

Oscillations and simple harmonic motion: worksheet with answers

This worksheet covers simple harmonic motion (SHM) for the Oscillations topic of the Cambridge International A Level Physics syllabus (9702), A2 content. It includes the defining condition and equation for SHM, displacement–time and velocity–displacement relationships, energy changes during SHM, worked examples for a simple pendulum and a mass–spring system, damped oscillations, and forced oscillations and resonance. The worksheet has 22 questions in three sections, worth 66 marks in total, with a full worked answer scheme.

Key ideas

- A system performs simple harmonic motion (SHM) if its acceleration is proportional to its displacement from a fixed point and is always directed towards that point: $a = -\omega^2 x$, where ω is the angular frequency in rad s^{-1} .
- The minus sign shows that the acceleration always acts to restore the object towards the equilibrium (centre) position.
- If the object starts at maximum displacement x_0 at $t = 0$, $x = x_0 \cos(\omega t)$ and $v = -\omega x_0 \sin(\omega t)$; if it starts at zero displacement moving through the centre, $x = x_0 \sin(\omega t)$ and $v = \omega x_0 \cos(\omega t)$.
- Angular frequency is related to period and frequency by $\omega = 2\pi/T = 2\pi f$.
- Velocity can also be written in terms of displacement as $v = \pm \omega \sqrt{(x_0^2 - x^2)}$; velocity is maximum at $x = 0$, where $v_0 = \omega x_0$, and zero at $x = \pm x_0$.
- Total energy of an undamped SHM oscillator is constant; it is continuously exchanged between kinetic energy (maximum at $x = 0$) and potential energy (maximum at $x = \pm x_0$).
- A simple pendulum of length l has period $T = 2\pi \sqrt{l/g}$, valid only for small angles of swing; a mass m on a spring of stiffness (force constant) k has period $T = 2\pi \sqrt{m/k}$.
- Damping removes energy from an oscillating system, reducing amplitude over time; the degree of damping is described as light, critical or heavy.
- A system driven at its own natural frequency undergoes resonance, where the amplitude of the forced oscillation is a maximum; increasing damping reduces and broadens this resonance peak. Everyday examples of resonance include a radio receiver circuit tuned to resonate at a broadcast frequency, and a bridge or tall building oscillating strongly when a driving frequency (such as footsteps or wind gusts) matches its natural frequency.

Section A: multiple choice

Choose the one correct answer, A, B, C or D. Each question is worth [1].

1. Which statement correctly defines the condition for simple harmonic motion? [1]

- A Acceleration is constant and directed towards a fixed point.
- B Acceleration is proportional to displacement and directed away from a fixed point.
- C Acceleration is proportional to displacement and always directed towards a fixed point.



- D Acceleration is proportional to velocity and directed towards a fixed point.

2. A particle performing SHM is released from rest at its maximum displacement x_0 at time $t = 0$. Which equation describes its displacement x at later times? [1]

- A $x = x_0 \sin(\omega t)$
- B $x = x_0 \cos(\omega t)$
- C $x = -x_0 \sin(\omega t)$
- D $x = -x_0 \cos(\omega t)$

3. An object oscillates with SHM of amplitude 5.0 cm and period 2.0 s. What is its maximum speed? [1]

- A 0.031 m s^{-1}
- B 0.16 m s^{-1}
- C 0.31 m s^{-1}
- D 1.6 m s^{-1}

4. A graph is plotted of velocity v against displacement x for an object performing SHM. The shape of this graph is: [1]

- A a straight line through the origin
- B a parabola
- C an ellipse centred on the origin
- D a circle of radius x_0

5. For an undamped oscillator performing SHM, which quantity stays constant as the oscillation proceeds? [1]

- A kinetic energy
- B potential energy
- C displacement
- D total energy (kinetic + potential)

6. The length of a simple pendulum is doubled while g stays constant. By what factor does the period change? [1]

- A $\times 2$
- B $\times \sqrt{2}$
- C $\times 0.5$
- D $\times 0.71$ ($1/\sqrt{2}$)

7. Which statement correctly describes critical damping? [1]

- A The system oscillates with slowly decreasing amplitude over many cycles.
- B The system returns to equilibrium in the shortest possible time without oscillating.
- C The system returns to equilibrium without oscillating, but more slowly than in B.
- D The system oscillates with constant amplitude indefinitely.

8. A system is driven by a periodic force of frequency f . At which value of f is the amplitude of the

forced oscillation a maximum? [1]

- A a frequency much lower than the natural frequency of the system
- B a frequency equal to the natural frequency of the system
- C a frequency much higher than the natural frequency of the system
- D any frequency — amplitude does not depend on the driving frequency

Section B: short answer

9. State the two conditions that must both be satisfied for an oscillation to be simple harmonic motion. [2]

10. A body performs SHM with amplitude 8.0 cm and angular frequency 4.0 rad s^{-1} . Calculate its speed when its displacement from the centre of oscillation is 5.0 cm. [3]

11. Describe, in words, how the acceleration of an object performing SHM varies with its displacement x , including the significance of the gradient of an acceleration–displacement graph. [2]

12. A mass of 0.20 kg is attached to a horizontal spring of force constant 50 N m^{-1} and is set oscillating with SHM. Calculate the period of oscillation. [3]

13. An oscillator of mass 0.50 kg has amplitude 3.0 cm and angular frequency 10 rad s^{-1} . Calculate the total energy of the oscillation. [3]

14. Describe how the kinetic energy and potential energy of an SHM oscillator each change as it moves from maximum displacement to the centre of oscillation. [2]

15. Sketch and describe, in words, the shapes of the graphs of kinetic energy against displacement and potential energy against displacement for an SHM oscillator, and state how the total-energy–displacement graph differs from these. [2]

16. An object performs SHM with amplitude 5.0 cm and angular frequency 8.0 rad s^{-1} . At time $t = 0$ the object is at its maximum positive displacement. Calculate its velocity at time $t = 0.15 \text{ s}$. [3]

17. A simple pendulum has a length of 0.90 m. Taking $g = 9.81 \text{ m s}^{-2}$, calculate (i) its period and (ii) its frequency of oscillation. [3]

18. Explain the effect of increasing the amount of damping in a system on the height and sharpness of its resonance peak when the system is driven at varying frequency. [4]

Section C: structured questions

19. A student investigates a simple pendulum of length 1.20 m, released from a small angle so that it swings with amplitude 4.0 cm.

(a) Define simple harmonic motion. [2]

(b) State the equation relating the period T of a simple pendulum to its length l and the acceleration of free fall g , and explain why this equation is only valid for small angles of swing. [2]

(c) Calculate the period of this pendulum, taking $g = 9.81 \text{ m s}^{-2}$. [2]

(d) Calculate the maximum speed of the pendulum bob. [2]

20. A mass of 0.25 kg is attached to a vertical spring of force constant 40 N m^{-1} and is displaced and

released so that it performs SHM with amplitude 6.0 cm.

- (a) Show that the angular frequency of the oscillation is about 12.6 rad s^{-1} . [2]
- (b) Calculate the maximum acceleration of the mass. [2]
- (c) Calculate the maximum kinetic energy of the mass, and state at what displacement this maximum occurs. [2]
- (d) Describe, in words, how the kinetic energy and the potential energy of the mass each vary with time over one complete period, starting from maximum displacement. [2]

21. A vibrating system can be damped to different degrees.

- (a) Distinguish between light damping and critical damping. [2]
- (b) A car suspension system is designed with damping close to critical damping. Explain, in terms of passenger comfort and control of the car, why this degree of damping is chosen. [2]
- (c) Describe, in words, how a displacement–time graph following a single disturbance would differ for a system that is lightly damped, critically damped and heavily damped. [2]
- (d) Give one example of a system that is deliberately made lightly damped and one example of a system that is deliberately made heavily damped. [2]

22. A footbridge can be set into oscillation by people walking across it.

- (a) State what is meant by a forced oscillation and by the natural frequency of a system. [2]
- (b) State the condition needed for resonance to occur in a driven system. [1]
- (c) Using ideas about resonance, explain why soldiers are traditionally ordered to break step (stop marching in time) when crossing a footbridge. [2]
- (d) Describe, in words, how the graph of the amplitude of oscillation against driving frequency changes as the amount of damping in the bridge structure is increased. [2]

Answers

1. C — acceleration proportional to displacement, always directed towards a fixed point (the defining condition of SHM). [1]

2. B — released from rest at maximum displacement corresponds to the cosine solution $x = x_0 \cos(\omega t)$, since at $t = 0$ this gives $x = x_0$ and $dx/dt = 0$. [1]

3. B. $\omega = 2\pi/T = 2\pi/2.0 = 3.14 \text{ rad s}^{-1}$. $v_0 = \omega x_0 = 3.14 \times 0.050 = 0.157 \text{ m s}^{-1} \approx 0.16 \text{ m s}^{-1}$. [1]

4. C — since $v = \omega\sqrt{(x_0^2 - x^2)}$, rearranging gives $(x/x_0)^2 + (v/\omega x_0)^2 = 1$, the equation of an ellipse. [1]

5. D — total energy is constant in an undamped oscillator; KE and PE individually change with displacement. [1]

6. B. $T = 2\pi\sqrt{l/g}$, so $T \propto \sqrt{l}$. Doubling l multiplies T by $\sqrt{2}$. [1]

7. B — critical damping returns the system to equilibrium in the shortest time possible without any oscillation occurring. [1]

8. B — resonance occurs, and the amplitude of the forced oscillation is a maximum, when the driving frequency is equal to the natural frequency of the system. [1]

9. Award 1 mark for each: (i) the acceleration of the object is proportional to its displacement from a fixed (equilibrium) point; (ii) the acceleration is always directed towards that fixed point (opposite in direction to the displacement). [2]

10. $x_0 = 0.080$ m, $\omega = 4.0$ rad s⁻¹, $x = 0.050$ m [1 for substitution]

$$v = \omega\sqrt{x_0^2 - x^2} = 4.0 \times \sqrt{(0.080^2 - 0.050^2)} = 4.0 \times \sqrt{(0.0064 - 0.0025)} = 4.0 \times \sqrt{0.0039} \text{ [1]}$$

$$v = 4.0 \times 0.06245 = 0.25 \text{ m s}^{-1} \text{ [1 for final answer with correct unit]}$$

11. The acceleration is directly proportional to the displacement x and always acts in the opposite direction to x ($a = -\omega^2 x$), so a graph of a against x is a straight line through the origin with a negative gradient. [1] The magnitude of the gradient of this line is equal to ω^2 . [1]

12. $m = 0.20$ kg, $k = 50$ N m⁻¹

$$T = 2\pi\sqrt{m/k} \text{ [1 for correct formula]}$$

$$T = 2\pi\sqrt{(0.20/50)} = 2\pi\sqrt{0.0040} = 2\pi \times 0.0632 \text{ [1]}$$

$$T = 0.40 \text{ s [1 for final answer]}$$

13. $m = 0.50$ kg, $x_0 = 0.030$ m, $\omega = 10$ rad s⁻¹

$$E = \frac{1}{2}m\omega^2 x_0^2 \text{ [1 for correct formula]}$$

$$E = \frac{1}{2} \times 0.50 \times 10^2 \times 0.030^2 = 0.25 \times 100 \times 0.00090 \text{ [1]}$$

$$E = 0.0225 \text{ J } (2.3 \times 10^{-2} \text{ J}) \text{ [1 for final answer]}$$

14. As the object moves from maximum displacement towards the centre, its potential energy decreases from a maximum to zero, while its kinetic energy increases from zero to a maximum [1]; the sum of kinetic and potential energy remains constant throughout the motion (energy is transferred between the two forms, none is lost, since the system is undamped). [1]

15. The kinetic-energy–displacement graph is a curve that is a maximum at $x = 0$ and falls symmetrically to zero at $x = \pm x_0$ (a dome-shaped curve); the potential-energy–displacement graph is the mirror image of this — zero at $x = 0$ and rising symmetrically to a maximum at $x = \pm x_0$ (a U-shaped curve). [1]

The total-energy–displacement graph is a horizontal straight line at every value of x , since the sum of kinetic and potential energy is constant (equal to the maximum KE and also equal to the maximum PE). [1]

16. $x_0 = 0.050$ m, $\omega = 8.0$ rad s⁻¹, $t = 0.15$ s. Since the object starts at maximum positive

displacement, $x = x_0 \cos(\omega t)$ and the paired velocity equation is $v = -\omega x_0 \sin(\omega t)$ [1 for correct equation]

$$v = -8.0 \times 0.050 \times \sin(8.0 \times 0.15) = -0.40 \times \sin(1.2 \text{ rad})$$
 [1 for substitution]

$$v = -0.40 \times 0.932 = -0.37 \text{ m s}^{-1}$$
 (the negative sign shows the object is moving back towards the centre of oscillation) [1 for final answer with correct sign and unit]

17. $l = 0.90 \text{ m}$, $g = 9.81 \text{ m s}^{-2}$

(i) $T = 2\pi\sqrt{l/g} = 2\pi\sqrt{0.90/9.81} = 2\pi\sqrt{0.0917} = 2\pi \times 0.3029$ [1]

$$T = 1.90 \text{ s}$$
 [1 for final answer]

(ii) $f = 1/T = 1/1.90 = 0.53 \text{ Hz}$ [1]

18. Award up to 4 marks for a clear explanation covering: as damping increases, the maximum (peak) amplitude of the resonance response decreases [1]; the resonance peak becomes broader / less sharply defined, so the system responds significantly over a wider range of driving frequencies [1]; at driving frequencies well below the natural frequency, the amplitude tends towards the same value regardless of the amount of damping [1]; with very heavy damping the resonance peak may disappear almost entirely, so amplitude changes only gradually with driving frequency [1].

19(a). Simple harmonic motion is oscillatory motion in which the acceleration of the object is proportional to its displacement from a fixed point [1] and is always directed towards that fixed point (opposite in direction to the displacement). [1]

19(b). $T = 2\pi\sqrt{l/g}$. [1] This relationship is derived assuming the restoring force component is proportional to $\sin\theta \approx \theta$ (in radians); this approximation is only accurate for small angles of swing, so the equation only holds when the amplitude of swing is small. [1]

19(c). $T = 2\pi\sqrt{l/g} = 2\pi\sqrt{1.20/9.81} = 2\pi\sqrt{0.1223} = 2\pi \times 0.3497$ [1]

$$T = 2.20 \text{ s}$$
 [1 for final answer]

19(d). $\omega = 2\pi/T = 2\pi/2.20 = 2.86 \text{ rad s}^{-1}$ [1]

$$v_0 = \omega x_0 = 2.86 \times 0.040 = 0.11 \text{ m s}^{-1}$$
 [1 for final answer]

20(a). $\omega = \sqrt{k/m}$ [1 for correct formula]

$$\omega = \sqrt{40/0.25} = \sqrt{160} = 12.6 \text{ rad s}^{-1}$$
 [1 for correct substitution and value shown]

20(b). $a_{\max} = \omega^2 x_0 = 160 \times 0.060$ [1]

$$a_{\max} = 9.6 \text{ m s}^{-2}$$
 [1 for final answer]

20(c). $KE_{\max} = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2} \times 0.25 \times 160 \times 0.060^2 = 0.072 \text{ J}$ [1 for correct value]

This maximum kinetic energy occurs at the centre of oscillation, where displacement $x = 0$. [1]

20(d). Starting from maximum displacement, the potential energy begins at its maximum value and the kinetic energy begins at zero. [1] As the mass moves towards the centre (a quarter period later) potential energy falls to zero while kinetic energy rises to its maximum; this pattern then repeats twice per period as the mass passes through the centre in each direction, with kinetic and potential energy each oscillating at twice the frequency of the displacement, while their sum stays constant. [1]

21(a). In light damping, the amplitude of oscillation decreases gradually (exponentially) over many complete cycles before the system comes to rest [1]; in critical damping, the system returns directly to its equilibrium position in the shortest possible time, without completing even one oscillation [1].

21(b). With damping close to critical, the suspension returns the car body to its equilibrium position quickly after a bump, without prolonged oscillation, which keeps the tyres in good contact with the road and gives the driver control of the car [1]; at the same time it avoids the sharp, sudden jolt of overdamping and the repeated bouncing of light damping, giving passengers a comfortable ride [1].

21(c). For light damping, the graph shows an oscillating curve (a decaying sine/cosine shape) with amplitude decreasing gradually cycle by cycle [1]; for critical damping, the displacement falls smoothly from its initial value to zero in the shortest time with no oscillation, while for heavy damping the displacement also falls smoothly to zero without oscillating, but takes noticeably longer to reach zero than in the critically damped case [1].

21(d). One mark for each correctly matched example: a swinging pendulum or a guitar string is designed to be lightly damped (so it keeps oscillating audibly/visibly for a useful time) [1]; a door closer or an analogue measuring instrument pointer is designed to be heavily or critically damped (so it settles quickly without overshooting or oscillating) [1].

22(a). A forced oscillation is one in which a system is made to oscillate at a frequency imposed by an external periodic driving force, rather than at its own natural frequency [1]; the natural frequency of a system is the frequency at which it would oscillate freely if displaced and released with no driving force and no damping [1].

22(b). Resonance occurs when the frequency of the driving force is equal (or very close) to the natural frequency of the system. [1]

22(c). If a large number of soldiers march in step, the regular force they exert on the bridge acts as a periodic driving force at the frequency of their footsteps [1]; if this frequency happens to match the natural frequency of the bridge, resonance occurs and the amplitude of the bridge's oscillation can build up to a dangerously large value, so breaking step removes this regular driving frequency and prevents resonance. [1]

22(d). As damping increases, the peak of the amplitude–driving frequency graph becomes lower [1] and broader, spreading the response over a wider range of frequencies rather than a sharp peak at one particular frequency close to the natural frequency. [1]