

Fluids: drag, upthrust, viscosity and terminal velocity (worksheet with answers)

What you need to know

A fluid is any substance that can flow: a liquid or a gas. Objects moving through fluids, or sitting in them, experience forces from the fluid. Use $g = 9.81 \text{ N kg}^{-1}$ throughout unless a question tells you otherwise.

Density and pressure

Density is mass per unit volume: $\rho = m / V$. Unit: kg m^{-3} .

Pressure is the normal force per unit area: $p = F / A$. Unit: pascal, Pa ($1 \text{ Pa} = 1 \text{ N m}^{-2}$).

Pressure in a fluid: the extra pressure at depth h below the surface of a fluid of density ρ is $\Delta p = \rho gh$. The total pressure is the surface (usually atmospheric) pressure plus ρgh .

- Pressure in a fluid acts equally in all directions at a given depth.
- Remember to convert cm^3 to m^3 : $1 \text{ cm}^3 = 1 \times 10^{-6} \text{ m}^3$. Water has a density of about 1000 kg m^{-3} .

Upthrust and Archimedes' principle

Archimedes' principle: when an object is wholly or partly immersed in a fluid, the upthrust on it is equal to the weight of the fluid it displaces.

$$\text{Upthrust } U = \rho_{\text{fluid}} \times V_{\text{displaced}} \times g$$

- Upthrust exists because pressure increases with depth. The bottom of an immersed object is deeper than the top, so the upward force on the bottom is larger than the downward force on the top. The difference is the upthrust.
- An object **floats** when the upthrust equals its weight. A floating object displaces its own weight of fluid, so it sinks until enough fluid is displaced.
- An object that is fully submerged and denser than the fluid sinks, because its weight is greater than the largest upthrust it can receive.
- Upthrust acts in air too, but it is usually tiny compared with weight because air is so much less dense than solids.

Drag

- Drag is the resistive force on an object moving through a fluid. It always acts opposite to the velocity of the object relative to the fluid.
- Drag increases with: the speed of the object, its cross-sectional area at right angles to the motion, the density of the fluid and the viscosity of the fluid. It also depends on shape: streamlined shapes have less drag.



- In **laminar** (streamline) flow the fluid moves in smooth layers that do not cross. In **turbulent** flow the fluid forms eddies and the layers mix. Laminar flow happens at low speeds and around small, streamlined objects.
- For laminar flow around a small sphere, drag is proportional to speed (Stokes' law). For fast-moving objects such as cars and skydivers, the flow is turbulent and drag is roughly proportional to speed squared: $F = kv^2$.

Viscosity and Stokes' law

Viscosity (symbol η , eta) is a measure of a fluid's resistance to flow. Unit: Pa s (equivalent to $\text{kg m}^{-1} \text{s}^{-1}$).

Stokes' law: the viscous drag on a small sphere of radius r moving at speed v through a fluid of viscosity η , in laminar flow, is $F = 6\pi\eta rv$.

- Stokes' law only applies to small, smooth spheres moving slowly so that the flow is laminar. It does not apply to turbulent flow.
- The viscosity of a liquid decreases sharply as its temperature rises. Experiments must therefore be done at a known, steady temperature.

Terminal velocity

- When an object is released in a fluid, its weight is larger than the upward forces, so it accelerates downwards.
- As its speed increases, drag increases, so the resultant force and the acceleration decrease.
- Eventually weight = upthrust + drag. The resultant force is zero, the acceleration is zero and the object moves at a constant speed called the **terminal velocity**.
- Free-body diagram for a sphere falling through a liquid: weight (W) acting down from the centre; upthrust (U) acting up; drag (D) acting up (opposite to the downward motion). At terminal velocity the two upward arrows together are the same length as the downward arrow.
- A velocity-time graph for a falling object starts at the origin with gradient equal to the initial acceleration, then curves over with decreasing gradient and levels off at the terminal velocity.

Worked example: terminal velocity of a sphere

At terminal velocity: $W = U + F$

$$(4/3)\pi r^3 \rho_s g = (4/3)\pi r^3 \rho_f g + 6\pi\eta r v_t$$

$$\text{Rearranging: } v_t = 2r^2 g (\rho_s - \rho_f) / (9\eta)$$

A glass bead of radius 0.50 mm and density 2500 kg m^{-3} falls through oil of density 900 kg m^{-3} and viscosity 0.30 Pa s .

$$v_t = 2 \times (0.50 \times 10^{-3})^2 \times 9.81 \times (2500 - 900) / (9 \times 0.30)$$

$$= 2 \times 2.5 \times 10^{-7} \times 9.81 \times 1600 / 2.7 = 7.848 \times 10^{-3} / 2.7 = 2.9 \times 10^{-3} \text{ m s}^{-1}$$

Note that v_t is proportional to r^2 : doubling the radius makes the terminal velocity four times larger.

The falling-ball experiment

- A tall, wide tube is filled with a viscous liquid such as glycerol. Rubber bands are placed round the tube at measured intervals well below the surface.
- Measure the diameter of a small steel ball with a micrometer in several directions and average. Measure the liquid's temperature with a thermometer.
- Release the ball at the centre of the surface and time it between bands with a stopwatch (or film it next to a ruler and step through the frames).
- Terminal velocity has been reached when the ball takes equal times to cover equal distances between successive pairs of bands.
- Use $v = \text{distance} / \text{time}$, then $\eta = 2r^2g(\rho_s - \rho_f) / (9v)$. Densities can be measured with a balance and a measuring cylinder, or taken from data.

Questions

Questions get harder as you go. Take $g = 9.81 \text{ N kg}^{-1}$ and the density of water as 1000 kg m^{-3} unless told otherwise.

1. A rectangular block has dimensions $2.0 \text{ cm} \times 4.0 \text{ cm} \times 5.0 \text{ cm}$ and a mass of 50 g . Calculate its density in kg m^{-3} . [2]
2. State Archimedes' principle. [1]
3. A diver is 12 m below the surface of the sea. The density of sea water is 1030 kg m^{-3} and atmospheric pressure is $1.01 \times 10^5 \text{ Pa}$.
 - (a) Calculate the pressure due to the water alone. [2]
 - (b) Calculate the total pressure on the diver. [1]
4. An aluminium block of volume $2.0 \times 10^{-4} \text{ m}^3$ hangs from a newton-meter, fully under water. The density of aluminium is 2700 kg m^{-3} .
 - (a) Calculate the weight of the block. [1]
 - (b) Calculate the upthrust on the block. [1]
 - (c) State the reading on the newton-meter. [1]
5. A wooden block of density 600 kg m^{-3} and volume $5.0 \times 10^{-3} \text{ m}^3$ floats in water.
 - (a) Explain why the block floats. [2]
 - (b) Calculate the volume of the block below the water surface. [2]
6. A cube of side 0.10 m is held under water with its top face horizontal and 0.20 m below the surface.
 - (a) Calculate the pressure due to the water on the top face and on the bottom face. [2]
 - (b) Use your answers to calculate the upthrust on the cube. [2]
 - (c) Show that your answer agrees with Archimedes' principle. [1]
7. A small sphere of radius 1.5 mm moves at a steady 0.040 m s^{-1} through oil of viscosity 0.80 Pa s . Calculate the viscous drag on the sphere. State one assumption you have made. [3]
8. A car moving at 20 m s^{-1} experiences a drag force of 400 N . Assume the drag is proportional to the square of the speed.
 - (a) Calculate the drag at 30 m s^{-1} . [2]
 - (b) Calculate the power the engine must supply to overcome drag at 30 m s^{-1} . [2]
9. A skydiver jumps from a plane and later opens her parachute. Describe and explain how her velocity changes from the moment she leaves the plane until she reaches the ground. Refer to the forces on

her at each stage. [6]

10. A skydiver of total mass 75 kg reaches a terminal velocity of 55 m s^{-1} . The drag force is given by $F = kv^2$. Ignore upthrust.

(a) Calculate k and give its unit. [3]

(b) Calculate his acceleration when his speed is 30 m s^{-1} . [2]

11. A helium balloon is tied to the ground by a string. The balloon contains 0.010 m^3 of helium of density 0.18 kg m^{-3} , and the rubber has a mass of 3.0 g. The density of air is 1.29 kg m^{-3} . Ignore the volume of the rubber and the weight of the string.

(a) Calculate the upthrust on the balloon. [1]

(b) Calculate the tension in the string. [2]

12. Starting from the forces on a sphere falling at terminal velocity through a liquid, show that $v_t = 2r^2g(\rho_s - \rho_f) / (9\eta)$, where ρ_s is the density of the sphere and ρ_f the density of the liquid. [3]

13. A steel ball of radius 1.0 mm and density 7800 kg m^{-3} is dropped into glycerol of density 1260 kg m^{-3} and viscosity 1.4 Pa s .

(a) Calculate the terminal velocity of the ball. [2]

(b) Calculate the acceleration of the ball at the instant it is released from rest just below the surface. [3]

(c) Sketch and describe the velocity-time graph for the ball. [2]

14. Describe how you would use a falling-ball experiment to determine the viscosity of glycerol. Include how you would make sure the ball has reached terminal velocity and how you would make your result accurate. [6]

15. In a falling-ball experiment a steel ball of diameter $(4.00 \pm 0.01) \text{ mm}$ falls $(0.200 \pm 0.001) \text{ m}$ between two bands in $(2.50 \pm 0.05) \text{ s}$ at terminal velocity. The density of steel is 7850 kg m^{-3} and the density of the liquid is 1260 kg m^{-3} .

(a) Calculate the viscosity of the liquid. [3]

(b) Estimate the percentage uncertainty in your answer. Treat the densities as exact. [3]

16. A student tries to use Stokes' law to calculate the drag on a skydiver falling at 50 m s^{-1} . Explain why this is not valid, and explain why a large raindrop also does not obey Stokes' law even though a tiny droplet in a cloud does. [3]

Answers

1. $V = 2.0 \times 4.0 \times 5.0 = 40 \text{ cm}^3 = 40 \times 10^{-6} = 4.0 \times 10^{-5} \text{ m}^3$. $m = 0.050 \text{ kg}$.
 $\rho = m / V = 0.050 / 4.0 \times 10^{-5} = 1250 \text{ kg m}^{-3}$ ($1.3 \times 10^3 \text{ kg m}^{-3}$ to 2 s.f.).

2. The upthrust on an object wholly or partly immersed in a fluid is equal to the weight of the fluid it displaces.

3. (a) $\Delta p = \rho gh = 1030 \times 9.81 \times 12 = 1.21 \times 10^5 \text{ Pa}$.

(b) $p = 1.01 \times 10^5 + 1.21 \times 10^5 = 2.22 \times 10^5 \text{ Pa}$.

4. (a) $W = \rho Vg = 2700 \times 2.0 \times 10^{-4} \times 9.81 = 5.30 \text{ N}$.

(b) $U = \rho_{\text{water}} Vg = 1000 \times 2.0 \times 10^{-4} \times 9.81 = 1.96 \text{ N}$.

(c) The newton-meter supports the rest of the weight: $5.30 - 1.96 = 3.34 \text{ N}$ (3.3 N).

5. (a) The block is less dense than water. If fully submerged, the upthrust (weight of displaced water) would be greater than its weight, so it rises until only part of it is under water and the

upthrust equals its weight.

(b) Weight = $600 \times 5.0 \times 10^{-3} \times 9.81 = 29.4 \text{ N}$. Floating, so upthrust = $29.4 \text{ N} = 1000 \times V_{\text{sub}} \times 9.81$.

$$V_{\text{sub}} = 29.4 / 9810 = 3.0 \times 10^{-3} \text{ m}^3.$$

6. (a) Top: $p = 1000 \times 9.81 \times 0.20 = 1962 \text{ Pa}$ ($\approx 1.96 \times 10^3 \text{ Pa}$). Bottom (depth 0.30 m): $p = 1000 \times 9.81 \times 0.30 = 2943 \text{ Pa}$ ($\approx 2.94 \times 10^3 \text{ Pa}$).

(b) Area of a face = $0.10 \times 0.10 = 0.010 \text{ m}^2$. Downward force on top = $1962 \times 0.010 = 19.62 \text{ N}$. Upward force on bottom = $2943 \times 0.010 = 29.43 \text{ N}$. Forces on the sides cancel. Upthrust = $29.43 - 19.62 = 9.81 \text{ N}$.

(c) Displaced volume = $0.10^3 = 1.0 \times 10^{-3} \text{ m}^3$. Weight of water displaced = $1000 \times 1.0 \times 10^{-3} \times 9.81 = 9.81 \text{ N}$, the same as (b).

7. $F = 6\pi\eta rv = 6\pi \times 0.80 \times 1.5 \times 10^{-3} \times 0.040 = 9.0 \times 10^{-4} \text{ N}$.

Assumption: the flow is laminar (the sphere is small, smooth and moving slowly), or the sphere is far enough from the container walls for them not to affect the flow.

8. (a) $F \propto v^2$, so $F_2 = 400 \times (30/20)^2 = 400 \times 2.25 = 900 \text{ N}$.

(b) At constant speed, power = force $\times$ velocity = $900 \times 30 = 2.7 \times 10^4 \text{ W}$ (27 kW).

9. (Up to 6 marks for a clear sequence.)

On leaving the plane her speed is low, so drag is small; weight is much larger than drag, so the resultant force is downwards and she accelerates at close to g .

As her speed increases, drag increases, so the resultant force falls and her acceleration decreases.

Eventually drag equals weight; the resultant force is zero, so she falls at constant (terminal) velocity.

When the parachute opens, the cross-sectional area increases greatly, so drag becomes much larger than weight. The resultant force is upwards, so she decelerates.

As she slows, drag decreases until it again equals her weight.

She reaches a new, much lower terminal velocity and lands at this speed.

10. (a) At terminal velocity, drag = weight: $kv^2 = mg$.

$$k = mg / v^2 = (75 \times 9.81) / 55^2 = 735.75 / 3025 = 0.243 \text{ kg m}^{-1} \text{ (unit from } \text{N} / (\text{m s}^{-1})^2 = \text{kg m s}^{-2} / \text{m}^2 \text{ s}^{-2} = \text{kg m}^{-1}\text{)}.$$

(b) Drag at $30 \text{ m s}^{-1} = 0.243 \times 30^2 = 219 \text{ N}$. Resultant = $735.75 - 219 = 517 \text{ N}$.

$$a = F / m = 517 / 75 = 6.9 \text{ m s}^{-2}.$$

11. (a) $U = \rho_{\text{air}} Vg = 1.29 \times 0.010 \times 9.81 = 0.127 \text{ N}$.

(b) Mass of balloon = mass of helium + rubber = $0.18 \times 0.010 + 0.0030 = 0.0018 + 0.0030 = 0.0048 \text{ kg}$. Weight = $0.0048 \times 9.81 = 0.0471 \text{ N}$.

The balloon is in equilibrium: $U = W + T$, so $T = 0.1265 - 0.0471 = 0.0794 \text{ N}$ (0.079 N).

12. At terminal velocity the resultant force is zero, so weight = upthrust + drag.

$$\text{Weight} = \text{mass} \times g = (4/3)\pi r^3 \rho_s g. \text{ Upthrust} = \text{weight of liquid displaced} = (4/3)\pi r^3 \rho_f g. \text{ Drag (Stokes)} = 6\pi\eta r v_t.$$

$$(4/3)\pi r^3 \rho_s g = (4/3)\pi r^3 \rho_f g + 6\pi\eta r v_t$$

$$6\pi\eta r v_t = (4/3)\pi r^3 g (\rho_s - \rho_f)$$

$$v_t = (4/3)\pi r^3 g (\rho_s - \rho_f) / (6\pi\eta r) = 4r^2 g (\rho_s - \rho_f) / (18\eta) = 2r^2 g (\rho_s - \rho_f) / (9\eta).$$

13. (a) $v_t = 2 \times (1.0 \times 10^{-3})^2 \times 9.81 \times (7800 - 1260) / (9 \times 1.4)$

$$= 2 \times 1.0 \times 10^{-6} \times 9.81 \times 6540 / 12.6 = 0.1283 / 12.6 = 1.0 \times 10^{-2} \text{ m s}^{-1}.$$

(b) At release, $v = 0$ so drag = 0. Resultant force = $W - U = Vg(\rho_s - \rho_f)$. Mass = $V\rho_s$.

$$a = Vg(\rho_s - \rho_f) / (V\rho_s) = g(1 - \rho_f/\rho_s) = 9.81 \times (1 - 1260/7800) = 9.81 \times 0.838 = 8.2 \text{ m s}^{-2}.$$

(c) The graph starts at the origin with a gradient of 8.2 m s^{-2} . The gradient decreases as drag grows, so the line curves over and levels off at a horizontal line at $1.0 \times 10^{-2} \text{ m s}^{-1}$.

14. (Up to 6 marks.)

Fill a tall, wide tube with glycerol and fix rubber bands round it at equal measured intervals (for example every 10 cm), starting well below the surface. Measure the band spacing with a metre rule.

Measure the diameter of the ball with a micrometer at several orientations; average and halve to get r .

Record the temperature of the glycerol.

Release the ball from rest at the centre of the surface, along the axis of the tube, and time it between successive bands with a stopwatch, or film it beside a scale and use frame-by-frame analysis.

Terminal velocity is reached when the times for successive equal intervals are the same (within uncertainty). Use only those intervals.

Calculate $v = \text{distance} / \text{time}$. Find ρ_s (mass from a balance, volume from $(4/3)\pi r^3$) and ρ_f (mass of a known volume in a measuring cylinder).

$$\text{Calculate } \eta = 2r^2g(\rho_s - \rho_f) / (9v).$$

Accuracy: repeat timings and average; use a wide tube so wall effects are small; view bands at eye level to avoid parallax; keep the temperature constant; long distances between bands reduce the percentage timing error.

$$15. (a) r = 2.00 \times 10^{-3} \text{ m. } v = 0.200 / 2.50 = 0.0800 \text{ m s}^{-1}.$$

$$\eta = 2r^2g(\rho_s - \rho_f) / (9v) = 2 \times (2.00 \times 10^{-3})^2 \times 9.81 \times (7850 - 1260) / (9 \times 0.0800)$$

$$= 2 \times 4.00 \times 10^{-6} \times 9.81 \times 6590 / 0.720 = 0.5172 / 0.720 = 0.718 \text{ Pa s (0.72 Pa s)}.$$

(b) Percentage uncertainties: diameter (and so r) = $0.01/4.00 \times 100 = 0.25\%$, so $r^2 = 0.50\%$.

Distance = $0.001/0.200 \times 100 = 0.50\%$. Time = $0.05/2.50 \times 100 = 2.0\%$. $v = d/t$, so $v = 0.50 + 2.0 = 2.5\%$.

$\eta \propto r^2/v$, so total = $0.50 + 2.5 = 3.0\%$. $\eta = 0.72 \pm 0.02 \text{ Pa s}$. The timing dominates the uncertainty.

16. Stokes' law only applies to small spheres in laminar flow at low speed. A skydiver is large, not spherical and moves fast, so the air flow around him is turbulent and drag is roughly proportional to v^2 , not v .

A tiny cloud droplet has a very small radius and a very low terminal velocity ($v_t \propto r^2$), so the flow round it is laminar and Stokes' law works. A large raindrop has a much larger radius and terminal velocity, so the flow becomes turbulent and the drop changes shape, so Stokes' law no longer applies.