# IB DP PHYSICS

## Theme E: Nuclear and Quantum Physics

## E.4 Fission

Revision Notes · Standard and Higher Level

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# What the syllabus requires

E.4 is common to SL and HL. By the end of this topic you should be able to do each of the following. Use this list as a final checklist before the exam.

| Understanding | You should be able to... |
|---|---|
| Spontaneous and induced fission | Describe fission as the splitting of a heavy nucleus into two mid-mass nuclei plus neutrons, usually induced by neutron capture. |
| Energy release | Explain the energy released using the curve of binding energy per nucleon, and calculate it from nuclear masses or binding energies. |
| Chain reactions | Explain the role of the released neutrons in sustaining a chain reaction, and the meaning of critical mass. |
| The nuclear power station | State the function of fuel rods, moderator, control rods, coolant, heat exchanger and shielding, and trace the energy transfer chain to the grid. |
| Properties of fission products | Explain why fragments are radioactive (neutron-rich), and discuss the storage and disposal of nuclear waste. |
| Risk and benefit | Evaluate nuclear power against alternatives: energy density and reliability versus waste, accident risk and proliferation. |

> **Exam note:** E.4 leans on E.3 (binding energy, mass defect, $\Delta E = \Delta mc^2$) and on E.1 (half-life) for waste questions. Calculation marks almost always come from a mass-to-energy conversion using 1 u = 931.5 MeV $c^{-2}$.

# 1. What is nuclear fission?

**Nuclear fission** is the splitting of a heavy nucleus into two nuclei of intermediate mass (the **fission fragments**), together with two or three free neutrons and a large amount of energy. A few very heavy nuclides fission spontaneously, but the process that matters for power generation is **induced fission**: a slow neutron is absorbed by a fissile nucleus such as uranium-235, forming an unstable, highly excited compound nucleus (U-236) that deforms, necks and splits in about $10^{-12}$ s.

A typical reaction — one of many possible fragment pairs — is:

$$^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{236}_{92}\text{U}^{*} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,^{1}_{0}\text{n} + \text{energy}$$

Check the balance: mass number 1 + 235 = 141 + 92 + 3, and charge 0 + 92 = 56 + 36 + 0. In every nuclear equation both the total nucleon number $A$ and the total charge $Z$ are conserved.

## Why is energy released?

The curve of **binding energy per nucleon** against nucleon number rises steeply, peaks near iron-56 (about 8.8 MeV per nucleon) and falls gently towards the heaviest nuclei (uranium is at about 7.6 MeV per nucleon). When a nucleus of $A \approx 236$ splits into two fragments of $A \approx$ 90-140, the fragments sit higher on the curve (about 8.5 MeV per nucleon). Each nucleon is therefore **more tightly bound** after the split, and the difference in total binding energy — roughly (8.5 - 7.6) MeV × 236 nucleons ≈ 200 MeV — is released. Equivalently, the total rest mass of the products is less than that of the reactants, and the missing mass appears as energy through $\Delta E = \Delta mc^2$.

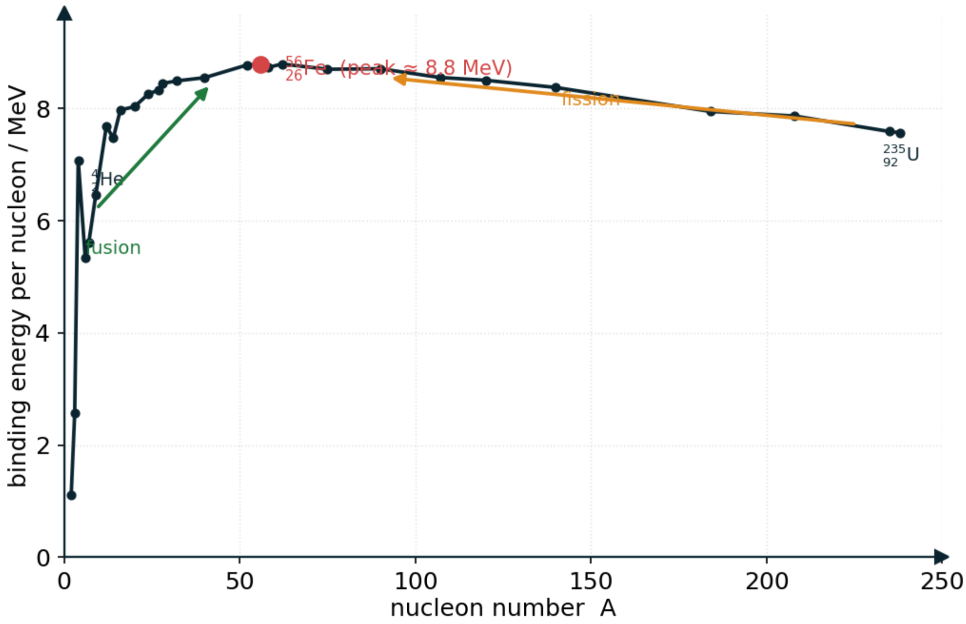


*Figure 1. Binding energy per nucleon against nucleon number A. The curve rises steeply for light nuclei, peaks near iron-56 ($^{56}_{26}$Fe, ≈ 8.8 MeV per nucleon) and falls gently for heavy nuclei. Because $^{235}_{92}$U (7.59 MeV) lies below the peak, splitting it into mid-mass fragments (near 8.5 MeV per nucleon) raises the binding energy per nucleon and so releases energy.*

Most of the released energy (about 85%) becomes kinetic energy of the two fragments, which recoil apart because of their enormous mutual electrostatic repulsion; the rest is shared among the neutrons, prompt γ-ray photons, and later β and γ decays of the fragments.

**Worked example 1 — energy released in one fission**

Using the masses m($^{235}_{92}$U) = 235.043930 u, m($^{141}_{56}$Ba) = 140.914411 u, m($^{92}_{36}$Kr) = 91.926156 u and m(n) = 1.008665 u, find the energy released in the reaction above.

Mass before = 235.043930 + 1.008665 = 236.052595 u.

Mass after = 140.914411 + 91.926156 + 3 × 1.008665 = 235.866562 u.

Δm = 236.052595 - 235.866562 = 0.186033 u (reactants minus products — positive, because mass is lost).

E = 0.186033 × 931.5 MeV = **173 MeV** ≈ 173 × 1.602 × $10^{-13}$ J = **2.8 × $10^{-11}$ J**.

Adding the energy later released by the radioactive fragments brings the total to roughly 200 MeV per fission — the figure normally quoted.

**Common pitfall — the sign of Δm:** Always compute Δm = (total mass of reactants) - (total mass of products). If your Δm comes out negative for a reaction that clearly releases energy, you have subtracted the wrong way round. Mass is *lost*; the products are lighter.

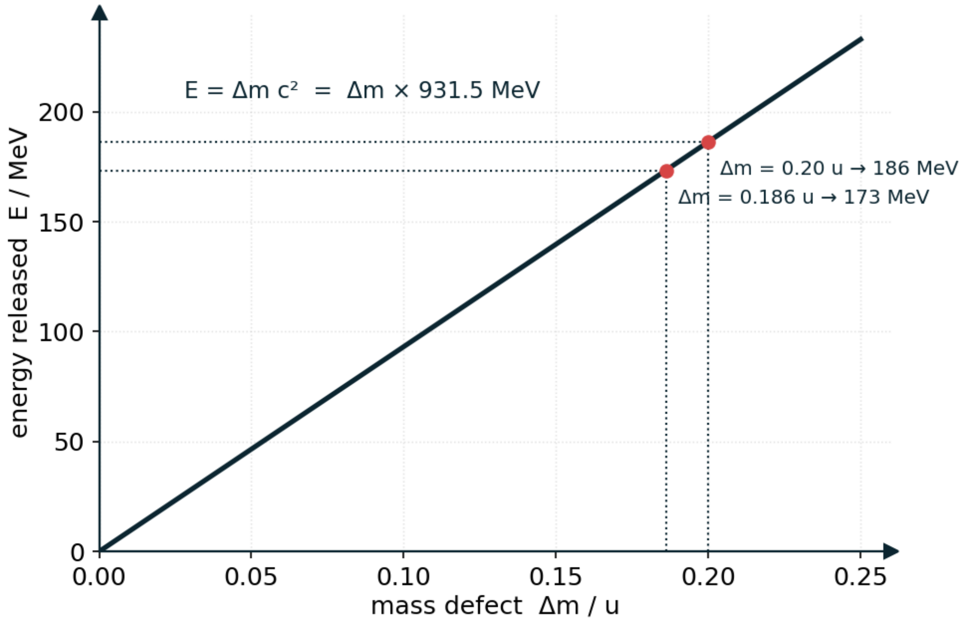


*Figure 3. Mass−energy equivalence, $E = \Delta m c^2 = \Delta m \times 931.5$ MeV. A fission mass defect of $\Delta m \approx 0.20$ u releases ≈ 186 MeV ($3.0 \times 10^{-11}$ J); the worked example above, $\Delta m = 0.186$ u, gives 173 MeV.*

## 2. Chain reactions and critical mass

Each fission releases two or three neutrons. If at least one of these, on average, goes on to cause another fission, the process is self-sustaining: a **chain reaction**. Define the **neutron multiplication factor** $k$ as the average number of neutrons from one fission that cause a further fission.

| Condition | Name | Behaviour |
|---|---|---|
| $k < 1$ | Subcritical | Each generation has fewer fissions; the reaction dies away. |
| $k = 1$ | Critical | Steady rate: exactly one neutron per fission causes the next fission. This is a reactor at constant power. |
| $k > 1$ | Supercritical | Fission rate grows exponentially - used briefly to raise reactor power; uncontrolled, it is a bomb. |

Neutrons are lost in three ways: they escape through the surface of the fuel, they are captured by nuclei that do not fission (notably U-238), or they are absorbed by control materials. Since fission happens throughout the volume but escape happens at the surface, a larger lump loses a smaller *fraction* of its neutrons. The minimum mass for which $k$ reaches 1 is the **critical mass** — about 52 kg for a bare sphere of pure U-235, much less with a neutron-reflecting shell.

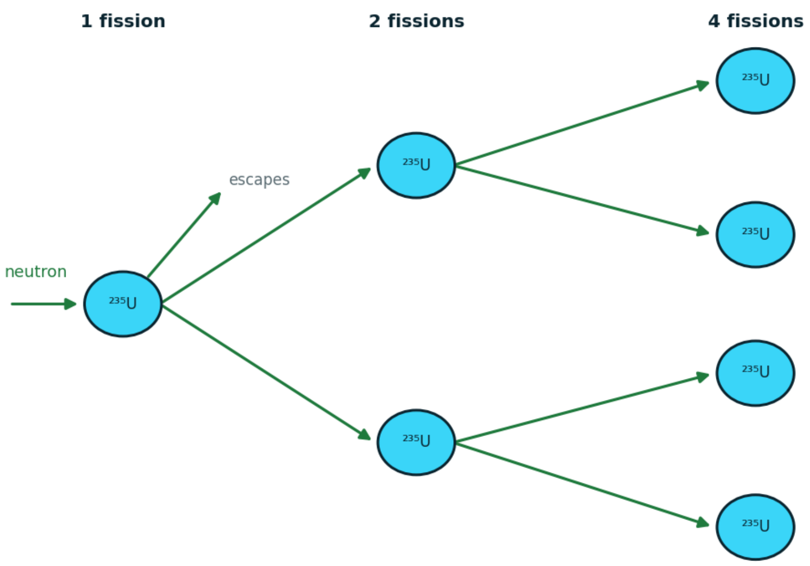


*Figure 2. Chain reaction. One neutron induces fission of $^{235}_{92}U$, which releases 2−3 fast neutrons; if on average at least one goes on to cause a further fission the reaction is self-sustaining. With two neutrons per fission propagating, the number of fissions doubles each generation (1 → 2 → 4); the rest escape or are absorbed.*

**Worked example 2 — growth of a chain reaction**

In a reactor being brought up to power, $k$ = 1.02. By what factor does the fission rate grow over 50 neutron generations? What if $k$ = 0.98?

Each generation multiplies the rate by $k$, so after 50 generations the factor is $1.02^{50}$ ≈ **2.7**.

With $k$ = 0.98 the factor is $0.98^{50}$ ≈ **0.36** — the reaction dies away.

Neutron generations are about $10^{-4}$ s apart in a thermal reactor, so even tiny departures of $k$ from 1 change the power quickly. Fine control is possible only because a small fraction of neutrons ('delayed neutrons') are emitted seconds later by the fragments.

## Why natural uranium must be enriched

Natural uranium is 99.3% U-238 and only **0.7% U-235**. Only U-235 is fissile with slow neutrons; U-238 usually just captures a neutron (becoming U-239) without splitting, especially at intermediate neutron energies. With so little U-235, too many neutrons are captured uselessly and $k$ cannot reach 1 in a simple assembly. Reactor fuel is therefore **enriched** to about 3-5% U-235 (weapons require > 90%). Enrichment is difficult precisely because the two isotopes are chemically identical — separation must exploit the tiny mass difference, for example in gas centrifuges.

# 3. Inside a nuclear power station

A nuclear power station is a heat engine: the reactor core simply replaces the furnace of a coal-fired station. You must know the job of each component.

| Component | Typical material | Function |
|---|---|---|
| Fuel rods | Enriched $UO_2$ pellets in metal cladding | Contain the fissile U-235. Fission of U-235 releases the energy, almost all as kinetic energy of fragments, which becomes thermal energy of the fuel. |
| Moderator | Water, heavy water or graphite | Slows fast fission neutrons (≈ 2 MeV) to thermal energies (≈ 0.025 eV) by repeated collisions, because slow neutrons are far more likely to induce fission in U-235. |
| Control rods | Boron or cadmium | Absorb neutrons without fissioning. Raising or lowering the rods tunes *k*: fully inserted they shut the reactor down (scram). |
| Coolant | Pressurised water, $CO_2$ gas or liquid sodium | Carries thermal energy away from the core to the heat exchanger; in a pressurised-water reactor the water is both coolant and moderator. |
| Heat exchanger / steam generator | - | Transfers energy from the (radioactive) primary coolant loop to a separate clean water loop, boiling it to steam. Keeps radioactive material out of the turbine. |
| Shielding | Thick concrete and steel | Absorbs neutrons and γ radiation, protecting workers; the steel pressure vessel and containment building also confine any release. |

### Why must the neutrons be slowed?

Fission neutrons are born fast. A slow ('thermal') neutron spends far longer near a nucleus as it passes, so its capture probability by U-235 is hundreds of times greater. A good moderator has light nuclei — a neutron colliding with a nucleus of similar mass (hydrogen in water is ideal) transfers a large fraction of its kinetic energy per elastic collision, just as a snooker ball stops dead when it hits an equal ball head-on. It must also be a poor neutron *absorber*, or the neutrons would be lost rather than slowed.

### The energy transfer chain

Nuclear (rest-mass) energy → kinetic energy of fragments and neutrons → thermal energy of fuel and coolant → internal energy of steam → kinetic energy of turbine → electrical energy from the generator. Like any heat engine the station is limited by thermodynamics: typical overall efficiency is about 30-35%, the rest being dissipated in cooling towers or the sea.

> **Classic exam confusion — moderator vs control rods:** The **moderator slows** neutrons so they cause *more* fission; the **control rods absorb** neutrons so they cause *less*. Removing the moderator does not speed the reaction up — it kills it, because fast neutrons hardly fission U-235 at all. Removing control rods increases the fission rate.

## 4. Energy accounting: why fission fuel is so concentrated

One fission releases about 200 MeV. Burning one carbon atom in a chemical reaction releases about 4 eV. Nuclear processes are therefore some 50 million times more energetic per atom than chemical ones — the single most important number in any discussion of nuclear power.

**Worked example 3 — energy per fission in SI units**

Convert 200 MeV to joules.

200 MeV = $200 \times 10^{6} \times 1.602 \times 10^{-19}$ J = **$3.2 \times 10^{-11}$ J** per fission.

Tiny per event — but a gram of U-235 contains about $2.6 \times 10^{21}$ nuclei.

**Worked example 4 — 1 kg of U-235 versus coal**

Estimate the energy released by complete fission of 1.0 kg of U-235, and the mass of coal (energy density 30 MJ $kg^{-1}$) giving the same energy.

Number of nuclei: $N = (1000 / 235) \times 6.02 \times 10^{23} = 2.6 \times 10^{24}$.

Energy: $E = 2.6 \times 10^{24} \times 3.2 \times 10^{-11}$ J ≈ **$8.2 \times 10^{13}$ J**.

Equivalent coal: $m = 8.2 \times 10^{13} / 3.0 \times 10^{7} \approx 2.7 \times 10^{6}$ kg ≈ **2700 tonnes of coal** — about 45 railway wagons, replaced by a fuel pellet stack you could carry.

**Worked example 5 — fuel consumption of a power station**

A station delivers 1.0 GW of electrical power at 33% efficiency. Find (a) the number of fissions per second, (b) the mass of U-235 fissioned per day, (c) the rest mass converted to energy per day.

(a) Thermal power needed = 1.0 / 0.33 = 3.0 GW. Fission rate = $3.0 \times 10^{9} / 3.2 \times 10^{-11}$ ≈ **$9.4 \times 10^{19}$ $s^{-1}$**.

(b) Fissions per day = $9.4 \times 10^{19} \times 86\,400 = 8.1 \times 10^{24}$. Mass = $(8.1 \times 10^{24} / 6.02 \times 10^{23}) \times 235$ g ≈ **3.2 kg per day**.

(c) E per day = $3.0 \times 10^{9} \times 86\,400 = 2.6 \times 10^{14}$ J. $\Delta m = E / c^2 = 2.6 \times 10^{14} / (3.0 \times 10^{8})^2$ ≈ **2.9 g per day**.

Note the distinction: about 3.2 kg of uranium *undergoes fission*, but only about 3 g of mass (0.09%) actually *disappears* as energy.

**Unit conversions you must be fluent in:** 1 u = 931.5 MeV $c^{-2}$; 1 eV = $1.602 \times 10^{-19}$ J; 1 MeV = $1.602 \times 10^{-13}$ J. The factor 931.5 converts *mass in u* directly to *energy in MeV* — do not multiply by $c^2$ again afterwards.

# 5. Nuclear waste, plutonium and decommissioning

## Why the fragments are radioactive

Heavy nuclei need a much higher neutron-to-proton ratio than mid-mass nuclei (U-235 has N/Z ≈ 1.55; stable nuclei near A = 100 need about 1.3). The fragments inherit the parent's ratio, so they are strongly **neutron-rich** and decay by $\beta^-$ emission ($n \rightarrow p + e^- +$ anti-ν), often through chains of several decays with γ emission, until a stable nuclide is reached. This is why spent fuel stays hot and dangerous long after removal from the reactor.

| Waste / product | Examples and half-lives | Handling |
|---|---|---|
| Short-lived fission products | I-131 (8.0 days), Xe-133 (5.2 days) | Intensely active but decay within months; the main hazard in an accident release. |
| Medium-lived fission products | Sr-90 (28.8 years), Cs-137 (30.1 years) | Dominate the activity of spent fuel for the first few centuries; need isolation for roughly 10 half-lives (≈ 300 years). |
| Long-lived products and actinides | Tc-99 ($2.1 \times 10^5$ years), Pu-239 ($2.4 \times 10^4$ years) | Low activity but essentially permanent on human timescales; require geological disposal. |

## Storage strategy

- **Short term (years):** spent fuel rods are stored under several metres of water in cooling ponds at the reactor site; the water removes decay heat and absorbs the radiation.
- **Medium term (decades):** once cool enough, fuel is sealed in steel-and-concrete dry casks on guarded sites.
- **Long term (millennia):** high-level waste is vitrified (fused into glass), sealed in corrosion-resistant canisters and buried in deep, geologically stable rock formations far from groundwater.
- **Decommissioning:** at end of life the reactor structure itself is radioactive (neutron activation of the steel and concrete); dismantling and site clean-up take decades and form a significant part of the true cost of nuclear electricity.

## Plutonium production

Neutron capture by the abundant U-238 in the fuel breeds plutonium:

$$^{238}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{239}_{92}\mathrm{U} \rightarrow (\beta^-, \text{23 min})\ {}^{239}_{93}\mathrm{Np} \rightarrow (\beta^-, \text{2.4 days})\ {}^{239}_{94}\mathrm{Pu}$$

Pu-239 is itself fissile: some of it fissions in place (contributing roughly a third of the energy of a fuel rod late in its life), and the rest remains in the spent fuel. It can be chemically separated and reused as fuel — but the same material can be used in weapons, which is the heart of the **proliferation** concern.

**Worked example 6 — how long must waste be stored?**

Cs-137 has a half-life of 30 years. How long until its activity falls to 1% of its initial value?

$(1/2)^n = 0.01$ gives $n = \ln 100 / \ln 2 \approx 6.6$ half-lives.

$t = 6.6 \times 30 \approx$ **200 years**.

This is why the few-centuries timescale is quoted for fission-product waste, while plutonium (half-life 24 000 years) forces the geological, 100 000-year perspective.

# 6. Safety systems, risk and the case for and against

## Engineered safety

- **Defence in depth:** multiple independent barriers — ceramic fuel pellets, sealed metal cladding, the steel pressure vessel, and finally the reinforced concrete **containment building** designed to hold any release.

- **Negative feedback by design:** in water-moderated reactors, if the core overheats the water expands or boils, moderation drops, fewer neutrons are thermalised and the fission rate falls automatically. (The Chernobyl design had the opposite, positive, feedback in part of its operating range.)
- **Automatic shutdown (scram):** control rods drop into the core under gravity or springs within seconds if any sensor trips.
- **Redundant cooling:** decay heat continues after shutdown (≈ 1% of full power hours later - megawatts in a large reactor), so independent, often passive, emergency cooling systems are required. Loss of decay-heat cooling caused the Fukushima meltdowns.

### Evaluating nuclear power — the balanced view IB expects

| Arguments for | Arguments against |
|---|---|
| Enormous energy density: very little fuel and mining per joule delivered. | Spent fuel is highly radioactive; safe storage is needed for centuries to millennia and no country has yet completed a permanent repository. |
| No $CO_2$ or particulate emission during operation - a low-carbon baseload source. | Accidents are rare but can be severe and long-lasting (Chernobyl 1986, Fukushima 2011), with large exclusion zones. |
| Reliable, continuous output independent of weather, complementing intermittent renewables. | Very high capital and decommissioning costs; long construction times. |
| Large fuel reserves; energy security for countries without fossil fuels. | Enrichment and plutonium technologies can be diverted to weapons (proliferation risk); plants are potential targets. |
| Modern designs have passive safety and can recycle fuel. | Public opposition and perceived risk affect siting; thermal pollution of rivers and seas from cooling. |

> **Exam technique:** 'Discuss' or 'evaluate' questions are marked for balance. Give at least two distinct points on each side, make them physical (energy density, half-lives, feedback) rather than purely political, and finish with a one-line judgement.

## 7. Common pitfalls

- Computing Δm the wrong way round: it is reactants minus products, and it must come out positive for an energy-releasing reaction.
- Confusing the **moderator** (slows neutrons, enables fission) with the **control rods** (absorb neutrons, throttle fission) - the classic E.4 error.
- Multiplying by $c^2$ after already using 1 u = 931.5 MeV $c^{-2}$: the 931.5 factor has done the conversion for you.
- MeV ↔ J slips: 1 MeV = $1.602 \times 10^{-13}$ J (not $10^{-19}$, which is for eV).
- Saying energy is released because binding energy *decreases*: total binding energy **increases**; it is the rest mass that decreases. 'Higher binding energy per nucleon = more stable' is the safe phrasing.
- Forgetting that the emitted neutrons are **fast** and must be moderated before they can efficiently cause further fissions of U-235.

- Treating fission fragments as stable: they are neutron-rich $\beta^-$ emitters, which is the whole origin of the waste problem.
- Balancing only mass numbers in a nuclear equation: check charge (Z) as well.

## 8. Quick reference

| Item | Value / statement |
|---|---|
| Typical fission | ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + \approx 200$ MeV |
| Energy per fission | $\approx 200$ MeV $\approx 3.2 \times 10^{-11}$ J (about $5 \times 10^{7}$ times a chemical bond) |
| Mass-energy conversion | $\Delta E = \Delta mc^2$; 1 u = 931.5 MeV $c^{-2}$; 1 MeV = $1.602 \times 10^{-13}$ J |
| Chain reaction | sustained when k = 1 (critical); needs mass ≥ critical mass |
| Natural / enriched uranium | 0.7% U-235 naturally; 3-5% in reactor fuel; only U-235 is fissile with thermal neutrons |
| Moderator | light nuclei (water, graphite): slow fast neutrons to thermal (≈ 0.025 eV) |
| Control rods | boron / cadmium: absorb neutrons to hold k = 1 or shut down |
| Fission fragments | neutron-rich → $\beta^-$ and γ decay; half-lives from days (I-131) to $10^{5}$ years (Tc-99) |
| Plutonium breeding | ${}^{238}_{92}U(n,\gamma){}^{239}_{92}U \rightarrow {}^{239}_{93}Np \rightarrow {}^{239}_{94}Pu$ (two $\beta^-$ decays) |

## 9. Test yourself

Attempt these without notes; full answers below. Take 1 u = 931.5 MeV $c^{-2}$.

1. Define induced nuclear fission and explain, with reference to the binding energy per nucleon curve, why it releases energy.

2. Complete and balance: ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + x\,{}^{1}_{0}n$. Find x.

3. For the fission ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{139}_{54}Xe + {}^{95}_{38}Sr + 2{}^{1}_{0}n$, the masses are m(U-235) = 235.043930 u, m(Xe-139) = 138.918793 u, m(Sr-95) = 94.919359 u, m(n) = 1.008665 u. Find the energy released in MeV and in joules.

4. Distinguish the roles of the moderator and the control rods, and state what happens to the fission rate when the control rods are (a) withdrawn slightly, (b) fully inserted.

5. Explain why natural uranium cannot sustain a chain reaction in a simple assembly and why enrichment is difficult.

6. A reactor produces 2.4 GW of thermal power with an average of 200 MeV per fission. Calculate (a) the fission rate, (b) the mass of U-235 consumed per year ($3.16 \times 10^{7}$ s).

7. Sr-90 has a half-life of 28.8 years. After how long is its activity 6.25% of the initial value? Explain why fission waste needs both century-scale and millennium-scale storage plans.

8. Outline two physical advantages and two physical disadvantages of fission power compared with a coal-fired station of the same output.

### Answers

1. A slow neutron is absorbed by a heavy fissile nucleus (e.g. U-235), forming an excited compound nucleus that splits into two mid-mass fragments plus 2-3 neutrons. The fragments lie nearer the maximum of the binding-energy-per-nucleon curve (near Fe-56) than uranium does, so total binding energy increases; the corresponding decrease in rest mass is released as energy, ≈ 200 MeV per fission, mostly as fragment kinetic energy.

2. Mass numbers: 1 + 235 = 140 + 94 + x, so **x = 2**. Check charge: 0 + 92 = 54 + 38 + 0. Both conserved.

3. Mass before = 236.052595 u; mass after = 138.918793 + 94.919359 + 2 × 1.008665 = 235.855482 u. Δm = 0.197113 u. E = 0.197113 × 931.5 = **184 MeV** = 184 × 1.602 × $10^{-13}$ ≈ **2.9 × $10^{-11}$ J**.

4. The moderator slows fast fission neutrons to thermal energies so that they can efficiently induce further fissions in U-235; the control rods absorb neutrons to regulate how many survive to cause fission. (a) Withdrawing rods absorbs fewer neutrons, k rises above 1 and the fission rate (and power) increases until the rods are reset; (b) full insertion drives k well below 1 and the chain reaction stops (only decay heat remains).

5. Only 0.7% of natural uranium is U-235; the dominant U-238 captures neutrons without fissioning, so too few neutrons cause new fissions and k < 1. Enrichment cannot use chemistry because isotopes are chemically identical; it relies on the 1% mass difference between $^{235}UF_6$ and $^{238}UF_6$ molecules in centrifuges, which is slow and expensive.

6. (a) Rate = 2.4 × $10^9$ / 3.2 × $10^{-11}$ = **7.5 × $10^{19}$ fissions $s^{-1}$**. (b) Fissions per year = 7.5 × $10^{19}$ × 3.16 × $10^7$ = 2.4 × $10^{27}$; mass = (2.4 × $10^{27}$ / 6.02 × $10^{23}$) × 235 g ≈ **9.2 × $10^2$ kg ≈ 1 tonne per year**.

7. 6.25% = 1/16 = $(1/2)^4$, so 4 half-lives: t = 4 × 28.8 ≈ **115 years**. Fission products such as Sr-90 and Cs-137 (half-lives ≈ 30 y) decay to safe levels within a few centuries, but actinides such as Pu-239 (2.4 × $10^4$ y) remain hazardous for tens of millennia, requiring deep geological disposal.

8. Advantages: energy density ≈ $10^6$ times greater, so far less fuel is mined and transported per joule; no $CO_2$ or particulates released in operation, and output is steady baseload. Disadvantages: the neutron-rich fission products make spent fuel intensely radioactive, requiring cooling and isolation for centuries; rare accidents can release long-lived isotopes over wide areas, and decommissioning the activated plant is slow and costly (plus proliferation risk from enrichment and plutonium).