

Mathematics formula and quick-reference sheet

This sheet brings together the formulae, rules and facts you need at your fingertips for Cambridge O Level Mathematics (4024) and Cambridge IGCSE Mathematics (0580). It covers number, algebra, geometry, mensuration, coordinate geometry, vectors, transformations, statistics and probability. Each formula is marked either **Learn** (not printed in the exam, so you must know it by heart) or **Usually printed** (it normally appears on the formula list at the front of the question paper). Items tagged **[E]** are tested on 0580 Extended and on 4024, but not on 0580 Core. If you are a 0580 Core candidate you can leave them until last. A short worked example follows each main idea, and the sheet ends with the mistakes that cost students the most marks.

About the printed formula list. The list at the front of the paper normally gives the areas and volumes of the harder shapes (for example the cone, sphere and pyramid). The quadratic formula, sine rule, cosine rule and area $\frac{1}{2}ab \sin C$ are on the 4024 list and the 0580 Extended list. They are not on the 0580 Core list. In the first minute of the exam, read the list on your own paper.

Anything on this sheet without a Status marking is NOT printed, so learn it. Learn the printed formulae too: remembering a formula saves time, and you still have to know *when* to use it.

1. Number

Standard form, rounding and bounds

Rule	Meaning	Status
$A \times 10^n$, where $1 \leq A < 10$ and n is an integer	Standard form. For example, $0.000\,47 = 4.7 \times 10^{-4}$	Learn
Bounds of a value rounded to the nearest u	Lower bound = value $- \frac{1}{2}u$, upper bound = value $+ \frac{1}{2}u$	Learn
Sum $a + b$ [E]	$UB = UB(a) + UB(b)$; $LB = LB(a) + LB(b)$	Learn
Difference $a - b$ [E]	$UB = UB(a) - LB(b)$; $LB = LB(a) - UB(b)$	Learn
Product $a \times b$ [E]	$UB = UB(a) \times UB(b)$; $LB = LB(a) \times LB(b)$ (positive values)	Learn
Quotient $a \div b$ [E]	$UB = UB(a) \div LB(b)$; $LB = LB(a) \div UB(b)$	Learn

Worked example (bounds) [E]. A runner covers 120 m, measured to the nearest 10 m, in 14.2 s, measured to the nearest 0.1 s. Calculate the upper bound of the runner's average speed.

Distance: upper bound = $120 + 5 = 125$ m.

Time: lower bound = $14.2 - 0.05 = 14.15$ s.

Maximum speed = largest distance $\div$ smallest time = $125 \div 14.15 = 8.8339\dots$ m/s

Upper bound of the speed = 8.83 m/s (3 s.f.)

Standard form calculation: $(3.2 \times 10^5) \times (4 \times 10^{-2}) = 12.8 \times 10^3 = 1.28 \times 10^4$. Check that the first number is between 1 and 10.

Percentages and interest

Formula	Meaning of symbols	Status
Percentage change = $(\text{change} \div \text{original}) \times 100$	Always divide by the <i>original</i> value	Learn
New value = original $\times$ multiplier	Increase by r%: multiplier $1 + r/100$. Decrease by r%: multiplier $1 - r/100$	Learn
Original = new value $\div$ multiplier	Reverse percentage	Learn
$I = PRT \div 100$	Simple interest: I = interest, P = principal, R = rate % per year, T = time in years	Learn
$A = P(1 + r/100)^n$	Compound interest: A = final amount, P = principal, r = rate % per period, n = number of periods	Learn
$A = P(1 - r/100)^n$	Compound depreciation (value falls by r% each period)	Learn

Worked example (compound interest). Ayesha invests Rs 24 000 at 3.5% per year compound interest. Calculate the total value of the investment after 4 years, correct to the nearest rupee.

$$\text{Multiplier} = 1 + 3.5/100 = 1.035$$

$$A = 24\,000 \times 1.035^4 = 24\,000 \times 1.147\,523\dots = 27\,540.55\dots$$

Rs 27 541 (nearest rupee). The interest earned is Rs 3 541.

Worked example (reverse percentage). A jacket costs Rs 3 570 after a 15% discount. Find the original price.

$$\text{Multiplier for a 15% decrease} = 1 - 0.15 = 0.85$$

$$\text{Original} = 3\,570 \div 0.85 = 4\,200$$

Rs 4 200. Do *not* add 15% of Rs 3 570, because that 15% is of the wrong amount.

Ratio, rates and measures

Formula	Meaning of symbols	Units
Sharing in the ratio a : b : c	Total parts = $a + b + c$, then one part = $\text{amount} \div \text{total parts}$	as given
speed = distance $\div$ time	Average speed = $\text{total distance} \div \text{total time}$	m/s, km/h
density = mass $\div$ volume	$\rho = m \div V$	g/cm ³ , kg/m ³
pressure = force $\div$ area	$P = F \div A$	N/m ²

Distance–time graph	gradient = speed	m/s
Speed–time graph	gradient = acceleration; area under the graph = distance travelled	m/s ² ; m

All the rates above are Learn. Gradient of a curve at a point [E]: draw a tangent at that point and find the gradient of the tangent.

Example: Rs 4 500 shared in the ratio 2 : 3 : 4. Total parts = 9, one part = Rs 500, so the shares are Rs 1 000, Rs 1 500 and Rs 2 000.

Unit conversions that students forget

- 1 m = 100 cm, but $1 \text{ m}^2 = 10\,000 \text{ cm}^2$ and $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$.
- 1 litre = $1\,000 \text{ cm}^3$, and $1 \text{ m}^3 = 1\,000$ litres.
- To change km/h to m/s, divide by 3.6. To change m/s to km/h, multiply by 3.6.
- $1 \text{ g/cm}^3 = 1\,000 \text{ kg/m}^3$.
- Time: 2.25 hours = 2 hours 15 minutes, *not* 2 hours 25 minutes.

2. Algebra

Laws of indices

Law	Example
$a^m \times a^n = a^{m+n}$	$x^3 \times x^5 = x^8$
$a^m \div a^n = a^{m-n}$	$y^7 \div y^2 = y^5$
$(a^m)^n = a^{mn}$	$(2p^3)^4 = 16p^{12}$
$a^0 = 1$ ($a \neq 0$)	$7^0 = 1$
$a^{-n} = 1 \div a^n$	$5^{-2} = 1/25$
$a^{1/n} = n\sqrt[n]{a}$ [E]	$64^{1/3} = 4$
$a^{m/n} = (n\sqrt[n]{a})^m$ [E]	$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$

Combining two laws [E]: $27^{-2/3} = 1 \div 27^{2/3} = 1 \div (\sqrt[3]{27})^2 = 1 \div 3^2 = 1/9$.

Expanding and factorising

- $(a + b)^2 = a^2 + 2ab + b^2$, which is **not** $a^2 + b^2$.
- $(a - b)^2 = a^2 - 2ab + b^2$
- Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$. Example: $9x^2 - 49 = (3x + 7)(3x - 7)$.
- Common factor first: $6x^2 - 15x = 3x(2x - 5)$.
- Grouping: $ax + ay + bx + by = a(x + y) + b(x + y) = (a + b)(x + y)$.
- Quadratic trinomial $x^2 + bx + c$: find two numbers that multiply to c and add to b. Example: $x^2 - 2x - 15 = (x - 5)(x + 3)$.
- Three brackets [E]: expand two of them first, then multiply the result by the third.

Quadratic equations

Formula	Meaning	Status
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ [E]	Solves $ax^2 + bx + c = 0$ ($a \neq 0$)	Usually printed: 4024 and 0580 Extended (not 0580 Core)
$x^2 + bx + c = (x + b/2)^2 - (b/2)^2 + c$ [E]	Completing the square	Learn
$y = (x + p)^2 + q$ has turning point $(-p, q)$ [E]	Minimum point when the x^2 coefficient is positive	Learn

Worked example (quadratic formula) [E]. Solve $2x^2 - 5x - 4 = 0$, giving your answers correct to 2 decimal places.

$$a = 2, b = -5, c = -4$$

$$b^2 - 4ac = (-5)^2 - 4(2)(-4) = 25 + 32 = 57$$

$$x = \frac{5 \pm \sqrt{57}}{4} = \frac{5 \pm 7.5498...}{4}$$

$$x = 12.5498... \div 4 = 3.137... \text{ or } x = -2.5498... \div 4 = -0.637...$$

$$x = 3.14 \text{ or } x = -0.64$$

Worked example (completing the square) [E]. Write $2x^2 - 8x + 3$ in the form $a(x + p)^2 + q$.

Take out the 2 from the x terms: $2(x^2 - 4x) + 3$

Complete the square inside the bracket: $x^2 - 4x = (x - 2)^2 - 4$

So $2[(x - 2)^2 - 4] + 3 = 2(x - 2)^2 - 8 + 3$

$$2(x - 2)^2 - 5. \text{ The minimum point of } y = 2x^2 - 8x + 3 \text{ is } (2, -5).$$

Sequences

Type	nth term	How to find it
Linear (constant first difference d)	$dn + (\text{first term} - d)$	7, 11, 15, 19, ...: $d = 4$, so n th term = $4n + 3$
Quadratic (constant second difference)	$an^2 + bn + c$	$a = \text{second difference} \div 2$. Subtract an^2 from each term, then find the linear rule for what is left
Geometric (constant ratio r) [E]	$a \times r^{n-1}$	2, 6, 18, 54, ...: n th term = $2 \times 3^{n-1}$
Cubic	often related to n^3	Compare with 1, 8, 27, 64, ...

Quadratic example: 3, 8, 15, 24, 35, ... First differences 5, 7, 9, 11. Second difference 2, so $a = 1$.
Terms - n^2 : 2, 4, 6, 8, 10, which is $2n$. So the n th term = $n^2 + 2n$.

Variation [E]

Statement	Equation
y is proportional to x	$y = kx$
y is proportional to the square of x	$y = kx^2$
y is proportional to the cube of x	$y = kx^3$
y is proportional to the square root of x	$y = k\sqrt{x}$
y is inversely proportional to x	$y = k \div x$
y is inversely proportional to the square of x	$y = k \div x^2$
y is inversely proportional to the square root of x	$y = k \div \sqrt{x}$

Method: write the equation with k, substitute the given pair of values to find k, then use the equation.

Example: y is inversely proportional to x^2 , and $y = 12$ when $x = 2$. Then $12 = k \div 4$, so $k = 48$ and $y = 48 \div x^2$. When $x = 4$, $y = 48 \div 16 = 3$.

Functions [E]

- $f(3)$ means substitute $x = 3$.
- **$fg(x)$ means do g first, then f:** $fg(x) = f(g(x))$.
- Inverse $f^{-1}(x)$: write $y = f(x)$, make x the subject, then replace y with x (swap the letters x and y).
- $ff^{-1}(x) = x$.

Worked example (inverse function). $f(x) = (3x - 2) \div 5$. Find $f^{-1}(x)$.

$$y = (3x - 2) \div 5$$

$$5y = 3x - 2$$

$$5y + 2 = 3x$$

$$x = (5y + 2) \div 3$$

$$f^{-1}(x) = (5x + 2) \div 3$$

3. Geometry

Angle facts (give the reason in words)

Fact	Result
Angles on a straight line	add up to 180°
Angles around a point	add up to 360°
Vertically opposite angles	are equal
Alternate angles (parallel lines)	are equal

Corresponding angles (parallel lines)	are equal
Co-interior angles (parallel lines)	add up to 180°
Angles in a triangle	add up to 180°
Exterior angle of a triangle	equals the sum of the two opposite interior angles
Base angles of an isosceles triangle	are equal
Equilateral triangle	each angle is 60°
Angles in a quadrilateral	add up to 360°

Quadrilateral	Angle and diagonal properties
Parallelogram	opposite angles equal; diagonals bisect each other
Rectangle	all angles 90° ; diagonals equal and bisect each other
Rhombus	opposite angles equal; diagonals bisect each other at 90° and bisect the angles
Kite	one pair of opposite angles equal; diagonals cross at 90°

Polygons

Formula	Meaning	Status
Sum of interior angles = $(n - 2) \times 180^\circ$	n = number of sides	Learn
Sum of exterior angles = 360°	true for any convex polygon	Learn
Exterior angle of a regular polygon = $360^\circ \div n$	number of sides $n = 360^\circ \div$ exterior angle	Learn
Interior angle + exterior angle = 180°	at each vertex	Learn

Example: a regular 12-sided polygon has exterior angle $360^\circ \div 12 = 30^\circ$, interior angle $180^\circ - 30^\circ = 150^\circ$, and interior angle sum $(12 - 2) \times 180^\circ = 1\,800^\circ$.

Circle theorems

- The angle in a semicircle is 90° .
- The angle at the centre is twice the angle at the circumference, when both stand on the same arc. **[E]**
- Angles in the same segment are equal. **[E]**
- Opposite angles of a cyclic quadrilateral add up to 180° . **[E]**
- The exterior angle of a cyclic quadrilateral equals the interior opposite angle. **[E]**
- A tangent is perpendicular to the radius at the point of contact.
- Two tangents from an external point are equal in length. **[E]**
- The perpendicular from the centre to a chord bisects the chord. **[E]**
- The perpendicular bisector of a chord passes through the centre. **[E]**
- Equal chords are equidistant from the centre. **[E]**

11. Alternate segment theorem: the angle between a tangent and a chord equals the angle in the alternate segment. [E]

Use the standard wording above when you give a reason, for example "angle in a semicircle", "tangent $\perp$ radius". An answer with no reason, or with a vague one, can lose the reasoning mark.

Similarity and congruence

If the length scale factor is k	then
corresponding lengths	are multiplied by k
corresponding areas [E]	are multiplied by k^2
corresponding volumes (and masses of the same material) [E]	are multiplied by k^3
angles	stay the same

Congruence conditions for triangles: SSS, SAS, ASA (or AAS) and RHS.

Worked example (similar solids) [E]. Two mathematically similar cones have heights 6 cm and 9 cm. The smaller cone has volume 80 cm^3 . Find the volume of the larger cone.

Length scale factor $k = 9 \div 6 = 1.5$

Volume scale factor $k^3 = 1.5^3 = 3.375$

Volume = $80 \times 3.375 = 270$

270 cm^3

If you are given areas or volumes and need a length, go backwards: length factor = $\sqrt[3]{\text{area factor}} = \sqrt[3]{\text{volume factor}}$.

Pythagoras and trigonometry

Formula	Meaning of symbols	Status
$a^2 + b^2 = c^2$	Right-angled triangle, c = hypotenuse (the side opposite the right angle)	Learn
$\sin \theta = \text{opp} \div \text{hyp}$	SOH. Right-angled triangles only	Learn
$\cos \theta = \text{adj} \div \text{hyp}$	CAH	Learn
$\tan \theta = \text{opp} \div \text{adj}$	TOA	Learn
$a \div \sin A = b \div \sin B = c \div \sin C$ [E]	Sine rule: side a is opposite angle A, and so on	Usually printed: 4024 and 0580 Extended (not 0580 Core)
$a^2 = b^2 + c^2 - 2bc \cos A$ [E]	Cosine rule	Usually printed: 4024 and 0580 Extended (not 0580 Core)
$\cos A = (b^2 + c^2 - a^2) \div 2bc$ [E]	Cosine rule rearranged to find an angle	Learn the rearrangement

Area = $\frac{1}{2}ab \sin C$ [E]	Two sides and the angle <i>between</i> them	Usually printed: 4024 and 0580 Extended (not 0580 Core)
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Exact trigonometric values [E]

θ	0°	30°	45°	60°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2} (= \frac{1}{\sqrt{2}})$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2} (= \frac{1}{\sqrt{2}})$	$\frac{1}{2}$
$\tan \theta$	0	$\frac{\sqrt{3}}{3} (= \frac{1}{\sqrt{3}})$	1	$\sqrt{3}$

θ	90°
$\sin \theta$	1
$\cos \theta$	0
$\tan \theta$	undefined

Status: Learn. These values are used in non-calculator questions.

Which rule?

- Right angle present: Pythagoras or SOHCAHTOA.
- No right angle, and you know a side with its opposite angle: sine rule.
- No right angle, and you know two sides with the angle between them, or all three sides: cosine rule.
- $\sin x = \sin(180^\circ - x)$. So a sine-rule angle may have a second, obtuse answer. Check whether the question says the angle is obtuse.
- Use degrees: make sure your calculator is in degree mode.

Worked example (sine rule) [E]. In triangle ABC, angle A = 48° , angle B = 67° and BC = 9.5 cm. Calculate AC.

BC = a (opposite A) and AC = b (opposite B).

$$b \div \sin 67^\circ = 9.5 \div \sin 48^\circ$$

$$b = 9.5 \times \sin 67^\circ \div \sin 48^\circ = 9.5 \times 0.920\ 50\dots \div 0.743\ 14\dots = 11.767\dots$$

$$\text{AC} = 11.8\text{ cm (3 s.f.)}$$

Worked example (cosine rule and area) [E]. A triangle has sides 7 cm, 8 cm and 10 cm. Calculate (a) the largest angle, (b) the area of the triangle.

(a) The largest angle, C, is opposite the longest side, 10 cm.

$$\cos C = (7^2 + 8^2 - 10^2) \div (2 \times 7 \times 8) = (49 + 64 - 100) \div 112 = 13 \div 112 = 0.116\ 07\dots$$

$$C = \cos^{-1}(0.116\ 07\dots) = 83.33\dots^\circ$$

(b) Area = $\frac{1}{2} \times 7 \times 8 \times \sin 83.33\dots^\circ = 28 \times 0.9932\dots = 27.81\dots$

(a) 83.3° ; (b) 27.8 cm^2 (3 s.f.). Keep the unrounded angle in your calculator for part (b).

Bearings

- A bearing is measured from **north, clockwise**, and written with **three figures**, for example 065° .
- "The bearing of A from B" means stand at B and face A.
- Back bearing: add 180° if the bearing is less than 180° , subtract 180° if it is more. If A from B is 065° , then B from A is 245° .
- Draw a north line at every point you use. Alternate and co-interior angles between north lines solve most bearing questions.
- Angle of elevation is measured up from the horizontal, angle of depression down from the horizontal.

4. Mensuration

Areas and perimeters

Shape	Formula	Symbols	Status
Rectangle	$A = lw$	l = length, w = width	Learn
Triangle	$A = \frac{1}{2}bh$	b = base, h = perpendicular height	Usually printed
Parallelogram	$A = bh$	h = perpendicular height, not the slanting side	Usually printed
Trapezium	$A = \frac{1}{2}(a + b)h$	a, b = parallel sides, h = distance between them	Usually printed
Circle	$A = \pi r^2$, $C = 2\pi r = \pi d$	r = radius, d = diameter	Usually printed
Arc length	$(\theta \div 360) \times 2\pi r$	θ = angle at centre in degrees	Learn
Sector area	$(\theta \div 360) \times \pi r^2$	θ = angle at centre in degrees	Learn

Worked example (sector). A sector has radius 9 cm and angle 140° . Calculate (a) the arc length, (b) the area, (c) the perimeter.

(a) Arc = $(140 \div 360) \times 2 \times \pi \times 9 = (140 \div 360) \times 18\pi = 7\pi = 21.99\dots$

(b) Area = $(140 \div 360) \times \pi \times 9^2 = (140 \div 360) \times 81\pi = 31.5\pi = 98.96\dots$

(c) Perimeter = arc + two radii = $7\pi + 9 + 9 = 39.99\dots$

(a) 22.0 cm ; (b) 99.0 cm^2 ; (c) 40.0 cm (all 3 s.f.)

Volumes and surface areas

Solid	Volume	Surface area	Status
Cuboid	$V = lwh$	$2(lw + lh + wh)$	Learn
Prism	$V = A \times l$ (A = area of cross-section, l = length)	sum of all faces	Volume usually printed
Cylinder	$V = \pi r^2 h$	curved = $2\pi rh$; total = $2\pi rh + 2\pi r^2$	Volume and curved area usually printed; add the two ends yourself
Pyramid	$V = \frac{1}{3} \times \text{base area} \times h$	base + triangular faces	Volume usually printed
Cone	$V = \frac{1}{3}\pi r^2 h$	curved = πrl ; total = $\pi rl + \pi r^2$	Volume and curved area usually printed; add the base πr^2 yourself
Sphere	$V = \frac{4}{3}\pi r^3$	$4\pi r^2$	Usually printed
Hemisphere	$V = \frac{2}{3}\pi r^3$	curved = $2\pi r^2$; total solid = $3\pi r^2$	Learn (halve the sphere)

In a cone, h is the perpendicular height and l is the slant height, with $l^2 = r^2 + h^2$ (Learn). Remember that "total surface area" of a closed solid includes the flat faces.

Worked example (cone). A solid cone has base radius 5 cm and perpendicular height 12 cm. Calculate (a) its volume, (b) its total surface area.

(a) $V = \frac{1}{3} \times \pi \times 5^2 \times 12 = \frac{1}{3} \times \pi \times 300 = 100\pi = 314.15\dots$

(b) Slant height $l = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ cm

Total surface area = $\pi rl + \pi r^2 = \pi \times 5 \times 13 + \pi \times 25 = 65\pi + 25\pi = 90\pi = 282.74\dots$

(a) 314 cm³; (b) 283 cm² (3 s.f.)

5. Coordinate geometry

Formula	Meaning	Status
$m = (y_2 - y_1) \div (x_2 - x_1)$	Gradient of the line through (x_1, y_1) and (x_2, y_2)	Learn
Midpoint = $((x_1 + x_2) \div 2, (y_1 + y_2) \div 2)$	Average the coordinates	Learn
Length = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Pythagoras on the grid	Learn
$y = mx + c$	m = gradient, c = y-intercept	Learn
Parallel lines	equal gradients: $m_1 = m_2$	Learn
Perpendicular lines [E]	$m_1 \times m_2 = -1$, so $m_2 = -1 \div m_1$	Learn

Worked example. A is $(-2, 5)$ and B is $(4, -3)$. Find (a) the gradient of AB, (b) the midpoint of AB, (c) the length of AB, (d) the equation of the perpendicular bisector of AB [E].

(a) $m = (-3 - 5) \div (4 - (-2)) = -8 \div 6 = -4/3$

(b) Midpoint = $((-2 + 4) \div 2, (5 + (-3)) \div 2) = (1, 1)$

(c) Length = $\sqrt{(6^2 + (-8)^2)} = \sqrt{(36 + 64)} = \sqrt{100} = 10$

(d) Perpendicular gradient = $-1 \div (-4/3) = 3/4$. The line passes through the midpoint (1, 1): $1 = \frac{3}{4}(1) + c$, so $c = \frac{1}{4}$.

(a) $-4/3$; (b) (1, 1); (c) 10 units; (d) $y = \frac{3}{4}x + \frac{1}{4}$, or $4y = 3x + 1$

6. Vectors

- Column vector (x over y): x units right (left if negative), y units up (down if negative).
- Magnitude of the vector (x over y) = $\sqrt{(x^2 + y^2)}$ [E]. Example: the vector (5 over -12) has magnitude $\sqrt{(25 + 144)} = 13$.
- Add or subtract vectors component by component. Multiplying by a scalar k multiplies each component by k.
- If $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$, then $\mathbf{AB} = \mathbf{b} - \mathbf{a}$ (go back along **a**, then along **b**). [E]
- If M is the midpoint of AB, then $\mathbf{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{a} + \mathbf{b})$. [E]
- If P divides AB in the ratio 1 : 3, then $\mathbf{AP} = \frac{1}{4}\mathbf{AB}$. [E]
- **Parallel** vectors: one is a scalar multiple of the other. If $\mathbf{PQ} = k\mathbf{QR}$ as well, P, Q and R are **collinear** (they lie on one straight line). [E]

In written work, underline a vector (**a** in print is written a with a line underneath).

7. Transformations

Transformation	To describe it fully you must state	What stays the same
Reflection	the mirror line, as an equation (for example $y = -x$ or $x = 3$)	shape and size
Rotation	the centre, the angle, and the direction (clockwise or anticlockwise; not needed for 180°)	shape and size
Translation	the column vector	shape, size and orientation
Enlargement	the centre and the scale factor	shape and angles (lengths $\times k$, area $\times k^2$)

Enlargement facts

- Scale factor between 0 and 1: the image is smaller, but the transformation is still called an enlargement.
- Negative scale factor [E]: the image is on the opposite side of the centre and is inverted (turned through 180° about the centre). It is not a reflection.
- To find the centre, join corresponding points on the object and image and extend the lines until they meet.
- "Describe fully the **single** transformation": name one transformation only, with all its details.

Naming two transformations scores nothing.

8. Statistics

Averages and spread

Measure	Formula or method
Mean from a frequency table	$\Sigma fx \div \Sigma f$
Estimated mean from grouped data	$\Sigma fx \div \Sigma f$, where x = midpoint of each class
Median of n ordered values	the value in position $(n + 1) \div 2$
Mode or modal class	the value or class with the highest frequency. With unequal class widths, the modal class is usually taken as the one with the highest frequency density (the tallest histogram bar); read the question, which normally makes clear which is meant
Range	largest value – smallest value
Interquartile range [E]	upper quartile – lower quartile

Histograms and cumulative frequency [E]

Formula or rule	Meaning
frequency density = frequency $\div$ class width	Histogram bar heights. The area of each bar represents the frequency
frequency = frequency density $\times$ class width	Reading frequencies back from a histogram
Cumulative frequency: plot at the upper class boundary	Join the points with a smooth curve (or straight lines if told)
From a cumulative frequency curve with total n	median at $n/2$, lower quartile at $n/4$, upper quartile at $3n/4$; percentiles in the same way

Worked example (grouped data). The times, t minutes, that 30 students spent on homework are shown in the table.

Time t (min)	Frequency f	Midpoint x	fx	Class width
$0 < t \leq 10$	4	5	20	10
$10 < t \leq 20$	9	15	135	10
$20 < t \leq 40$	12	30	360	20
$40 < t \leq 60$	5	50	250	20

Total	30		765	
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Time t (min)	Frequency density
$0 < t \leq 10$	0.4
$10 < t \leq 20$	0.9
$20 < t \leq 40$	0.6
$40 < t \leq 60$	0.25
Total	

Estimated mean = $\Sigma fx \div \Sigma f = 765 \div 30 = 25.5$

Estimated mean = 25.5 minutes. For the histogram [E], the bar heights are the frequency densities 0.4, 0.9, 0.6 and 0.25. With unequal class widths, the modal class is the one with the highest frequency density (the tallest bar), $10 < t \leq 20$, even though the $20 < t \leq 40$ class has the largest frequency.

9. Probability

Rule	When to use it
$0 \leq P(A) \leq 1$	Every probability is between 0 and 1
$P(\text{not } A) = 1 - P(A)$	Complement. Very useful for "at least one"
$P(A \text{ or } B) = P(A) + P(B)$	Only for mutually exclusive events (they cannot both happen)
$P(A \text{ and } B) = P(A) \times P(B)$	Only for independent events
Expected frequency = $P(A) \times \text{number of trials}$	Predicting outcomes
Relative frequency = $\text{number of successes} \div \text{number of trials}$	Experimental probability
$P(A B) = n(A \cap B) \div n(B)$ [E]	Conditional probability from a Venn diagram or table

On a tree diagram, **multiply along** the branches and **add down** between the outcomes you want. The probabilities on each set of branches from one point add up to 1.

Worked example (without replacement). A bag contains 5 red and 3 blue counters. Two counters are taken at random without replacement. Find the probability that (a) both counters are the same colour, (b) at least one counter is blue.

First pick: $P(R) = 5/8$, $P(B) = 3/8$. After a red has been taken: $P(R) = 4/7$, $P(B) = 3/7$. After a blue has been taken: $P(R) = 5/7$, $P(B) = 2/7$.

(a) $P(RR) + P(BB) = (5/8 \times 4/7) + (3/8 \times 2/7) = 20/56 + 6/56 = 26/56 = 13/28$

(b) $P(\text{at least one blue}) = 1 - P(RR) = 1 - 20/56 = 36/56 = 9/14$

(a) $13/28$; (b) $9/14$

Common mistakes

- Rounding too early. Keep full calculator values until the final answer. Give answers to 3 significant figures (angles in degrees to 1 decimal place) unless the question says otherwise.
- Finding a percentage change by dividing by the new value instead of the original.
- Answering a reverse-percentage question by adding the percentage to the reduced price.
- Using simple interest when the question says compound, or giving only the interest when it asks for the total amount (or the other way round).
- Getting the bounds wrong in a division or subtraction. The largest $a - b$ and the largest $a \div b$ both use LB(b).
- Expanding $(x + 3)^2$ as $x^2 + 9$.
- Sign errors in the quadratic formula when b is negative: $-b$ becomes positive, and b^2 is always positive.
- Doing $fg(x)$ in the wrong order. g is applied first.
- Using SOHCAHTOA in a triangle with no right angle, or leaving the calculator in radians.
- Stating a circle theorem or angle fact with no reason, or a vague one ("because of the circle").
- Multiplying areas or volumes by the length scale factor instead of k^2 or k^3 .
- Using slant height where perpendicular height is needed (cone and pyramid volume), or the reverse for curved surface area.
- Mixing units, for example cm and m in one calculation, or converting m^2 to cm^2 by multiplying by 100.
- Writing bearings with two figures (65° instead of 065°) or measuring them anticlockwise.
- Getting the perpendicular gradient wrong: the perpendicular to $m = -4/3$ is $3/4$, not $4/3$ or $-3/4$.
- Describing a transformation only partly, for example "rotation 90° " with no centre or direction, or giving two transformations.
- Drawing histogram bars with frequency, not frequency density, on the vertical axis, or plotting cumulative frequency at class midpoints instead of upper class boundaries.
- Using class limits instead of midpoints for an estimated mean, or dividing by the number of classes instead of Σf .
- In "without replacement" questions, forgetting that the second-pick fractions have a denominator one less.