IB DP PHYSICS

Theme E: Nuclear and Quantum Physics

E.3 Radioactive Decay

Revision Notes · Standard and Higher Level

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Original notes prepared for the IB Diploma Programme Physics course (first assessment 2025)

What the syllabus requires

By the end of E.3 you should be able to work confidently with each of the following. Use this list as a final checklist before the exam. Items marked (HL) are Higher Level only.

Understanding	You should be able to...
Nuclear stability	Interpret the N-Z (neutron number against proton number) curve; explain why heavy stable nuclei need more neutrons than protons.
Isotopes	Recognise nuclides with the same Z but different N, and that unstable isotopes decay towards the stability curve.
α, β^-, β^+ and γ radiation	State their nature, charge and mass; compare ionising power, penetration and behaviour in electric and magnetic fields; write balanced decay equations conserving A and Z.
Neutrinos and antineutrinos	Explain how the continuous energy spectrum of beta particles is evidence for a third particle sharing the decay energy.
Mass defect and binding energy	Use Δm and $E = mc^2$; sketch and interpret the binding energy per nucleon curve (maximum near Fe-56).
Background radiation	List its sources and correct measured count rates for background before any analysis.
Half-life	Define half-life; determine it from a decay curve; solve problems with whole numbers of half-lives; apply it to radioactive dating.
(HL) The decay law	Use $N = N_0e^{-\lambda t}$, $A = \lambda N$, $A = A_0e^{-\lambda t}$ and $\lambda = \ln 2 / T_{1/2}$ for any elapsed time, not just whole half-lives.
(HL) Measuring half-lives	Outline how long and short half-lives are determined experimentally.

> **Exam note:** Sections 1-6 of these notes are common to SL and HL. Section 7 (the exponential decay law) is HL only, but SL students may still find it useful background for the iterative method.

1. Nuclear stability and the N-Z curve

A nucleus is held together by the **strong nuclear force**, an attractive, very short range ($\approx 10^{-15}$ m) force acting between all nucleons - proton-proton, proton-neutron and neutron-neutron alike. Working against it is the **electrostatic (Coulomb) repulsion** between the protons, which is weaker at short range but is long range and acts between *every* pair of protons in the nucleus. A nucleus is stable only if these effects balance.

Plotting neutron number N against proton number Z for all known stable nuclides gives the **curve (band) of stability**:

- For light nuclei (Z up to about 20) the stable nuclides lie close to the line N = Z (equal numbers of protons and neutrons, e.g. $^{12}_{6}C$, $^{16}_{8}O$).
- As Z increases the curve bends **above** the N = Z line: heavy stable nuclei have N/Z ratios approaching about 1.5 (e.g. $^{208}_{82}Pb$ has 126 neutrons to 82 protons).

- No nuclide with Z > 83 (bismuth) is completely stable; the heaviest nuclei all decay, most of them by alpha emission.

Why do heavy nuclei need extra neutrons?

Each added proton repels *all* the other protons electrostatically, no matter how large the nucleus grows, whereas the strong force saturates: a nucleon only attracts its nearest neighbours. Extra neutrons add strong-force attraction (and spacing between protons) *without* adding any repulsion, so as Z rises an increasing surplus of neutrons is needed to hold the nucleus together. Eventually even this fails, which is why the very heaviest nuclei are unstable.

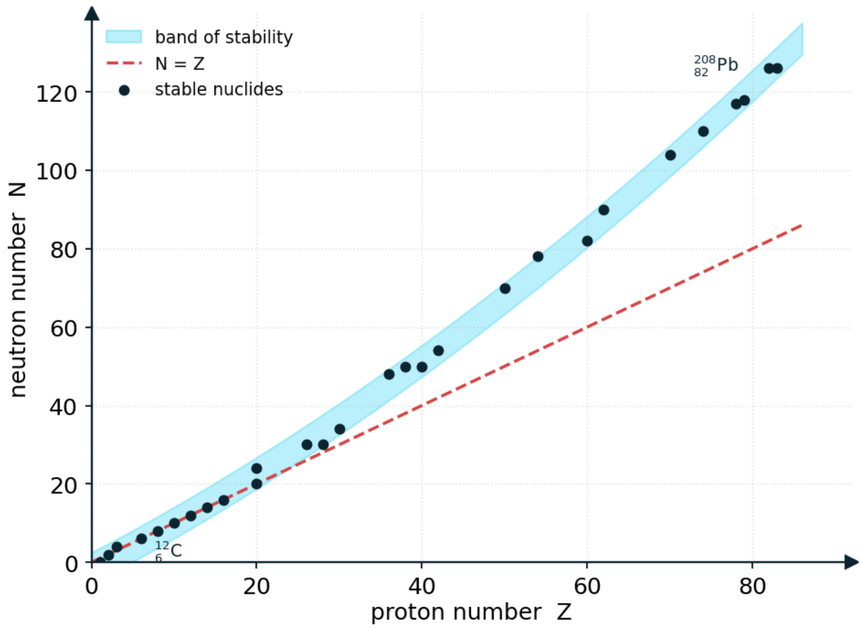

Figure 1. Band of stability: neutron number N against proton number Z for stable nuclides. Light stable nuclei sit on the line N = Z (dashed); as Z grows the stable band curves above it, because heavy nuclei need a neutron surplus. For example $^{208}_{82}$Pb has N/Z ≈ 1.54, whereas $^{12}_{6}$C has N/Z = 1.00.

Nuclides that lie **off** the stability curve are unstable: they transform spontaneously, by one decay after another if necessary, until the daughter nuclide lands on the curve. The type of decay depends on *which side* of the curve the nuclide sits (Section 2.4).

> **Vocabulary check:** Nuclide = a particular nuclear species, specified by both Z and A. Isotopes = nuclides of the same element (same Z) with different N and therefore different A. Radioactive decay is both **random** (we cannot predict when a given nucleus will decay) and **spontaneous** (unaffected by temperature, pressure or chemical state).

2. Alpha, beta and gamma radiation

2.1 The three (four) radiations compared

Property	Alpha (α)	Beta-minus (β^-)	Beta-plus (β^+)	Gamma (γ)
Nature	Helium-4 nucleus, ${}^{4}_{2}He$	Fast electron emitted by the nucleus	Positron (anti-electron) emitted by the nucleus	High-frequency photon (electromagnetic radiation)
Charge	+2e	-e	+e	0
Rest mass	≈ 4 u (6.6×10^{-27} kg)	m_e (9.1×10^{-31} kg)	m_e	0
Typical speed	≈ 0.05c	up to ≈ 0.99c	up to ≈ 0.99c	c
Ionising power	Very high	Moderate	Moderate	Low
Penetration	Stopped by paper or a few cm of air	Stopped by ≈ 5 mm of aluminium or ≈ 1 m of air	Annihilates with an electron soon after emission, producing two γ photons	Intensity reduced by several cm of lead or ≈ 1 m of concrete; never fully stopped
Deflection in E and B fields	Deflected (as a positive charge); small deflection because of large mass	Deflected opposite to α; large deflection (small mass)	Deflected same way as α but much more strongly	Undeflected

Ionising power and penetration are inversely related: the alpha particle, being massive, slow and doubly charged, interacts strongly with atoms in its path, creating something like 10^5 ion pairs per cm of air - and for the same reason it runs out of energy after only a few centimetres.

> **Exam technique:** In a magnetic field, use the right-hand slap / F = qv×B rule remembering that β^- is negative. Examiners often draw all three radiations entering the same field: γ goes straight through, α and β^- curve opposite ways, and β^- curves far more tightly because its mass is about 7300 times smaller.

2.2 Decay equations: conserving A and Z

In every nuclear equation the total nucleon number A and the total charge (proton number Z) are conserved. Balancing these two numbers is how you identify the daughter nuclide.

Decay	General equation	What happens inside the nucleus
Alpha	${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$	The nucleus ejects a tightly bound cluster of 2 protons + 2 neutrons
Beta-minus	${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}$	A neutron becomes a proton: $n \rightarrow p + e^- + \bar{\nu}$
Beta-plus	${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e + \nu$	A proton becomes a neutron: $p \rightarrow n + e^+ + \nu$
Gamma	${}^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + \gamma$	An excited nucleus (often left excited by a previous α or β decay) drops to a lower energy state; no change in A or Z

Worked example 1 - alpha decay of uranium-238

Uranium-238 decays by alpha emission. Write the decay equation and identify the daughter nuclide.

A: 238 = 234 + 4. Z: 92 = 90 + 2. Element 90 is thorium.

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

The alpha particle carries a discrete kinetic energy of about 4.2 MeV; the massive daughter recoils with a small share of the energy (momentum conservation).

Worked example 2 - beta-minus decay of carbon-14

Carbon-14 is neutron-rich. Write its decay equation.

A is unchanged (the electron has A = 0); Z rises by 1: 6 → 7 (nitrogen).

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}$$

Check the charge line: 6 = 7 + (-1) + 0. The antineutrino is needed both to balance lepton number and to share the decay energy (Section 3).

Worked example 3 - beta-plus decay of fluorine-18

Fluorine-18, the proton-rich tracer used in PET medical scans, decays by β^+ emission. Write the equation.

A is unchanged; Z falls by 1: 9 → 8 (oxygen).

$$^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + {}^{0}_{+1}\mathrm{e} + \nu$$

The emitted positron annihilates with a nearby electron, producing a back-to-back pair of 0.511 MeV gamma photons - exactly what the PET scanner detects.

2.3 Gamma emission

Gamma decay usually follows another decay. For example cobalt-60 decays by β^- to an **excited state** of nickel-60 (written $^{60}_{28}\mathrm{Ni}^*$), which immediately de-excites by emitting gamma photons of discrete energies (1.17 MeV and 1.33 MeV). The discrete gamma energies are evidence that the **nucleus itself has discrete energy levels**, just as atomic spectra are evidence for electron energy levels.

2.4 Which decay happens where on the N-Z curve?

Position of nuclide	Problem	Decay that fixes it	Effect on the N-Z plot
Above / left of the stability curve	Too many neutrons (N/Z too high)	β^- decay: n → p	N falls by 1, Z rises by 1: moves diagonally down-right towards the curve
Below / right of the curve	Too many protons (N/Z too low)	β^+ decay (or electron capture): p → n	N rises by 1, Z falls by 1: moves diagonally up-left towards the curve

Position of nuclide	Problem	Decay that fixes it	Effect on the N-Z plot
Beyond the top of the curve (A ≈ > 210)	Nucleus simply too big	α decay	Loses 2 protons and 2 neutrons; repeated α (and β^-) steps form a decay chain ending at stable lead

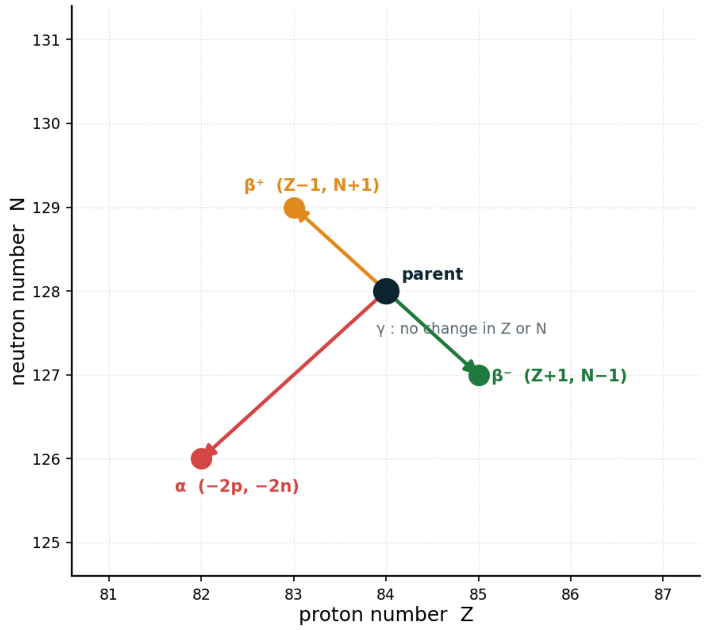

Figure 2. How each decay moves a nuclide on the N−Z chart. α decay removes 2 protons and 2 neutrons (down−left); β^- decay turns a neutron into a proton (Z +1, N −1: down−right); β^+ decay turns a proton into a neutron (Z −1, N +1: up−left); γ emission changes neither Z nor N.

3. The neutrino and the beta energy spectrum

Alpha particles from a given nuclide are emitted with **discrete** energies: the decay is a two-body process, so conservation of energy and momentum fixes exactly how the released energy is shared between the alpha particle and the recoiling daughter. Every alpha from ${}^{238}_{92}U$ carries essentially the same kinetic energy.

Beta particles are different. Measurements show a **continuous spectrum**: beta particles from the same nuclide emerge with any kinetic energy from almost zero up to a definite maximum E_{max}, which equals the total energy released by the decay. If the decay produced only the daughter nucleus and the electron, this would be impossible - a two-body decay must give the electron a single fixed energy, and energy would appear to be missing whenever the electron carried less than E_{max}.

Pauli's solution (1930) was a third, unseen particle: the **neutrino** (in β^- decay, strictly an antineutrino $\bar{\nu}$). It is neutral, has almost no mass and interacts extremely weakly with matter, so it escaped detection until 1956. The decay energy is shared, in random proportions, among the electron, the antineutrino and the recoiling daughter:

- Electron near E_{max}: the antineutrino carries almost nothing.
- Electron with little energy: the antineutrino carries almost all of E_{max}.
- Every intermediate split occurs, producing the smooth continuous spectrum.

> **Classic exam question:** "Explain how the beta energy spectrum provides evidence for the existence of the neutrino." Answer in three steps: (1) beta particles show a continuous range of energies up to a maximum; (2) the energy released in the decay is fixed, so a two-body decay would give the beta particle one discrete energy; (3) a third particle must carry away the remaining energy (and momentum) - the (anti)neutrino.

4. Mass defect and binding energy

4.1 Mass defect

Every nucleus has **less** mass than the total mass of its separated protons and neutrons. The difference is the **mass defect**:

$$\Delta m = Z\, m_p + (A - Z)\, m_n - m_{nucleus}$$

The missing mass was released as energy when the nucleus formed, according to Einstein's relation $E = mc^2$. Equivalently, the **binding energy** $E_b = \Delta m\, c^2$ is the energy that would have to be *supplied* to pull the nucleus apart into free nucleons. A large binding energy means a strongly bound, hard-to-break nucleus.

Nuclear masses are usually quoted in **unified atomic mass units**: 1 u = 1.661×10^{-27} kg, defined as one twelfth of the mass of a carbon-12 atom. The energy equivalent, which you should use in almost every calculation, is:

$$\mathbf{1\ u = 931.5\ MeV\ c^{-2}}\ \text{(so a mass defect of } \Delta m \text{ u releases } \Delta m \times 931.5 \text{ MeV)}$$

Worked example 4 - binding energy of helium-4

Data: m_p = 1.007276 u, m_n = 1.008665 u, nuclear mass of ${}^{4}_{2}He$ = 4.001506 u. Find the binding energy in MeV and the binding energy per nucleon.

Mass of separate nucleons = 2(1.007276) + 2(1.008665) = 2.014552 + 2.017330 = 4.031882 u.

Δm = 4.031882 - 4.001506 = 0.030376 u.

E_b = 0.030376 × 931.5 = **28.3 MeV**.

Binding energy per nucleon = 28.3 / 4 = **7.07 MeV per nucleon** - remarkably high for such a light nucleus, which is why the alpha particle is emitted as a ready-made unit in alpha decay.

4.2 The binding energy per nucleon curve

Binding energy per nucleon, E_b/A, measures how tightly bound the *average* nucleon is, and is the correct measure of nuclear stability. Plotted against nucleon number A, the curve:

- rises steeply for light nuclei (with a pronounced local spike at ${}^{4}_{2}He$),
- reaches a broad **maximum of about 8.8 MeV per nucleon near iron-56** (${}^{56}_{26}Fe$, and its neighbour ${}^{62}_{28}Ni$) - the most stable nuclei in nature,
- then falls gently to about 7.6 MeV per nucleon at uranium-238.

Any nuclear change that moves the products **towards the peak** increases the binding energy per nucleon and therefore **releases energy**: fusion of light nuclei climbs the steep left side (the Sun, Topic E.4-style contexts), while fission of heavy nuclei and radioactive decay creep down from the right side towards iron. The energy released in any reaction equals the total increase in binding energy, or equivalently $\Delta m c^2$ computed from the mass difference between the two sides of the equation.

> **Common misconception:** It is binding energy **per nucleon**, not total binding energy, that measures stability. Uranium-238 has a far larger total binding energy than iron-56 simply because it has more nucleons, yet it is less stable.

5. Background radiation

A Geiger counter clicks even with no source in the room. This ever-present **background radiation** is mostly natural:

Source	Notes
Radon gas (largest natural share)	Radioactive $^{222}_{86}Rn$ seeps out of rocks and soil and accumulates in buildings; it and its daughters are alpha emitters that can be inhaled
Rocks, soil and building materials	Gamma radiation from traces of uranium, thorium and their decay chains
Cosmic rays	High-energy particles from the Sun and beyond; the dose roughly doubles for every ≈ 1500 m of altitude, so aircrew receive more
Food and drink / our own bodies	Mainly potassium-40 and carbon-14 incorporated into living tissue
Artificial sources	Dominated by medical uses (X-rays, CT scans, nuclear medicine); tiny contributions from weapons-test fallout and the nuclear industry

Correcting for background

Because the detector cannot tell background counts from source counts, **every quantitative experiment must begin by measuring the background count rate with the source removed**, then subtracting it from every reading:

corrected count rate = measured count rate - background count rate

Half-lives, decay constants and absorption measurements must always be computed from **corrected** rates. Forgetting the subtraction is one of the most common ways to lose marks in Paper 2 and in the collaborative sciences/IA context: an uncorrected decay curve flattens out at the background level instead of tending to zero, and half-lives read from it are systematically too long.

6. Half-life and activity

6.1 Definitions

- **Activity** A of a sample = number of decays per second. Unit: the becquerel, 1 Bq = 1 decay s^{-1}. (A count rate is generally *less* than the activity, because the detector intercepts only a fraction of the emitted radiation.)
- **Half-life** $T_{1/2}$ = the time taken for half of the radioactive nuclei in a sample to decay; equivalently, the time for the activity (or corrected count rate) to halve.

Because decay is random, the half-life is a *statistical* statement about very large numbers of nuclei - it says nothing about when an individual nucleus will decay. Whatever moment you start timing, half of what is present decays in the next $T_{1/2}$: after 1, 2, 3, 4 ... half-lives the fractions remaining are 1/2, 1/4, 1/8, 1/16, ...

6.2 Reading a decay curve

A graph of corrected count rate (or activity, or N) against time is an exponential decay curve. To find $T_{1/2}$: pick a convenient starting value on the y-axis, read off the time at which the curve reaches half that value, and repeat from a different start to check - a true exponential gives the same half-life every time, wherever you begin. Averaging several such readings reduces random error.

Worked example 5 - half-life with a background correction

With no source present a detector records 10 counts per minute. With a radioactive sample in place it records 210 counts per minute, and 6.0 hours later it records 35 counts per minute. Find the half-life.

Corrected initial rate = 210 - 10 = 200 min^{-1}. Corrected final rate = 35 - 10 = 25 min^{-1}.

200 → 100 → 50 → 25: the rate has halved 3 times ($200/25 = 8 = 2^3$).

So 3 $T_{1/2}$ = 6.0 h, giving **$T_{1/2}$ = 2.0 hours**.

Without the background correction you would use 210 → 35, a factor of 6 - not a whole number of halvings - and get the wrong answer.

Worked example 6 - radiocarbon dating

Living material maintains a constant, known proportion of $^{14}_{6}C$ ($T_{1/2}$ = 5730 years) because it constantly exchanges carbon with the atmosphere, where C-14 is replenished by cosmic rays. At death the exchange stops and the C-14 decays. A wooden bowl is found to have a C-14 activity per gram of carbon equal to 1/8 of that of living wood. Estimate its age.

$1/8 = (1/2)^3$, so exactly 3 half-lives have passed.

Age = 3 × 5730 = 17 190 ≈ **1.7×10^4 years**.

The same logic dates rocks over billions of years using long-lived nuclides such as $^{238}_{92}U$ ($T_{1/2}$ = 4.5×10^9 yr) or $^{40}_{19}K$: comparing the amount of parent remaining with the amount of stable daughter trapped in the rock gives the number of half-lives since the rock solidified.

> **Choosing an isotope:** A useful dating clock must have a half-life comparable to the age being measured. C-14 (5730 yr) is useless beyond about 50 000 years - too little remains to measure - and useless for rocks, which exchange no carbon; U-238 is useless for archaeology because essentially none of it has decayed in a few thousand years.

7. (HL) The exponential decay law

7.1 From randomness to an equation

Each nucleus has a fixed probability of decaying per unit time, the **decay constant** λ (unit s^{-1}, yr^{-1}, ...). It never changes: nuclei do not age. For N undecayed nuclei the expected number of decays per second - the activity - is therefore proportional to N:

$$A = \lambda N \text{ (equivalently } dN/dt = -\lambda N)$$

A quantity whose rate of loss is proportional to the amount remaining decays exponentially. Solving the equation gives the **decay law** and, since activity is proportional to N, the same law for activity and for corrected count rate C:

$$N = N_0\, e^{-\lambda t} \quad A = A_0\, e^{-\lambda t} \quad C = C_0\, e^{-\lambda t}$$

7.2 Linking λ and the half-life

Set $N = N_0/2$ at $t = T_{1/2}$: $N_0/2 = N_0\, e^{-\lambda T}$. Taking natural logs: $\ln(1/2) = -\lambda T_{1/2}$, so $\lambda T_{1/2} = \ln 2$ and

$$\lambda = \ln 2 / T_{1/2} = 0.693 / T_{1/2}$$

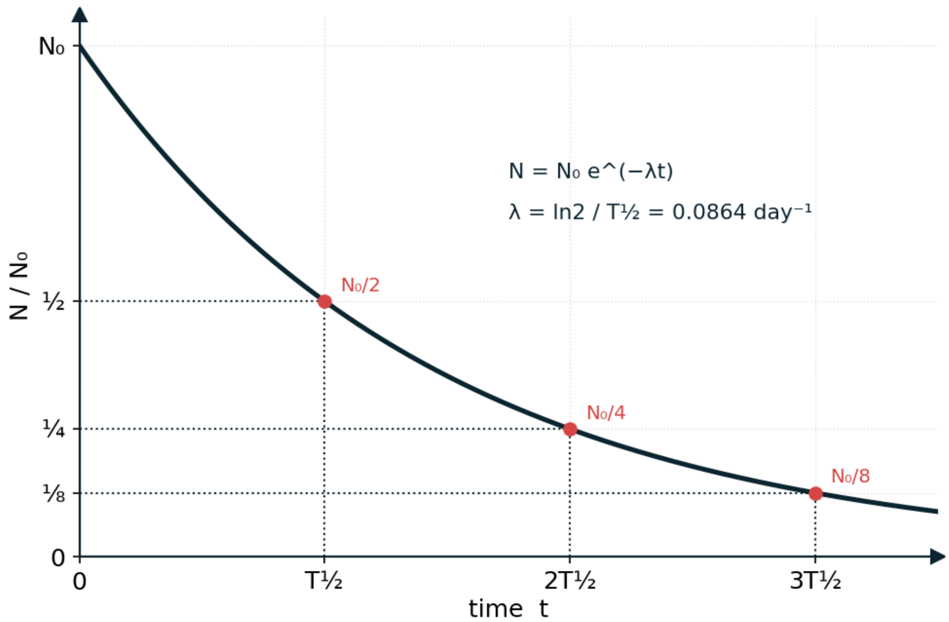

Figure 3. Exponential decay $N = N_0 e^{-\lambda t}$, shown for a worked half-life $T_{1/2}$ = 8.02 days. The number of nuclei halves every half-life, reaching $N_0/2$, $N_0/4$ and $N_0/8$ at $t = T_{1/2}$, $2T_{1/2}$, $3T_{1/2}$. Here $\lambda = \ln 2 / T_{1/2} \approx 0.0864\ \text{day}^{-1}$.

A large decay constant means a short half-life and vice versa: λ is the probability per unit time that any one nucleus decays, so it is also the fraction of a large sample decaying per unit time (for times much shorter than $T_{1/2}$).

Worked example 7 (HL) - activity of an iodine-131 sample

Iodine-131 (used in thyroid treatment) has $T_{1/2}$ = 8.02 days. A sample initially contains 8.0×10^{14} nuclei. Find (a) the decay constant in s^{-1}, (b) the initial activity, (c) the number of nuclei remaining after 20.0 days.

(a) $T_{1/2} = 8.02 \times 86\,400 = 6.93 \times 10^5$ s, so $\lambda = 0.693 / (6.93 \times 10^5) = \mathbf{1.00 \times 10^{-6}\ s^{-1}}$.

(b) $A_0 = \lambda N_0 = 1.00 \times 10^{-6} \times 8.0 \times 10^{14} = \mathbf{8.0 \times 10^8\ Bq}$.

(c) In days: $\lambda = 0.693/8.02 = 0.0864\ \text{day}^{-1}$, so $\lambda t = 0.0864 \times 20.0 = 1.73$.

$N = N_0\, e^{-1.73} = 8.0 \times 10^{14} \times 0.178 = \mathbf{1.4 \times 10^{14}}$ **nuclei**.

Note 20.0 days is 2.49 half-lives - not a whole number, so the SL halving method cannot finish the job; the exponential can.

Worked example 8 (HL) - non-integer half-lives and inverse problems

Strontium-90 has $T_{1/2}$ = 28.8 years. (a) What fraction of a sample remains after 100 years? (b) How long until only 1.0% remains?

(a) $\lambda = 0.693/28.8 = 0.0241\ \text{yr}^{-1}$. $\lambda t = 0.0241 \times 100 = 2.41$.

$N/N_0 = e^{-2.41} =$ **0.090, i.e. 9.0%**. (Check: $(1/2)^{100/28.8} = (1/2)^{3.47} \approx 0.090$.)

(b) $0.010 = e^{-\lambda t} \rightarrow \ln 0.010 = -\lambda t \rightarrow t = 4.61 / 0.0241 =$ **190 years** (about 6.6 half-lives).

7.3 Measuring half-lives (qualitative)

- **Short half-lives** (seconds to days): record the corrected count rate at regular intervals as the sample decays. Either read repeated halvings straight off the decay curve, or - better - plot ln C against t: the decay law gives $\ln C = \ln C_0 - \lambda t$, a straight line of gradient $-\lambda$, and then $T_{1/2} = \ln 2 / \lambda$.
- **Long half-lives** (years to billions of years): the activity never changes measurably during an experiment, so decay cannot be watched. Instead measure both quantities in $A = \lambda N$ directly: find N from the sample's mass, molar mass and Avogadro's constant (or by mass spectrometry), measure the (small) activity A with a calibrated detector, and compute $\lambda = A/N$.

> **Symbol warning:** λ in this topic is the **decay constant**, not a wavelength - and both meanings can appear on the same exam paper in Theme E. Read the context. Similarly A may mean activity or nucleon number: activity is italic *A* with units Bq; nucleon number is a pure number.

8. Common pitfalls

- Forgetting to subtract background from every count rate before finding a half-life.
- Treating count rate as if it were activity: the detector catches only a fraction of the decays. Ratios of count rates are fine; absolute values need care.

- Confusing λ the decay constant with λ the wavelength, or Bq with counts per minute (watch the units of λ and t - they must match before you compute $e^{-\lambda t}$).
- Writing beta decay without the antineutrino (or β^+ without the neutrino), or putting the wrong one in.
- Balancing A but not Z (or vice versa) in decay equations; remember the electron in β^- decay carries Z = -1.
- Saying half-life means “all gone after two half-lives”: a quarter remains, and a sample is never exactly zero.
- Using total binding energy instead of binding energy per nucleon when discussing stability.
- In mass defect calculations, mixing atomic masses (which include electrons) with nuclear masses; be consistent with the data given.
- Claiming decay rate depends on temperature or chemical state - it does not; decay is spontaneous.

9. Quick reference

Result	Statement
Stability	Light stable nuclei: N ≈ Z; heavy stable nuclei: N/Z up to ≈ 1.5; no stable nuclei beyond Z = 83
Alpha decay	${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$ (discrete α energies)
Beta-minus decay	${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}$ (continuous β spectrum)
Beta-plus decay	${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e + \nu$
Gamma decay	Excited nucleus emits a photon; A and Z unchanged
Mass defect	$\Delta m = Z\,m_p + (A - Z)\,m_n - m_{nucleus}$; $E_b = \Delta m\,c^2$; 1 u = 931.5 MeV c^{-2}
BE per nucleon curve	Peak ≈ 8.8 MeV/nucleon at Fe-56; fusion (left side) and fission/decay (right side) both move products towards the peak and release energy
Background	corrected rate = measured rate - background rate (always!)
Half-life	Time for N, A or corrected count rate to halve; after n half-lives a fraction $(1/2)^n$ remains
(HL) Decay law	$N = N_0e^{-\lambda t}$; $A = \lambda N = A_0e^{-\lambda t}$; $\lambda = \ln 2 / T_{1/2}$

10. Test yourself

Attempt these without notes; full answers below. Questions 8-10 are HL only.

1. Radium-226 (${}^{226}_{88}Ra$) decays by alpha emission. Write the full decay equation, identifying the daughter (element 86 is radon).

2. Strontium-90 (${}^{90}_{38}Sr$) is neutron-rich. Write its decay equation (element 39 is yttrium) and state where Sr-90 lies relative to the N-Z stability curve.

3. A source emits radiation that is undeflected by a strong magnetic field and passes through 10 cm of aluminium with only partial loss of intensity. Identify the radiation and justify your answer.

4. Using m_p = 1.007276 u, m_n = 1.008665 u and a nuclear mass of 11.996706 u for ${}^{12}_{6}C$, find its binding energy and binding energy per nucleon.

5. A detector near a sample reads 92 counts per minute; the background is 20 counts per minute. What will the detector read after two half-lives of the source?

6. The corrected activity of a sample falls from 6400 Bq to 400 Bq in 32 days. Find the half-life.

7. Charcoal from an ancient fire pit has a C-14 activity per gram 25% of that of living wood ($T_{1/2}$ = 5730 yr). How old is the fire pit?

8. (HL) Technetium-99m ($T_{1/2}$ = 6.0 h) is injected for a medical scan. What fraction of the injected nuclei remain after 15 hours?

9. (HL) A sample's corrected count rate falls from 3.2×10^5 s^{-1} to 2.0×10^4 s^{-1} in 20.0 minutes. Find the half-life and the decay constant in s^{-1}.

10. (HL) A pure sample has activity 5.0×10^6 Bq and decay constant 1.0×10^{-6} s^{-1}. How many undecayed nuclei does it contain, and what (approximately) is its half-life in days?

Answers

1. ${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$. Check: A: 226 = 222 + 4; Z: 88 = 86 + 2.

2. ${}^{90}_{38}Sr \rightarrow {}^{90}_{39}Y + {}^{0}_{-1}e + \bar{\nu}$. Neutron-rich nuclides lie **above** the stability curve; β^- decay converts a neutron to a proton, moving the nuclide down-right towards the curve.

3. Gamma. No deflection → uncharged (rules out α and β); high penetration through aluminium with only gradual attenuation is characteristic of γ photons.

4. Separate nucleons: 6(1.007276) + 6(1.008665) = 6.043656 + 6.051990 = 12.095646 u. Δm = 12.095646 - 11.996706 = 0.098940 u. $E_b = 0.098940 \times 931.5$ = 92.2 MeV; per nucleon: 92.2/12 = 7.68 MeV.

5. Corrected rate now: 92 - 20 = 72 min^{-1}. After two half-lives: 72/4 = 18 min^{-1}. The detector reads 18 + 20 = **38 counts per minute**. (Adding the background back on is the step most students miss.)

6. 6400/400 = 16 = 2^4, so 4 half-lives = 32 days and $T_{1/2}$ = 8.0 days.

7. 25% = $(1/2)^2$: two half-lives, so age = 2 × 5730 = 11 460 ≈ 1.1×10^4 years.

8. λ = 0.693/6.0 = 0.116 h^{-1}; λt = 0.116 × 15 = 1.73. $N/N_0 = e^{-1.73}$ = 0.18, so about **18%** remain (15 h = 2.5 half-lives).

9. Ratio = $3.2 \times 10^5 / 2.0 \times 10^4 = 16 = 2^4$, so $T_{1/2}$ = 20.0/4 = 5.0 min = 300 s. λ = 0.693/300 = 2.3×10^{-3} s^{-1}.

10. N = A/λ = $5.0 \times 10^6 / 1.0 \times 10^{-6} = 5.0 \times 10^{12}$ nuclei. $T_{1/2}$ = 0.693/λ = 6.93×10^5 s ≈ 8.0 days.