $x = \frac{7}{22}$.

Check: $22 = 2 \times 11$, and the factor 11 explains why the decimal recurs.

Exam tip

On the non-calculator paper, the instruction "show all your working" means the examiner must see the common denominator or the reciprocal step. A correct final answer with no method can lose

marks. When a question says "show that", you must reach the given answer exactly, so write every line.

Questions

Do not use a calculator. Give fractions in their simplest form.

1. Work out $\frac{3}{4} + \frac{2}{5}$.
2. Work out $\frac{5}{6} - \frac{3}{8}$.
3. Work out $\frac{2}{3} \times \frac{9}{14}$.
4. Work out $\frac{4}{9} \div \frac{2}{3}$.
5. Write $\frac{7}{8}$ as (a) a decimal, (b) a percentage.
6. Write 0.36 as a fraction in its simplest form.
7. Work out $3.7 + 12.48 - 5.9$.
8. Work out 0.6×0.07 .
9. Work out $4.8 \div 0.06$.
10. (a) Write 12.5% as a fraction. (b) Write 0.045 as a percentage. (c) Write $\frac{3}{40}$ as a decimal.
11. Work out $2\frac{1}{3} + 1\frac{3}{4}$. Give your answer as a mixed number.
12. Work out $5\frac{1}{4} - 2\frac{2}{3}$. Give your answer as a mixed number.
13. Work out $1\frac{3}{5} \times 2\frac{1}{2}$.
14. Work out $3\frac{3}{4} \div 1\frac{1}{8}$. Give your answer as a mixed number.
15. Write these numbers in order of size, starting with the smallest: 0.68, $\frac{2}{3}$, 65%, $\frac{7}{11}$.
16. Work out $\frac{3}{5}$ of 2.4 kg.
17. Write down the reciprocal of (a) $2\frac{2}{5}$, (b) 0.125.
18. Work out $(\frac{2}{3} + \frac{1}{4}) \div 1\frac{5}{6}$.
19. Work out $(1.2 \times 0.15) \div 0.09$.
20. You are given that $37 \times 64 = 2368$. Use this to write down the value of (a) 3.7×0.64 , (b) $236.8 \div 6.4$, (c) $23.68 \div 0.37$.
21. Ali spends $\frac{1}{3}$ of his money on books. He then spends $\frac{2}{5}$ of the money he has left on food.
 - (a) What fraction of his original money does he have left?
 - (b) He has \$36 left. How much money did he have at the start?
22. A rectangle measures $2\frac{1}{4}$ m by $1\frac{2}{3}$ m. Calculate (a) its area, (b) its perimeter. Give each answer as a mixed number.
23. Show that the recurring decimal 0.272727... can be written as $\frac{3}{11}$.
24. Convert 0.1333... (the 3 recurs) to a fraction in its simplest form.
25. (a) Convert 2.0363636... (the block 36 recurs) to a mixed number in its simplest form.
(b) Work out $0.181818... + 0.333...$, giving your answer as a fraction in its simplest form.

Answers

1. The LCM of 4 and 5 is 20. $\frac{3}{4} = \frac{15}{20}$ and $\frac{2}{5} = \frac{8}{20}$.
 $\frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1 \frac{3}{20}$.
2. The LCM of 6 and 8 is 24. $\frac{5}{6} = \frac{20}{24}$ and $\frac{3}{8} = \frac{9}{24}$.
 $\frac{20}{24} - \frac{9}{24} = \frac{11}{24}$.
3. Cancel first: 9 and 3 share a factor of 3, and 2 and 14 share a factor of 2.
 $\frac{2}{3} \times \frac{9}{14} = \frac{1}{1} \times \frac{3}{7} = \frac{3}{7}$.
4. Multiply by the reciprocal: $\frac{4}{9} \times \frac{3}{2} = \frac{12}{18} = \frac{2}{3}$.
5. (a) $7 \div 8$ by short division: $7.000 \div 8 = 0.875$.
(b) $0.875 \times 100 = 87.5\%$.
6. $0.36 = \frac{36}{100}$. Divide top and bottom by 4: $\frac{9}{25}$.
7. Line up the points: $3.70 + 12.48 = 16.18$.
 $16.18 - 5.90 = 10.28$.
8. $6 \times 7 = 42$. There are $1 + 2 = 3$ decimal places in total, so the answer is 0.042.
9. Multiply both numbers by 100: $4.8 \div 0.06 = 480 \div 6 = 80$.
10. (a) $12.5\% = \frac{12.5}{100} = \frac{125}{1000} = \frac{1}{8}$.
(b) $0.045 \times 100 = 4.5\%$.
(c) $\frac{3}{40} = \frac{75}{1000}$ (multiply top and bottom by 25) $= 0.075$.
11. $2 \frac{1}{3} = \frac{7}{3}$ and $1 \frac{3}{4} = \frac{7}{4}$. The LCM of 3 and 4 is 12.
 $\frac{7}{3} + \frac{7}{4} = \frac{28}{12} + \frac{21}{12} = \frac{49}{12} = 4 \frac{1}{12}$.
12. $5 \frac{1}{4} = \frac{21}{4}$ and $2 \frac{2}{3} = \frac{8}{3}$. The LCM is 12.
 $\frac{21}{4} - \frac{8}{3} = \frac{63}{12} - \frac{32}{12} = \frac{31}{12} = 2 \frac{7}{12}$.
13. $1 \frac{3}{5} = \frac{8}{5}$ and $2 \frac{1}{2} = \frac{5}{2}$.
 $\frac{8}{5} \times \frac{5}{2}$: cancel the 5s and divide 8 and 2 by 2, giving $\frac{4}{1} \times \frac{1}{1} = 4$.
14. $3 \frac{3}{4} = \frac{15}{4}$ and $1 \frac{1}{8} = \frac{9}{8}$.
 $\frac{15}{4} \div \frac{9}{8} = \frac{15}{4} \times \frac{8}{9}$. Cancel 15 and 9 by 3 (5 and 3), and 8 and 4 by 4 (2 and 1).
 $= \frac{5}{1} \times \frac{2}{3} = \frac{10}{3} = 3 \frac{1}{3}$.
15. Change every number to a decimal to three decimal places.
 $0.68 = 0.680$. $\frac{2}{3} = 0.666... = 0.667$ (3 d.p.). $65\% = 0.650$. $\frac{7}{11} = 0.636363... = 0.636$ (3 d.p.).
Order: $\frac{7}{11}$, 65%, $\frac{2}{3}$, 0.68.
16. $\frac{1}{5}$ of 2.4 kg $= 2.4 \div 5 = 0.48$ kg.
 $\frac{3}{5}$ of 2.4 kg $= 3 \times 0.48 = 1.44$ kg.
17. (a) $2 \frac{2}{5} = \frac{12}{5}$, so its reciprocal is $\frac{5}{12}$.
(b) $0.125 = \frac{1}{8}$, so its reciprocal is $\frac{8}{1} = 8$.
18. Brackets first: $\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$.
 $1 \frac{5}{6} = \frac{11}{6}$.
 $\frac{11}{12} \div \frac{11}{6} = \frac{11}{12} \times \frac{6}{11} = \frac{6}{12} = \frac{1}{2}$.
19. 1.2×0.15 : $12 \times 15 = 180$, with $1 + 2 = 3$ decimal places, gives $0.180 = 0.18$.
 $0.18 \div 0.09$: multiply both by 100, giving $18 \div 9 = 2$.

- 20.** (a) 3.7 is $37 \div 10$ and 0.64 is $64 \div 100$, so the product is $2368 \div 1000 = 2.368$.
 (b) $2368 \div 64 = 37$. Here $236.8 = 2368 \div 10$ and $6.4 = 64 \div 10$. Both are divided by 10, so the answer is unchanged: 37.
 (c) $2368 \div 37 = 64$. Here $23.68 = 2368 \div 100$ and $0.37 = 37 \div 100$. Both are divided by 100, so the answer is 64.
- 21.** (a) After the books he has $1 - 1/3 = 2/3$ of his money.
 Food: $2/5$ of $2/3 = 2/5 \times 2/3 = 4/15$ of the original amount.
 Left: $2/3 - 4/15 = 10/15 - 4/15 = 6/15 = 2/5$.
 (Check: after food he keeps $3/5$ of the $2/3$, and $3/5 \times 2/3 = 6/15 = 2/5$.)
 (b) $2/5$ of the original amount is \$36, so $1/5$ is \$18 and the whole amount is $5 \times \$18 = \$90$.
- 22.** $2 \frac{1}{4} = 9/4$ and $1 \frac{2}{3} = 5/3$.
 (a) Area = $9/4 \times 5/3$. Cancel 9 and 3 by 3: $3/4 \times 5/1 = 15/4 = 3 \frac{3}{4} \text{ m}^2$.
 (b) Perimeter = $2 \times (9/4 + 5/3) = 2 \times (27/12 + 20/12) = 2 \times 47/12 = 47/6 = 7 \frac{5}{6} \text{ m}$.
- 23.** Let $x = 0.272727\ldots$. The block 27 has two digits, so multiply by 100.
 $100x = 27.272727\ldots$
 Subtract: $100x - x = 27$, so $99x = 27$.
 $x = 27/99$. Divide top and bottom by 9: $x = 3/11$, as required.
- 24.** Let $x = 0.1333\ldots$. The 1 does not repeat, so multiply by 10 to move it in front of the point, then by a further 10 to line up the repeating 3s.
 $10x = 1.333\ldots$
 $100x = 13.333\ldots$
 Subtract: $100x - 10x = 13.333\ldots - 1.333\ldots$, so $90x = 12$.
 $x = 12/90 = 2/15$.
- 25.** (a) Let $x = 2.0363636\ldots$
 $10x = 20.363636\ldots$
 $1000x = 2036.363636\ldots$
 Subtract: $1000x - 10x = 2036.363636\ldots - 20.363636\ldots$, so $990x = 2016$.
 $x = 2016/990$. Divide top and bottom by 18: $x = 112/55 = 2 \frac{2}{55}$.
 (Check: $2/55 = 0.0363636\ldots$, which matches.)
 (b) Let $y = 0.181818\ldots$. Then $100y = 18.181818\ldots$, so $99y = 18$ and $y = 18/99 = 2/11$.
 Let $z = 0.333\ldots$. Then $10z = 3.333\ldots$, so $9z = 3$ and $z = 3/9 = 1/3$.
 $2/11 + 1/3 = 6/33 + 11/33 = 17/33$.