IB DP PHYSICS

Theme D: Fields

D.4 Induction

Revision Notes · Higher Level only

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Original notes prepared for the IB Diploma Programme Physics course (first assessment 2025)

What the syllabus requires

> **Higher Level only:** The whole of D.4 is examined at HL only. SL students are not required to study electromagnetic induction. HL students should expect D.4 to be combined with D.2 (motion in fields), D.3 (magnetic forces) and circuit ideas from B.5.

By the end of D.4 you should be able to work confidently with each of the following. Use this list as a final checklist before the exam.

Understanding	You should be able to...
Magnetic flux Φ	Calculate $\Phi = BA \cos \theta$, interpret the weber, and distinguish flux from flux density and from flux linkage $N\Phi$.
Electromagnetic induction	Describe the experimental facts: an emf is induced only while the flux linking a circuit is changing.
Faraday's law	Use $\varepsilon = -N \Delta\Phi / \Delta t$ to find average and instantaneous emfs, including from graphs.
Lenz's law	Predict the direction of induced currents and justify the minus sign using conservation of energy.
Motional emf	Derive and use $\varepsilon = BvL$ for a straight conductor, including rod-on-rails circuits with resistance.
AC generation	Explain qualitatively how a rotating coil generates a sinusoidal emf, and how N, B, A and ω control the peak value.
Applications	Discuss eddy currents (braking, induction hobs, metal detectors), mutual induction in transformers, and induction charging.

1. Magnetic flux and flux linkage

Magnetic flux measures *how much magnetic field passes through a surface*. Picture field lines threading a loop of wire: the flux counts the field lines captured by the loop. For a uniform field of flux density B passing through a flat area A,

$$\Phi = BA \cos \theta$$

where θ is the angle between the magnetic field and the **normal** (perpendicular) to the surface — not the angle to the surface itself. The SI unit is the **weber**: 1 Wb = 1 T m^2.

Quantity	Symbol and unit	What it measures
Magnetic flux density	B, tesla (T)	Strength of the field at a point — flux per unit perpendicular area, $B = \Phi / A$
Magnetic flux	Φ, weber (Wb)	Total field through a chosen surface: $\Phi = BA \cos \theta$
Flux linkage	$N\Phi$, weber (or weber-turns)	Flux multiplied by the number of turns N of the coil it threads

Three special orientations are worth memorising:

- Field perpendicular to the plane of the loop ($\theta = 0$): flux is maximum, $\Phi = BA$.

- Field at angle θ to the normal: $\Phi = BA\cos\theta$.
- Field parallel to the plane of the loop (θ = 90°): no field lines thread the loop, so Φ = 0.

> **Common pitfall:** If a question gives the angle between the field and the *plane* of the coil, convert it: the angle to the normal is (90° – that angle). A field "at 30° to the plane" makes 60° with the normal.

Worked example 1 — flux through a tilted coil

A square coil of side 4.0 cm sits in a uniform field of flux density 0.25 T. Find the flux through the coil when (a) the field is perpendicular to the plane of the coil, (b) the field makes an angle of 60° with the plane of the coil.

Area: $A = (0.040)^2 = 1.6 \times 10^{-3}$ m^2.

(a) θ = 0, so $\Phi = BA = 0.25 \times 1.6 \times 10^{-3} = \mathbf{4.0 \times 10^{-4}}$ **Wb**.

(b) The field makes 60° with the plane, so it makes 30° with the normal: $\Phi = BA\cos 30° = 4.0 \times 10^{-4} \times 0.866 = \mathbf{3.5 \times 10^{-4}}$ **Wb**.

Worked example 2 — flux linkage

A circular coil of 500 turns and radius 3.0 cm has its axis parallel to a uniform field of flux density 0.080 T. Calculate the flux linkage.

Area: $A = \pi r^2 = \pi \times (0.030)^2 = 2.83 \times 10^{-3}$ m^2.

Flux through one turn: $\Phi = BA = 0.080 \times 2.83 \times 10^{-3} = 2.26 \times 10^{-4}$ Wb.

Flux linkage: $N\Phi = 500 \times 2.26 \times 10^{-4} =$ **0.11 Wb** (often quoted as 0.11 weber-turns).

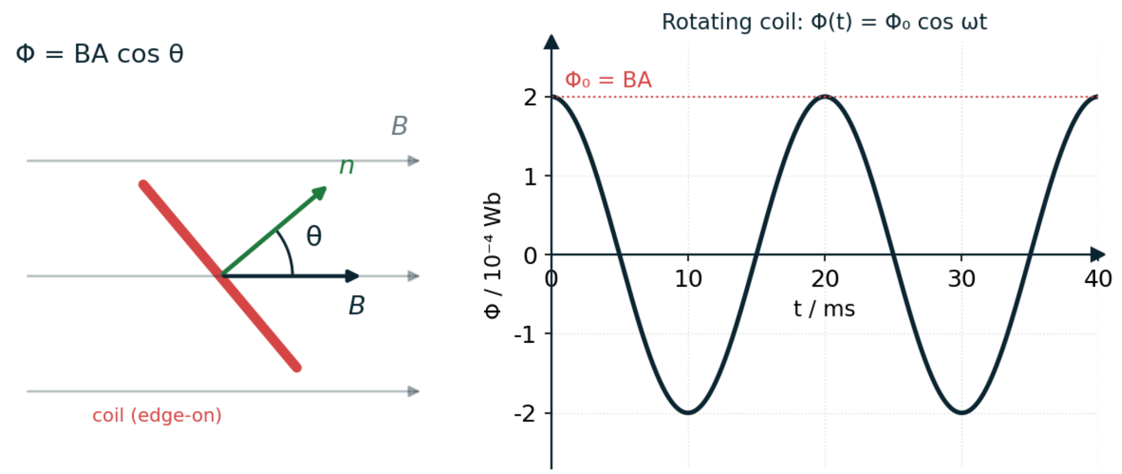

Figure 1. Magnetic flux Φ = BA cos θ, where θ is measured from the coil normal n to the field B. As the coil rotates at angular speed ω, θ = ωt and the flux varies sinusoidally, $\Phi(t) = \Phi_0 \cos \omega t$. Here $\Phi_0 = BA = 2.0\times10^{-4}$ Wb (B = 0.10 T, $A = 2.0\times10^{-3}$ m^2).

2. Electromagnetic induction: the experimental facts

Connect a coil to a sensitive galvanometer (no battery anywhere) and move a bar magnet near it. The observations below — first made by Faraday in 1831 — are the foundation of the whole topic.

- Push the magnet **into** the coil: the needle deflects one way — a current flows.
- Hold the magnet **stationary** inside the coil: the needle returns to zero — no current, however strong the field.
- Pull the magnet **out**: the needle deflects the **opposite** way.
- Move the magnet **faster**: larger deflection. Use a **stronger magnet** or a coil with **more turns**: larger deflection.
- Moving the **coil** instead of the magnet gives identical results — only relative motion matters.

The single idea that explains all of this: **an emf is induced in a circuit only while the magnetic flux linking it is changing**. The induced emf exists whether or not the circuit is complete; a current flows only if there is a closed conducting path.

Ways of changing the flux

Since $\Phi = BA \cos \theta$, the flux linking a coil can change in three distinct ways — and exam questions use all three:

Change	Example
Change *B*	Move a magnet toward or away from the coil; switch a neighbouring electromagnet on or off; vary the current in a nearby coil (mutual induction)
Change *A*	Slide a rod along rails so the circuit area grows; stretch or squash a flexible loop; pull a coil out of a field region
Change θ	Rotate a coil in a fixed field — the principle of the AC generator

> **Exam language:** Write “the flux (linkage) through the coil changes, so by Faraday’s law an emf is induced”. Statements such as “the field cuts the wire so charge flows” earn no marks without the change-of-flux idea.

3. Faraday’s law

Faraday’s law makes the observations quantitative: **the induced emf equals the rate of change of flux linkage**. For a coil of *N* turns,

$$\varepsilon = -N \, \Delta\Phi / \Delta t$$

The minus sign encodes Lenz’s law (Section 4) — the induced effects oppose the change. In calculations you normally work with magnitudes and assign the direction physically at the end. Over a finite interval the equation gives the **average** emf; the **instantaneous** emf is the gradient of the flux-linkage–time graph at that instant.

Graphs: emf as a gradient

- Flux constant (horizontal graph): zero emf, no matter how large the flux is.
- Flux changing at a steady rate (straight sloping graph): constant emf.
- Sinusoidal flux $\Phi = \Phi_0 \cos \omega t$: the emf is **cosinusoidal in shape but shifted by a quarter cycle** — $\varepsilon \propto \sin \omega t$. The emf is **zero when the flux is maximum** (gradient zero) and **maximum when the flux**

passes through zero (steepest gradient).

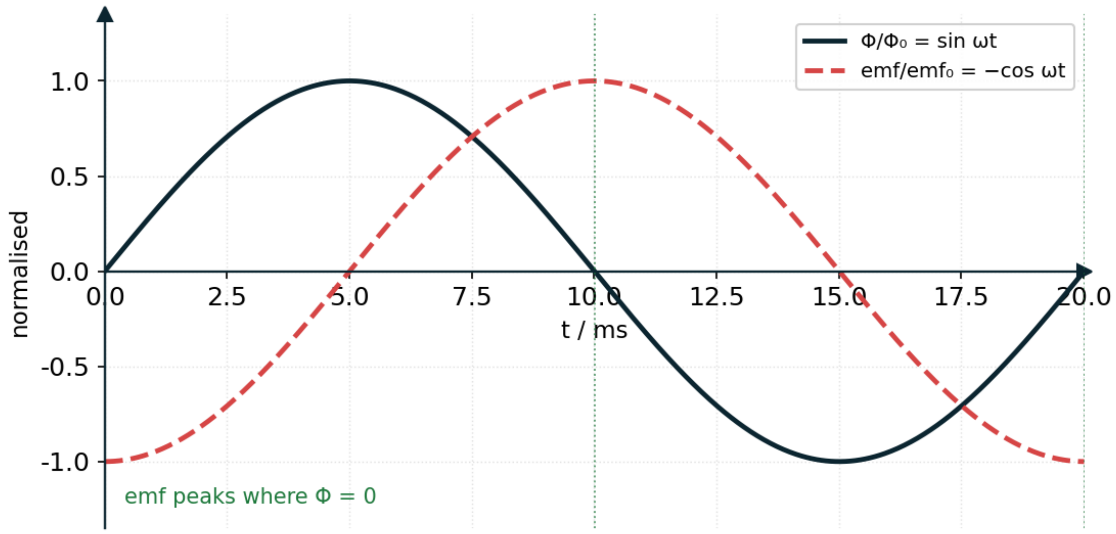

Figure 2. Faraday's law: emf = $-N\,d\Phi/dt$. With $\Phi(t) = \Phi_0 \sin \omega t$ the emf is $-N\Phi_0\omega \cos \omega t$, a quarter cycle (90°) out of phase: the emf peaks exactly where the flux crosses zero and is zero where the flux is greatest. Peak emf = $N\Phi_0\omega \approx 9.4$ V ($N = 150$, $\Phi_0 = 2.0{\times}10^{-4}$ Wb, $\omega = 314$ rad s^{-1}).

Worked example 3 — collapsing field

A 200-turn coil of area 5.0 cm^2 sits inside a solenoid whose field, 0.60 T, is switched off and falls steadily to zero in 0.30 s. The coil's resistance is 4.0 Ω. Find (a) the average induced emf, (b) the induced current.

(a) $\Delta\Phi = B A = 0.60 \times 5.0 \times 10^{-4} = 3.0 \times 10^{-4}$ Wb per turn.

$\varepsilon = N\,\Delta\Phi / \Delta t = 200 \times 3.0 \times 10^{-4} / 0.30 =$ **0.20 V**.

(b) $I = \varepsilon / R = 0.20 / 4.0 =$ **0.050 A**, flowing only during the 0.30 s while the field is collapsing.

Worked example 4 — coil pulled out of a field

A square coil of side 5.0 cm with 40 turns lies wholly inside a region of uniform field, $B = 0.30$ T, perpendicular to the coil. It is pulled completely out of the field in 0.20 s. Find the average emf induced.

Flux through one turn initially: $\Phi = 0.30 \times (0.050)^2 = 0.30 \times 2.5 \times 10^{-3} = 7.5 \times 10^{-4}$ Wb; finally zero.

$\varepsilon = N\,\Delta\Phi / \Delta t = 40 \times 7.5 \times 10^{-4} / 0.20 =$ **0.15 V**.

Note this is an *average*: the instantaneous emf is zero while the coil is still fully inside (no change of flux) and non-zero only while the coil is crossing the field boundary.

Worked example 5 — rotating coil at particular instants

A coil rotating at constant frequency has flux linkage $N\Phi = 0.048 \cos(\omega t)$ Wb with $\omega = 63$ rad s^{-1} ($f = 10$ Hz). Find the emf (a) when the coil plane is perpendicular to the field (flux maximum), (b) when the coil plane is parallel to the field (flux zero).

The emf is the (negative) gradient of the flux linkage: $\varepsilon = N\Phi_0\omega \sin(\omega t)$.

(a) Flux maximum → gradient zero → ε = **0**.

(b) Flux zero → gradient steepest → $\varepsilon = N\Phi_0\omega = 0.048 \times 63 =$ **3.0 V** (the peak value).

Classic exam point: For a rotating coil the emf is zero exactly when the flux linkage is greatest, and greatest when the flux linkage is zero. The emf depends on the *rate of change* of flux, not on the flux itself.

4. Lenz's law and energy conservation

Lenz's law: the induced emf (and any induced current) is directed so as to **oppose the change of flux that produces it**. This is the physical meaning of the minus sign in Faraday's law.

Why must it be so? Conservation of energy. Suppose the opposite were true: pushing a magnet into a coil would create a current whose field *pulled the magnet in faster*, inducing a still larger current — kinetic energy and electrical energy would both grow from nothing. Instead, the induced current opposes the motion, so an external agent must do work against the opposition, and it is **this work that becomes the electrical energy** dissipated in the circuit.

Predicting current directions

Situation	Reasoning	Result
N pole pushed toward the face of a coil	Flux into the coil increasing; the induced current opposes it by making the near face a N pole (repelling the magnet)	Current anticlockwise as seen from the magnet
N pole pulled away from the coil	Flux decreasing; induced current sustains it, making the near face a S pole (attracting the magnet back)	Current clockwise as seen from the magnet
Two coils side by side; current in coil 1 switched on	Growing flux from coil 1 threads coil 2; coil 2's current opposes the growth	Momentary current in coil 2 opposite in sense to coil 1's
Loop entering a field region (into the page)	Flux into the page increasing; induced current creates flux out of the page inside the loop	Anticlockwise current; the magnetic force on it opposes the loop's entry

A quick recipe: (1) decide whether the flux through the circuit is increasing or decreasing, and in which direction it points; (2) the induced current's own field must oppose that change inside the loop; (3) use the right-hand grip rule to turn that field direction into a current direction.

Eddy currents

When the flux through a **solid conductor** changes, circulating induced currents — **eddy currents** — swirl inside the metal. By Lenz's law they oppose the change, and because the metal has low resistance they can be large, dissipating energy as heat.

- **Induction (eddy-current) braking:** a moving conductor (train disc, roller-coaster fin) passes an electromagnet; eddy currents oppose the motion, giving smooth, contact-free braking. The braking force fades as the speed (and hence rate of flux change) falls, so friction brakes finish the stop.
- **Induction hobs:** an alternating current in a coil under the ceramic surface drives eddy currents directly in the steel pan base, heating the pan while the hob stays cool.
- **Metal detectors:** a transmitted alternating field induces eddy currents in buried metal; the detector senses the small field those currents radiate back.
- Where eddy currents are unwanted (transformer cores, motor cores) the iron is **laminated** — sliced into thin insulated sheets that interrupt the current loops.

Worked example 6 — applying Lenz's law

A copper ring lies flat on a table. A bar magnet, S pole downward, is dropped toward the centre of the ring. State and explain the direction of the induced current seen from above, and the effect on the magnet's fall.

The falling S pole sends increasing flux *upward* through the ring (field lines run into a S pole). The induced current opposes the increase by creating downward flux inside the ring, so by the right-hand grip rule the current flows **clockwise seen from above**.

The ring's upper face behaves as a S pole facing the approaching S pole: the magnet is repelled, so it falls with acceleration **less than g**. The lost kinetic energy appears as electrical heating in the ring — exactly what energy conservation demands.

5. Motional emf: $\varepsilon = BvL$

Where the derivation comes from

Move a straight conductor of length L at speed v perpendicular to a field B. Every free electron inside it moves with the rod, so each feels a magnetic force $F = qvB$ along the rod (D.3). Electrons pile up at one end, leaving the other positive, until the electric field they create exactly balances the magnetic push. The rod behaves as a battery of emf

$$\varepsilon = BvL$$

Equivalently, by Faraday's law: in time Δt the rod sweeps out area $\Delta A = Lv\Delta t$, so $\Delta\Phi / \Delta t = BLv$ — the same result. The formula needs B, v and the rod to be mutually perpendicular; if the velocity makes angle θ with the field, use the perpendicular component ($\varepsilon = BvL \sin\theta$).

Rod on rails: the standard circuit

A conducting rod slides on two frictionless rails joined by a resistor R, in a uniform field perpendicular to the circuit plane. This one setup ties together almost everything in D.4:

- emf: $\varepsilon = BvL$; current: $I = BvL / R$.
- The current-carrying rod feels a magnetic force $F = BIL = B^2L^2v / R$, which by Lenz's law **opposes the motion**.

- To keep the rod at constant speed, an external agent must pull with an equal and opposite force; the power supplied, $P = Fv$, exactly equals the electrical power I^2R dissipated.
- If the agent lets go, the rod decelerates — its kinetic energy is converted to heat and the rod exponentially approaches rest.

Worked example 7 — rod on rails with resistance

A rod of length L = 0.40 m slides at a steady v = 3.0 m s^{-1} along rails in a field B = 0.50 T perpendicular to the plane of the circuit. The circuit resistance is 2.0 Ω. Find (a) the emf, (b) the current, (c) the force needed to maintain the speed, (d) show the power balances.

(a) $\varepsilon = BvL$ = 0.50 × 3.0 × 0.40 = **0.60 V**.

(b) $I = \varepsilon / R$ = 0.60 / 2.0 = **0.30 A**.

(c) Magnetic force on the rod: $F = BIL$ = 0.50 × 0.30 × 0.40 = 0.060 N opposing the motion, so the agent must pull with **0.060 N** forward.

(d) Mechanical power in: $P = Fv$ = 0.060 × 3.0 = 0.18 W. Electrical power out: $I^2R = (0.30)^2 \times 2.0$ = 0.18 W. **They match** — the work done against the magnetic force is the source of the electrical energy.

Check the physics, not just the algebra: In every motional-emf problem the energy story must close: work done by the agent = electrical energy dissipated (+ any gain in kinetic energy). Quoting this balance is often worth a mark on its own.

6. Generating alternating current

Rotate a coil of N turns and area A at constant angular speed ω in a uniform field B. The flux linkage varies as $N\Phi = NBA \cos \omega t$, so by Faraday's law the emf is sinusoidal:

$$\varepsilon = \varepsilon_0 \sin \omega t, \text{ with peak value } \varepsilon_0 = NBA\omega$$

(The IB requires this *qualitatively*: you should know the shape of the graph, when the peaks occur, and what controls the peak value.) Slip rings and brushes connect the spinning coil to the external circuit; replacing them with a split-ring commutator would instead give a one-directional (DC) output.

- $\varepsilon = 0$ when the coil plane is perpendicular to the field (flux maximum, momentarily unchanging).
- ε is a peak when the coil plane is parallel to the field (flux zero but changing fastest — the sides cut field lines at the greatest rate).
- Peak emf grows with each of N, B, A and ω — more turns, stronger field, bigger coil, faster rotation.

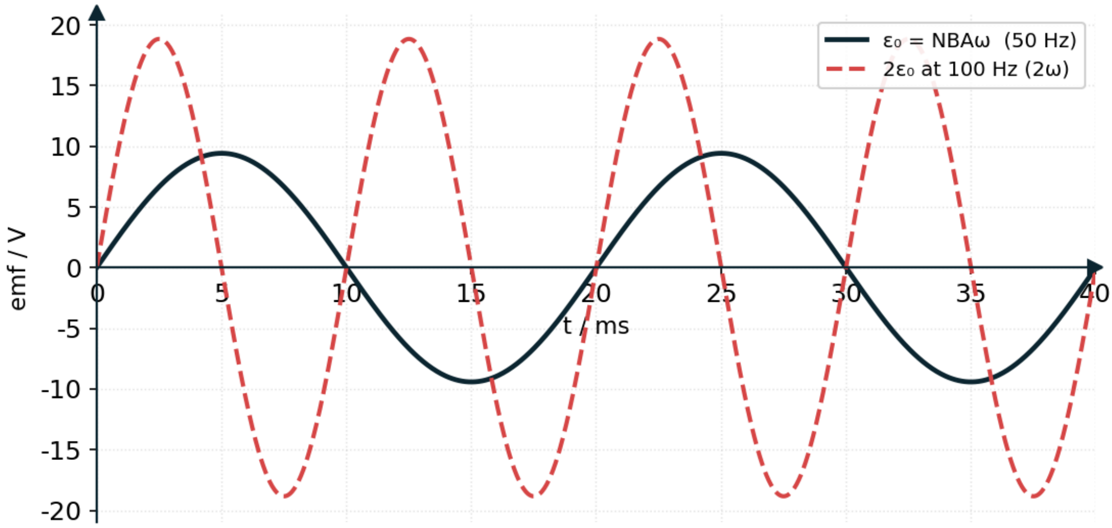

Figure 3. AC generator output $\varepsilon = \varepsilon_0 \sin \omega t$, with peak $\varepsilon_0 = NBA\omega$. Doubling the rotation rate doubles the peak emf ($\varepsilon_0 \propto \omega$) and halves the period (frequency doubles): 9.4 V at 50 Hz → 18.8 V at 100 Hz.

Changing the rotation rate: the classic exam question

Double the angular speed and **two** things happen at once: the peak emf doubles ($\varepsilon_0 \propto \omega$) **and** the period halves (frequency doubles). On a sketch, the new curve is twice as tall and twice as compressed horizontally. Candidates routinely change only one of the two — examiners award marks for each feature separately.

Worked example 8 — peak emf of a generator

A generator coil has 150 turns of area 2.0×10^{-3} m^2 and rotates at 50 revolutions per second in a field of 0.10 T. Find (a) the peak emf, (b) the peak emf and output frequency if the rotation rate is doubled.

(a) $\omega = 2\pi f = 2\pi \times 50 = 314$ rad s^{-1}. $\varepsilon_0 = NBA\omega = 150 \times 0.10 \times 2.0 \times 10^{-3} \times 314 =$ **9.4 V**, at 50 Hz.

(b) At 100 rev s^{-1}: ε_0 doubles to **19 V** (≈ 18.8 V) and the frequency doubles to **100 Hz**.

7. Induction at work: transformers and wireless charging

Mutual induction and the transformer (qualitative)

Wind two coils on the same soft-iron core. An **alternating** current in the primary coil produces a continuously changing flux; the core guides almost all of it through the secondary coil, where the changing flux linkage induces an alternating emf of the same frequency. Because energy is conserved, an ideal transformer transfers power unchanged — stepping the voltage up steps the current down in proportion. A steady (DC) current induces nothing: no change of flux, no emf — which is precisely why transformers, and the entire AC grid built around them, require alternating current.

Induction (wireless) charging

A charging pad contains a coil driven by a high-frequency alternating current. Placing a phone or toothbrush on the pad puts its small receiver coil in the pad's changing flux; the induced emf is rectified to charge the battery. It is a transformer with an air gap: convenient and sealed against water, but less efficient than a cable because flux escapes and eddy currents warm nearby metal — alignment matters for exactly this reason.

8. Common pitfalls

- Confusing flux (Φ, through one loop) with flux linkage ($N\Phi$) — or forgetting N entirely in Faraday's law.
- Using the angle to the plane instead of the angle to the normal in $\Phi = BA \cos \theta$.
- Saying a large flux induces a large emf. Only the *rate of change* matters: for a rotating coil the emf is zero at maximum flux.
- Claiming a current is always induced. The emf always appears when flux changes, but current needs a complete circuit.
- Dropping units: flux in Wb, flux density in T, and areas converted to m^2 ($1\ cm^2 = 10^{-4}\ m^2$ — a very common slip).
- When rotation speed changes, altering the amplitude of the ε–t sketch but not the period (or vice versa).
- Explaining Lenz's law as "opposing the motion" in every case — it opposes the *change of flux*; opposition to motion is one consequence.

9. Quick reference

Result	Statement
Magnetic flux	$\Phi = BA \cos \theta$ (θ measured from the normal); unit weber, $1\ Wb = 1\ T\ m^2$
Flux linkage	$N\Phi$ for a coil of N turns
Faraday's law	$\varepsilon = -N\ \Delta\Phi / \Delta t$ (magnitude = rate of change of flux linkage)
Lenz's law	Induced effects oppose the change producing them — required by energy conservation
Motional emf	$\varepsilon = BvL$ for B, v, L mutually perpendicular
Rod on rails	$I = BvL / R$; opposing force $F = B^2L^2v / R$; $P = Fv = I^2R$
Rotating coil	$\varepsilon = \varepsilon_0 \sin \omega t$ with $\varepsilon_0 = NBA\omega$; emf zero at maximum flux; doubling ω doubles both peak emf and frequency
Eddy currents	Induced loops in solid conductors: exploited in braking, hobs, detectors; suppressed by lamination

10. Test yourself

Attempt these without notes; full answers below.

1. A circular loop of radius 5.0 cm is in a uniform field of 0.040 T whose direction makes 25° with the normal to the loop. Calculate the flux through the loop.

2. A 1200-turn coil of area 4.0 cm^2 has the field along its axis increased steadily from zero to 0.15 T in 0.60 s. Find the change in flux linkage and the average induced emf.

3. An aircraft with wingspan 60 m flies horizontally at 250 m s^{-1} where the vertical component of the Earth's field is 5.0×10^{-5} T. Find the emf between the wingtips, and explain why no current would flow through a wire connecting them inside the aircraft.

4. The N pole of a magnet is pushed toward a coil. State the direction of the induced current as seen from the magnet, and explain your answer using Lenz's law.

5. A rod of length 0.50 m slides at 4.0 m s^{-1} on rails in a perpendicular field of 0.20 T; the circuit resistance is 0.80 Ω. Find the emf, the current, the force required for constant speed, and the power delivered.

6. A generator's rotation rate is doubled. State what happens to (a) the peak emf, (b) the frequency, and (c) sketch (describe) how the ε–*t* graph changes.

7. Explain why, for a coil rotating in a uniform field, the induced emf is zero at the instant the flux linkage is maximum.

8. An eddy-current brake slows a metal disc. Explain (a) how the braking force arises and (b) why this brake alone cannot bring the disc completely to rest.

9. A 500-turn coil of area 1.2×10^{-3} m^2 rotates at 60 Hz in a 0.20 T field. Calculate the peak emf.

10. Explain why a transformer works only with an alternating supply, referring to flux and Faraday's law.

Answers

1. $A = \pi \times (0.050)^2 = 7.85 \times 10^{-3}$ m^2. $\Phi = BA \cos\theta = 0.040 \times 7.85 \times 10^{-3} \times \cos 25° =$ **2.8×10^{-4} Wb**.

2. $\Delta(N\Phi) = 1200 \times 0.15 \times 4.0 \times 10^{-4} =$ **0.072 Wb**. $\varepsilon = 0.072 / 0.60 =$ **0.12 V**.

3. $\varepsilon = BvL = 5.0 \times 10^{-5} \times 250 \times 60 =$ **0.75 V**. A connecting wire moves through the same field with the same velocity, so an equal emf is induced in it; around the whole circuit the net emf is zero and no current flows.

4. **Anticlockwise** viewed from the magnet. The flux through the coil (toward it) is increasing, so the induced current must create opposing flux, making the near face a N pole to repel the approaching N pole; the right-hand grip rule then gives an anticlockwise current on that face.

5. $\varepsilon = BvL = 0.20 \times 4.0 \times 0.50 =$ **0.40 V**; $I = 0.40 / 0.80 =$ **0.50 A**; $F = BIL = 0.20 \times 0.50 \times 0.50 =$ **0.050 N**; $P = Fv = 0.050 \times 4.0 =$ **0.20 W** ($= I^2R = 0.25 \times 0.80 = 0.20$ W ✓).

6. (a) Peak emf doubles ($\varepsilon_0 \propto \omega$). (b) Frequency doubles (period halves). (c) The new curve has twice the amplitude and completes two cycles in the time the old curve completed one.

7. The emf equals the rate of change of flux linkage. At maximum flux linkage the Φ–*t* curve is at a turning point, so its gradient — and therefore the emf — is instantaneously zero, even though the flux itself is largest there.

8. (a) The changing flux through each region of the moving disc induces circulating eddy currents; by Lenz's law their magnetic interaction with the field opposes the disc's motion, producing a retarding force while the kinetic energy is dissipated as heat. (b) The induced currents — and hence the force — are proportional to the rate of change of flux, which falls with speed; as $v \to 0$ the braking force $\to 0$, so the disc creeps asymptotically and a friction brake is needed for the final stop.

9. $\omega = 2\pi \times 60 = 377$ rad s^{-1}. $\varepsilon_0 = NBA\omega = 500 \times 0.20 \times 1.2 \times 10^{-3} \times 377 =$ **45 V**.

10. The secondary emf exists only while the flux in the core is changing ($\varepsilon = -N\,\Delta\Phi / \Delta t$). An alternating primary current makes the core flux change continuously, sustaining a secondary emf; a steady DC current gives constant flux, zero rate of change, and therefore zero output (apart from brief pulses at switch-on and switch-off).