

Exponential growth and decay, including compound interest: IGCSE Maths worksheet with answers

This worksheet covers percentage multipliers, compound interest, depreciation, the general growth and decay formula, and finding a rate or a number of years. It is written for Cambridge IGCSE Mathematics 0580 (Extended). Work through the method notes first, then attempt the questions in order: they start easy and get harder. Calculators are allowed. Give money answers to the nearest cent unless the question says otherwise, and other answers to 3 significant figures.

What you need to know

1. Percentage multipliers

A multiplier is the single number you multiply by to make a percentage change.

- An increase of $r\%$ has multiplier $1 + r/100$. For example, an increase of 6% has multiplier 1.06.
- A decrease of $r\%$ has multiplier $1 - r/100$. For example, a decrease of 15% has multiplier 0.85.
- Repeated changes are combined by multiplying the multipliers. An increase of 10% followed by a decrease of 10% gives $1.1 \times 0.9 = 0.99$, which is an overall decrease of 1%, not zero.
- To reverse a percentage change (find the original amount), divide by the multiplier. Never subtract the percentage from the new amount.

Learn these

Simple interest: interest is calculated on the original amount only, so it is the same every year.
Interest = $P \times r \times n / 100$.

Compound interest: interest is added to the amount each year, and the next year's interest is calculated on the new total. Total amount = $P \times (1 + r/100)^n$.

Depreciation: a fall in value by a fixed percentage each year. Value = $P \times (1 - r/100)^n$.

Here P is the starting amount, r is the percentage rate per time period and n is the number of time periods.

2. Compound interest and depreciation

The formula gives the **total amount** at the end. If the question asks for the **interest**, subtract the starting amount from the total. Read the question carefully: "how much is in the account" and "how much interest is earned" are different answers.

If the rate is given per month, n must be the number of months. If the rate is per year, n must be in years.

3. The general growth and decay formula

Any quantity that changes by the same percentage in each equal time period follows



$$y = a \times k^t$$

- a is the starting value (the value when $t = 0$).
- k is the multiplier. If $k > 1$ the quantity is growing; if $0 < k < 1$ it is decaying.
- The percentage change per period is $(k - 1) \times 100$. For $k = 1.08$ this is an increase of 8%; for $k = 0.7$ it is a decrease of 30%.

Doubling and halving are special cases. A quantity that doubles every period has $k = 2$. A quantity that halves every period (for example a radioactive substance with a fixed half-life) has $k = 0.5$.

4. Finding the number of years by trial

At IGCSE you find an unknown number of periods by trying whole numbers. Work out the value for $n = 1, 2, 3$ and so on (or jump ahead with sensible guesses) until the value first passes the target. Always show at least the value just before and just after the target, so the examiner can see why your answer is correct.

5. Finding the rate

If you know the starting value, the final value and the number of years, find the multiplier first. Divide the final value by the starting value, then take the n th root (use the $x^{1/n}$ or root key). Then convert the multiplier back to a percentage.

Worked example

Amira invests \$3500 at 4% per year compound interest.

(a) Calculate the amount in the account after 6 years.

Multiplier = 1.04. Amount = $3500 \times 1.04^6 = 3500 \times 1.265319... = \$4428.62$ (to the nearest cent).

(b) Calculate the interest earned in 6 years.

Interest = $4428.62 - 3500 = \$928.62$.

(c) Find the number of complete years until the amount first exceeds \$5000.

After 9 years: $3500 \times 1.04^9 = \$4981.59$ (less than \$5000).

After 10 years: $3500 \times 1.04^{10} = \$5180.86$ (more than \$5000).

So the amount first exceeds \$5000 after **10 years**.

(d) Bilal invests \$3500 for 6 years and receives \$4690.33 at the end. Find his annual compound interest rate.

Multiplier for 6 years = $4690.33 \div 3500 = 1.340094$. Multiplier for 1 year = $1.340094^{1/6} = 1.05$.

Rate = 5% per year.

Exam tips

- Write down the multiplier before you calculate. It earns a method mark even if you press a wrong key later.
- Do not round in the middle of a calculation. Keep the full value on your calculator and round only the final answer.
- If a question asks for a percentage change overall, find the overall multiplier first, then

convert it.

- For a "reverse" question (the value after the change is given), divide by the multiplier.

Questions

Section A: multipliers and single changes

1. Write down the multiplier for

(a) an increase of 7%, (b) a decrease of 12%, (c) an increase of 0.5%. [3]

2. Describe the percentage change represented by each multiplier.

(a) 1.3 (b) 0.95 (c) 0.6 [3]

3. The price of a jacket is \$85. The price increases by 4%. Calculate the new price. [2]

4. After a price increase of 3%, a bicycle costs \$618. Calculate the price before the increase. [2]

5. The price of a phone increases by 20%. The new price is then reduced by 20%. Show that the final price is 4% less than the original price. [2]

Section B: simple and compound interest

6. Kofi invests \$2000 at a rate of 3% per year simple interest. Calculate the interest he receives after 4 years. [2]

7. Lena invests \$2000 at a rate of 3% per year compound interest.

(a) Calculate the total amount in her account after 4 years. [2]

(b) How much more interest does Lena receive than Kofi in Question 6? [1]

8. Sara borrows \$12 000 at 2.5% per year compound interest. She repays nothing for 2 years. Calculate the amount she owes after 2 years. [2]

9. A savings account pays 0.5% per month compound interest. Tomas puts \$3000 into the account. Calculate the amount in the account after 12 months. [2]

10. Account A pays 4.5% per year simple interest. Account B pays 4% per year compound interest. Hana has \$6000 to invest for 5 years. Which account gives her more money at the end, and by how much? Show all your working. [4]

11. An amount of money is invested at 4% per year compound interest. After 3 years the amount in the account is \$5624.32. Calculate the amount that was invested. [3]

Section C: depreciation and decay

12. A car costs \$18 000 when new. Its value depreciates by 15% each year. Calculate its value after 3 years. [2]

13. A radioactive sample has a mass of 80 g. Its mass decreases by 10% each hour. Calculate its mass after 5 hours. Give your answer correct to 3 significant figures. [2]

14. A substance has a half-life of 8 years, so its mass halves every 8 years. A sample has a mass of 640 mg.

(a) Calculate the mass after 40 years. [2]

(b) Find how many years it takes for the mass to fall below 1 mg for the first time. [2]

Section D: growth formula, years and rates

15. The population of a town is 45 000. It grows by 2.5% each year. Calculate the population after 6 years. Give your answer to the nearest whole number. [2]

16. The value, y dollars, of an antique after t years is given by $y = 200 \times 1.08^t$.

(a) Write down the value of the antique when $t = 0$. [1]

(b) Write down the annual percentage increase in its value. [1]

(c) Calculate the value after 5 years. [2]

17. A colony of bacteria doubles in number every 3 hours. At midday there are 500 bacteria. Calculate the number of bacteria 12 hours later. [2]

18. Ravi invests \$4000 at 5% per year compound interest. Find the number of complete years it takes for the amount in his account to exceed \$5000 for the first time. Show your trials. [3]

19. A machine loses 20% of its value each year. Find the number of complete years it takes for its value to fall below half of its original value. [3]

20. An investment of \$8000 grows to \$9261 after 3 years of compound interest. Calculate the annual rate of interest. [3]

21. The value of a delivery van falls from \$50 000 to \$32 000 in 2 years. The rate of depreciation is the same each year. Calculate the annual percentage rate of depreciation. [3]

Section E: structured problems

22. A family buys a house for \$240 000 at the start of 2020.

(a) The value of the house increases by 6% each year for 3 years. Calculate its value at the end of these 3 years. [2]

(b) The value then decreases by 4% each year for the next 2 years. Calculate the value at the end of the 5 years. Give your answer to the nearest dollar. [2]

(c) Calculate the overall percentage change in the value over the 5 years. [2]

(d) A second house was worth \$150 000 four years ago and is worth \$189 000 now. Its value increased by the same percentage each year. Calculate this percentage. Give your answer correct to 3 significant figures. [3]

23. The number of fish, P , in a lake t years after a survey is modelled by $P = 2400 \times 0.85^t$.

(a) Write down the number of fish at the time of the survey. [1]

(b) Write down the annual percentage decrease in the number of fish. [1]

(c) Calculate the number of fish after 4 years. Give your answer to the nearest whole number. [2]

(d) Find the number of complete years after the survey when the number of fish first falls below 1000. Show your trials. [3]

(e) Find the number of complete years it takes for the number of fish to fall below half of the number at the time of the survey. [2]

(f) Explain why the model $P = 2400 \times 0.85^t$ predicts that the number of fish never reaches zero. [1]

Answers

1. (a) $1 + 7/100 = 1.07$ (b) $1 - 12/100 = 0.88$ (c) $1 + 0.5/100 = 1.005$
2. (a) $1.3 - 1 = 0.3$, so an **increase of 30%**. (b) $1 - 0.95 = 0.05$, so a **decrease of 5%**. (c) $1 - 0.6 = 0.4$, so a **decrease of 40%**.
3. Multiplier = 1.04. New price = $85 \times 1.04 = \$88.40$.
4. Multiplier = 1.03. Original price = $618 \div 1.03 = \$600$. (Taking 3% of \$618 and subtracting gives \$599.46, which is wrong.)
5. Overall multiplier = $1.2 \times 0.8 = 0.96$. Since $0.96 = 1 - 0.04$, the final price is 4% less than the original.
6. Interest per year = 3% of 2000 = \$60. Interest for 4 years = $60 \times 4 = \$240$.
7. (a) Amount = $2000 \times 1.03^4 = 2000 \times 1.12550881 = \$2251.02$.
(b) Lena's interest = $2251.02 - 2000 = \$251.02$. Difference = $251.02 - 240 = \$11.02$ more.
8. Amount owed = $12\,000 \times 1.025^2 = 12\,000 \times 1.050625 = \$12\,607.50$.
9. The rate is per month, so $n = 12$. Amount = $3000 \times 1.005^{12} = 3000 \times 1.0616778 = \$3185.03$.
10. Account A: interest = $6000 \times 4.5 \times 5 / 100 = \$1350$, so the total = \$7350.
Account B: total = $6000 \times 1.04^5 = 6000 \times 1.2166529 = \$7299.92$.
Account A gives more, by $7350 - 7299.92 = \$50.08$.
11. Original amount $\times 1.04^3 = 5624.32$.
 $1.04^3 = 1.124864$.
Original amount = $5624.32 \div 1.124864 = \$5000$.
12. Multiplier = 0.85. Value = $18\,000 \times 0.85^3 = 18\,000 \times 0.614125 = \$11\,054.25$.
13. Multiplier = 0.9. Mass = $80 \times 0.9^5 = 80 \times 0.59049 = 47.2392 = 47.2 \text{ g}$ (3 s.f.).
14. (a) 40 years is $40 \div 8 = 5$ half-lives. Mass = $640 \times 0.5^5 = 640 \div 32 = 20 \text{ mg}$.
(b) After 9 half-lives: $640 \div 2^9 = 640 \div 512 = 1.25 \text{ mg}$ (not yet below 1 mg).
After 10 half-lives: $640 \div 2^{10} = 640 \div 1024 = 0.625 \text{ mg}$ (below 1 mg).
10 half-lives = $10 \times 8 = 80 \text{ years}$.
15. Population = $45\,000 \times 1.025^6 = 45\,000 \times 1.1596934 = 52\,186.2 = 52\,186$.
16. (a) When $t = 0$, $1.08^0 = 1$, so $y = \$200$.
(b) $k = 1.08$, so the increase is **8%** per year.
(c) $y = 200 \times 1.08^5 = 200 \times 1.4693281 = \$293.87$.
17. 12 hours is $12 \div 3 = 4$ doubling periods. Number = $500 \times 2^4 = 500 \times 16 = 8000$.
18. After 4 years: $4000 \times 1.05^4 = \$4862.03$ (not more than \$5000).
After 5 years: $4000 \times 1.05^5 = \$5105.13$ (more than \$5000).
Answer: **5 years**.
19. Let the original value be 1 (any value works). Multiplier = 0.8.
After 3 years: $0.8^3 = 0.512$ (still more than 0.5).
After 4 years: $0.8^4 = 0.4096$ (less than 0.5).
Answer: **4 years**.

20. Multiplier for 3 years = $9261 \div 8000 = 1.157625$.

Multiplier for 1 year = $1.157625^{1/3} = 1.05$.

Rate = **5% per year**. Check: $8000 \times 1.05^3 = 9261$.

21. Multiplier for 2 years = $32\,000 \div 50\,000 = 0.64$.

Multiplier for 1 year = $\sqrt{0.64} = 0.8$.

Rate of depreciation = $1 - 0.8 = 0.2 = \mathbf{20\% \text{ per year}}$.

22. (a) Value = $240\,000 \times 1.06^3 = 240\,000 \times 1.191016 = \mathbf{\$285\,843.84}$.

(b) Value = $285\,843.84 \times 0.96^2 = 285\,843.84 \times 0.9216 = 263\,433.68 = \mathbf{\$263\,434}$ (nearest dollar).

(c) Overall multiplier = $1.06^3 \times 0.96^2 = 1.191016 \times 0.9216 = 1.0976403$. This is an **increase of 9.76%** (3 s.f.).

Alternatively: $(263\,433.68 - 240\,000) \div 240\,000 \times 100 = 9.76\%$.

(d) Multiplier for 4 years = $189\,000 \div 150\,000 = 1.26$.

Multiplier for 1 year = $1.26^{1/4} = 1.0594797$.

Annual increase = **5.95%** (3 s.f.).

23. (a) When $t = 0$, $P = 2400 \times 1 = \mathbf{2400}$ fish.

(b) $k = 0.85 = 1 - 0.15$, so the decrease is **15%** per year.

(c) $P = 2400 \times 0.85^4 = 2400 \times 0.52200625 = 1252.8 = \mathbf{1253}$ fish.

(d) $t = 5$: $2400 \times 0.85^5 = 1064.9$ (still at least 1000).

$t = 6$: $2400 \times 0.85^6 = 905.2$ (below 1000).

Answer: **6 years**.

(e) Half of 2400 is 1200. $t = 4$ gives 1252.8 (above 1200). $t = 5$ gives 1064.9 (below 1200).

Answer: **5 years**.

Alternatively: $0.85^4 = 0.522$ and $0.85^5 = 0.444$, and 0.444 is the first value below 0.5.

(f) Each year the number is multiplied by 0.85, which is a positive number. A positive number multiplied by a positive number is always positive, so P gets smaller but never reaches 0. (In reality the population would end as a whole number of fish, so the model stops being useful when P is small.)