IB DP PHYSICS

Theme C: Wave Behaviour

C.4 Standing Waves and Resonance

Revision Notes · Standard and Higher Level

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What the syllabus requires

By the end of C.4 you should be able to work confidently with each of the following. Use this list as a final checklist before the exam.

Understanding	You should be able to...
Formation of standing waves	Explain a standing wave as the superposition of two identical waves travelling in opposite directions, usually a wave and its reflection.
Nodes and antinodes	Locate them, state that adjacent nodes are $\lambda/2$ apart, and describe the phase of points between and across nodes.
Strings and pipes	Sketch and interpret harmonic patterns; use $\lambda_n = 2L/n$ (string, open pipe) and $\lambda_n = 4L/n$, n odd (pipe closed at one end).
Standing vs travelling waves	Contrast energy transfer, amplitude and phase behaviour of the two types.
Natural frequency and resonance	Describe free, damped and forced oscillations, and the large-amplitude response when the driving frequency matches the natural frequency.
Damping	Distinguish light, critical and heavy damping and state the effect of damping on the resonance curve.

> **Exam note:** C.4 content is common to SL and HL. Standing-wave patterns are a favourite for data-based questions, and the resonance-curve description is frequently asked as a 3-4 mark explain question.

1. How standing waves form

A **standing (stationary) wave** is produced by the **superposition** of two waves of the **same frequency, wavelength and (ideally) amplitude** travelling through the same medium **in opposite directions**. In practice the two waves are almost always an incident wave and its **reflection** from a boundary - the fixed end of a string, or the end of an air column.

At every point the displacements of the two waves add (principle of superposition). At some points the two waves always arrive in antiphase and cancel permanently: these are **nodes**. Midway between them the waves arrive in phase and reinforce, giving points of maximum amplitude: **antinodes**. The result is a pattern that oscillates but does not travel - the wave profile stays in place while the string or air within it vibrates.

> **Key phrase for the exam:** “A standing wave forms when two waves of equal frequency and amplitude travelling in opposite directions superpose.” All four ideas - two waves, identical, opposite directions, superposition - are needed for full marks.

Standing waves compared with travelling waves

Property	Travelling (progressive) wave	Standing wave
Energy	Transfers energy through the medium in the direction of travel	Stores energy in the oscillation; no net energy is transferred along the wave

Property	Travelling (progressive) wave	Standing wave
Amplitude	Every point oscillates with the same amplitude (no absorption)	Amplitude varies with position: zero at nodes, maximum (2A) at antinodes
Phase	Phase changes continuously with position: points one wavelength apart are in phase	All points between two adjacent nodes are in phase; points on opposite sides of a node are in antiphase
Wave profile	Moves forward at the wave speed v	Does not move; the pattern of loops is fixed in space
Frequency	All points oscillate at the wave frequency	All points oscillate at the same frequency (except nodes, which do not move)

2. Nodes, antinodes and phase

- A **node** is a point of permanently zero displacement (complete destructive superposition).
- An **antinode** is a point of maximum amplitude, equal to **twice** the amplitude of each travelling wave.
- Adjacent nodes are separated by **λ/2**; adjacent antinodes are also λ/2 apart.
- A node and the nearest antinode are separated by **λ/4**.

Picture one “loop” of a vibrating string between two nodes: every particle in that loop moves up together and down together - they are **in phase** (phase difference zero) although their amplitudes differ. Particles in the **next** loop do the opposite: when one loop is at its highest, the neighbouring loop is at its lowest. Points separated by one node are therefore in **antiphase** (phase difference 180°), and points separated by two nodes are back in phase.

> **Contrast to remember:** In a travelling wave, phase difference depends on separation (Δx/λ of a cycle). In a standing wave there are only two possibilities: in phase (same loop, or an even number of nodes apart) or antiphase (an odd number of nodes apart).

The standing wave through one cycle

Freeze a vibrating string at successive instants. At t = 0 every point is at its extreme displacement; a quarter of a period later the whole string passes through the flat, undisplaced position - every particle is moving at its fastest, and the energy of the wave is entirely kinetic; at t = T/2 each loop has inverted. Twice in every cycle the string is momentarily completely straight. Each point oscillates inside a fixed **envelope**: its amplitude depends only on its position between the nodes, while its frequency is the same everywhere.

Because the node positions are fixed, standing waves only “fit” on a string or in a pipe for particular wavelengths - those that satisfy the **boundary conditions**. This is why each system has a discrete set of natural frequencies, the **harmonics**.

3. Standing waves on strings

A string fixed at both ends must have a **node at each end**. The simplest pattern that fits is a single loop - one antinode in the middle. This is the **fundamental** (first harmonic): the string length is half a wavelength, so $\lambda_1 = 2L$. Adding one more node at a time gives the higher harmonics.

Harmonic n	Pattern (both ends fixed)	Nodes	Antinodes	Wavelength λ_n	Frequency f_n
1 (fundamental)	one loop: N-A-N	2	1	2L	$f_1 = v / 2L$
2	two loops: N-A-N-A-N	3	2	L	$2f_1$
3	three loops	4	3	2L/3	$3f_1$
n	n loops	n + 1	n	2L/n	nf_1

The allowed frequencies follow from v = fλ with the wave speed v on the string fixed by its physical properties:

$$f_n = nv / 2L = nf_1,\ n = 1, 2, 3, ...$$

What decides the fundamental frequency?

Since $f_1 = v / 2L$, anything that changes the length or the wave speed changes the pitch. The speed of a transverse wave on a string is $v = \sqrt{(T/\mu)}$, where T is the tension and μ the mass per unit length (this speed formula supports the reasoning; the syllabus expects the qualitative conclusions):

- **Shorter string** (fretting a guitar): $f_1 \propto 1/L$, so pitch rises.
- **Greater tension** (tightening the tuning peg): v increases, so f_1 rises. Quadrupling T doubles f_1.
- **Thicker, heavier string** (larger μ): v decreases, so f_1 falls - bass strings are thick or wire-wound.

A real plucked string vibrates in a mixture of harmonics at once. The fundamental determines the note; the blend of higher harmonics determines the characteristic quality (timbre) that distinguishes a guitar from a violin playing the same note.

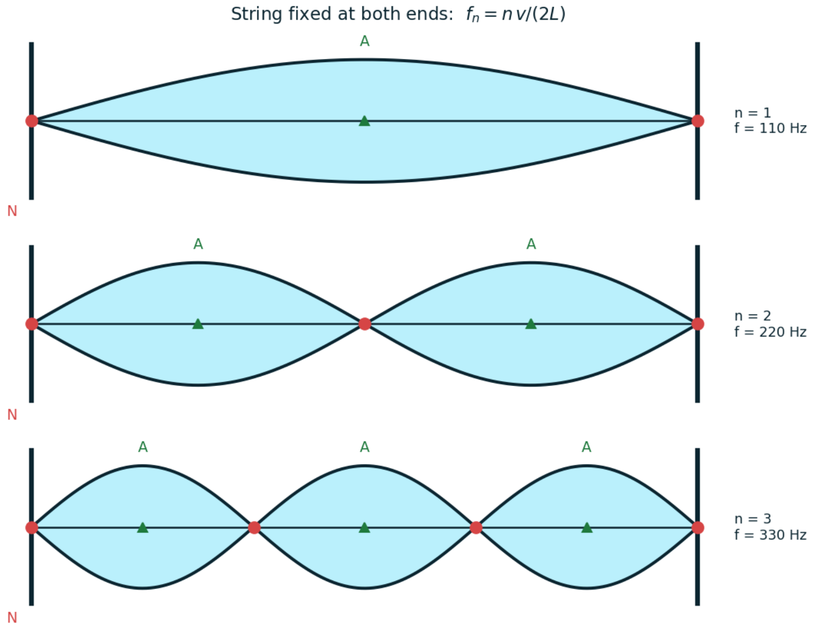

Figure 1. Harmonics of a string fixed at both ends (node N at each end). The fundamental has one loop; each higher harmonic adds a node. With $v = 143\ m\ s^{-1}$ and L = 0.65 m, $f_n = n \cdot v/(2L)$ gives $f_1 = 110$ Hz, $f_2 = 220$ Hz, $f_3 = 330$ Hz.

Worked example 1 - guitar string harmonics

A guitar string of length 0.65 m fixed at both ends sounds its fundamental at 110 Hz. Find (a) the speed of waves on the string, (b) the frequency and wavelength of the third harmonic.

(a) Fundamental: λ_1 = 2L = 1.30 m, so v = $f_1\lambda_1$ = 110 × 1.30 = **143 m s^{-1}**.

(b) f_3 = $3f_1$ = **330 Hz**; λ_3 = 2L/3 = **0.43 m**. Check: 330 × 0.433 ≈ 143 m s^{-1}, as required - the speed is a property of the string, not of the harmonic.

Worked example 2 - identifying a harmonic

A string of length 0.75 m fixed at both ends vibrates in a standing wave with four nodes in total (including the ends). The frequency is 480 Hz. Find the wavelength, the wave speed, and the fundamental frequency.

Four nodes means three loops: the third harmonic, so λ_3 = 2L/3 = 2 × 0.75 / 3 = **0.50 m**.

v = fλ = 480 × 0.50 = **240 m s^{-1}**.

f_1 = f_3 / 3 = **160 Hz**.

4. Standing waves in pipes (air columns)

Sound reflecting up and down an air column superposes with itself to form longitudinal standing waves. The boundary conditions are:

- **Closed end:** air cannot move along the pipe, so there is a displacement **node**.
- **Open end:** air moves freely, so there is a displacement **antinode** (in reality just beyond the rim - see the end correction in Section 5).

Pipe open at both ends

Antinode at each end. The simplest pattern is A-N-A: half a wavelength fits in the pipe, exactly as for a string (with nodes and antinodes swapped). So an open-open pipe supports **all harmonics**:

$$\lambda_n = 2L / n, \quad f_n = nv / 2L, \quad n = 1, 2, 3, \ldots$$

Pipe closed at one end

Node at the closed end, antinode at the open end. The simplest pattern is N-A: only a **quarter** of a wavelength fits, so λ_1 = 4L. The next pattern that satisfies both boundary conditions (N-A-N-A) fits three quarter-wavelengths, giving frequency $3f_1$. A closed pipe therefore produces **odd harmonics only**:

$$\lambda_n = 4L / n, \quad f_n = nv / 4L, \quad n = 1, 3, 5, \ldots$$

Most common C.4 error: For a closed pipe the harmonic number n counts 1, 3, 5, ... - there is no second harmonic. The “first overtone” of a closed pipe is the **third** harmonic at $3f_1$, not $2f_1$. Never write f_n = nv/4L with n = 2.

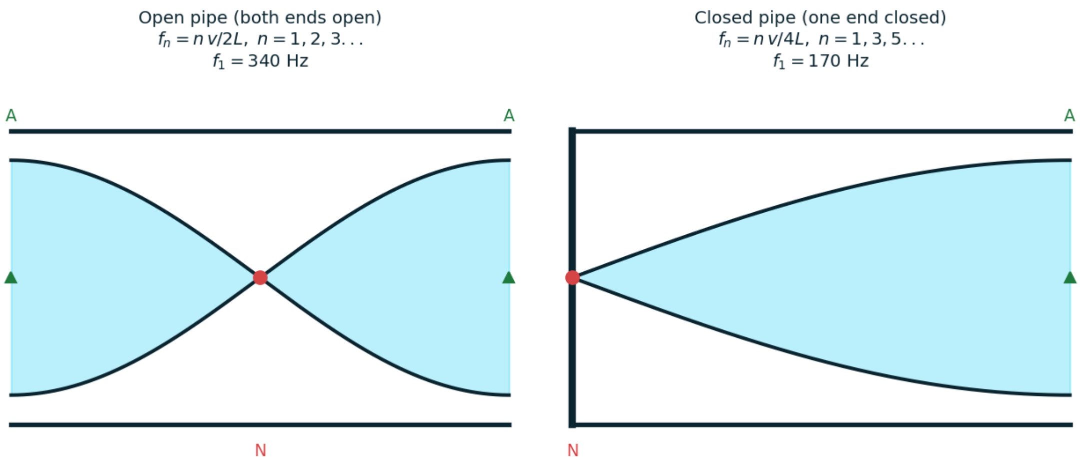

Figure 2. Fundamental modes of an open pipe (antinode A at both ends, all harmonics, $f_n = n \cdot v/2L$) versus a pipe closed at one end (node at the closed end, antinode at the open end, odd harmonics only, $f_n = n \cdot v/4L$, n odd). For $v = 340\ m\ s^{-1}$ and L = 0.50 m the open pipe sounds $f_1 = 340$ Hz but the closed pipe only 170 Hz - an octave lower.

Comparing the three systems

Feature	String, both ends fixed	Pipe, both ends open	Pipe, one end closed
Ends	Node - Node	Antinode - Antinode	Node (closed) - Antinode (open)
Fundamental	$\lambda_1 = 2L$, $f_1 = v/2L$	$\lambda_1 = 2L$, $f_1 = v/2L$	$\lambda_1 = 4L$, $f_1 = v/4L$
Harmonics present	all: f_1, $2f_1$, $3f_1$, ...	all: f_1, $2f_1$, $3f_1$, ...	odd only: f_1, $3f_1$, $5f_1$, ...
Wavelengths	2L/n, n = 1, 2, 3...	2L/n, n = 1, 2, 3...	4L/n, n = 1, 3, 5...
Which v?	speed of the wave on the string	speed of sound in the air	speed of sound in the air

Note the practical consequence: for the same length and the same wave speed, a closed pipe sounds an octave **lower** than an open pipe (f_1 is halved), and its missing even harmonics give it a distinctive hollow tone - the clarinet (effectively closed at the reed end) compared with the flute (open).

Worked example 3 - open pipe

An organ pipe open at both ends is 0.85 m long. The speed of sound is 340 m s^{-1}. Find the fundamental frequency and the frequencies of the next two harmonics.

$\lambda_1 = 2L = 1.70$ m, so $f_1 = v / \lambda_1 = 340 / 1.70 =$ **200 Hz**.

All harmonics are present: $f_2 =$ **400 Hz**, $f_3 =$ **600 Hz**.

Worked example 4 - closed pipe

A pipe 0.17 m long is closed at one end. Taking the speed of sound as 340 m s^{-1}, find the three lowest frequencies at which it resonates.

Fundamental: $\lambda_1 = 4L = 0.68$ m, so $f_1 = 340 / 0.68 =$ **500 Hz**.

Only odd harmonics exist: $f_3 = 3 \times 500 =$ **1500 Hz** and $f_5 = 5 \times 500 =$ **2500 Hz**. (There is no resonance at 1000 Hz or 2000 Hz.)

Worked example 5 - which harmonic is this?

A pipe of length 0.60 m closed at one end resonates strongly at 425 Hz (speed of sound 340 m s^{-1}). Which harmonic is sounding?

$\lambda = v / f = 340 / 425 = 0.80$ m.

For a closed pipe $\lambda_n = 4L/n$, so $n = 4L / \lambda = (4 \times 0.60) / 0.80 =$ **3: the third harmonic** - consistent, because n must be odd.

5. Measuring the speed of sound: the resonance tube

A classic experiment (and IA favourite): a tuning fork of known frequency f is held over a vertical tube whose air-column length can be changed by raising or lowering the water level inside it. The water surface acts as the closed end. As the column is lengthened, the sound becomes loud whenever the column resonates - that is, whenever a closed-pipe standing wave of the fork's frequency fits the column.

- First (shortest) resonance: $L_1 \approx \lambda/4$
- Second resonance: $L_2 \approx 3\lambda/4$
- Therefore $L_2 - L_1 = \lambda/2$ exactly, and $v = f\lambda = 2f(L_2 - L_1)$.

The antinode actually forms a small distance e (the **end correction**, roughly 0.6 × the tube radius) above the open end, so $L_1 + e = \lambda/4$. Using the **difference** of two resonance lengths eliminates e - which is why the two-resonance method is better than using L_1 alone.

Worked example 6 - speed of sound by resonance tube

A 512 Hz tuning fork is held over a resonance tube. The sound is loudest at column lengths of 16.0 cm and 49.2 cm. Find the speed of sound and the end correction.

$\lambda/2 = L_2 - L_1 = 49.2 - 16.0 = 33.2$ cm, so $\lambda = 0.664$ m.

$v = f\lambda = 512 \times 0.664 =$ **340 m s^{-1}**.

End correction: $L_1 + e = \lambda/4 = 16.6$ cm, so $e = 16.6 - 16.0 =$ **0.6 cm**.

6. Standing waves in the laboratory

Vibrating string (Melde's experiment)

A string runs from a signal-generator-driven **vibration generator** over a pulley to a hanging load, which sets the tension T. As the driving frequency is slowly increased, clear standing-wave patterns appear one after another whenever the frequency matches a harmonic of the stretched string - this is itself an example of resonance. Measuring the loop length gives the wavelength (one loop = $\lambda/2$), and changing the hanging mass tests how the wave speed depends on tension: doubling f_1 requires four times the tension, confirming $v = \sqrt{(T/\mu)}$.

Worked example 7 - vibration generator and pulley

A string of mass per unit length 4.0 g m^{-1} is stretched between a vibration generator and a pulley 1.5 m away, with tension 90 N provided by a hanging load. The frequency is raised until the string vibrates in three loops. Find the wave speed on the string and the generator frequency.

$v = \sqrt{(T/\mu)} = \sqrt{(90 / 0.0040)} = \sqrt{(22500)} =$ **150 m s^{-1}**.

Three loops on length 1.5 m: $\lambda = 2L/3 = 1.0$ m, so $f = v/\lambda =$ **150 Hz** (the third harmonic of this string).

Microwaves: measuring the speed of light in a kitchen

With the turntable removed, the microwave field inside an oven forms a standing wave. A bar of chocolate left in place melts first at the **antinodes**, which are $\lambda/2$ apart. Melted spots typically about 6.1 cm apart give $\lambda \approx 0.122$ m; with the oven frequency f = 2.45 GHz printed on the back panel, $c = f\lambda \approx 2.45 \times 10^9 \times 0.122 \approx$ **3.0×10^8 m s^{-1}**. The same physics - fixed hot and cold spots - is why ovens use rotating turntables at all.

Other classic demonstrations

- **Kundt's tube:** fine powder in a horizontal tube collects in small heaps at the displacement nodes of a sound standing wave, making the node spacing ($\lambda/2$) directly measurable.
- **Chladni plates:** sand on a vibrating plate gathers along nodal lines, showing two-dimensional standing-wave patterns.
- **Resonance tube** (Section 5): the standard IB method for the speed of sound.

> **Experiment logic to quote:** Every standing-wave measurement works the same way: locate two adjacent nodes or antinodes, take their separation as $\lambda/2$, then combine with a known frequency using $v = f\lambda$.

7. Resonance and forced oscillations

Free, damped and forced oscillations

- A **free oscillation** occurs when a system is displaced and released: it oscillates at its **natural frequency** f_0, set only by the system's own properties (mass and stiffness of a spring system; length of a pendulum; length, tension and μ of a string).
- A **damped oscillation** is a free oscillation losing energy to resistive forces, so its amplitude decreases with time.
- A **forced oscillation** occurs when a periodic external force (the **driver**) is applied: the system settles into oscillation at the **driver's frequency**, not its own.

Resonance

Resonance occurs when the driving frequency equals (or is very close to) the natural frequency of the system. Energy is then transferred from the driver to the oscillator most efficiently - each push arrives in step with the motion - so the amplitude builds up to a maximum, limited only by damping.

The amplitude-frequency (resonance) curve

Plot the steady amplitude of the driven oscillator against the driver frequency:

- At very low driver frequency, the oscillator simply follows the driver: amplitude roughly equals the driver amplitude.
- As the driver frequency approaches f_0, the amplitude rises steeply to a sharp **peak at (very near) f_0**.
- Beyond f_0 the amplitude falls away, tending to a very small value at high frequency - the system cannot keep up with the driver.

Phase of the driven oscillator

The phase relationship between driver and oscillator changes across the curve: well below f_0 the oscillator moves in phase with the driver; at resonance it lags the driver by a quarter of a cycle (90°), which puts the driving force in phase with the velocity - the condition for maximum power transfer; well above f_0 it approaches antiphase. **Barton's pendulums** show all of this at once: of several paper-cone pendulums hung from the same cord as a heavy driver pendulum, only the one whose length (and hence natural frequency) matches the driver builds up a large amplitude.

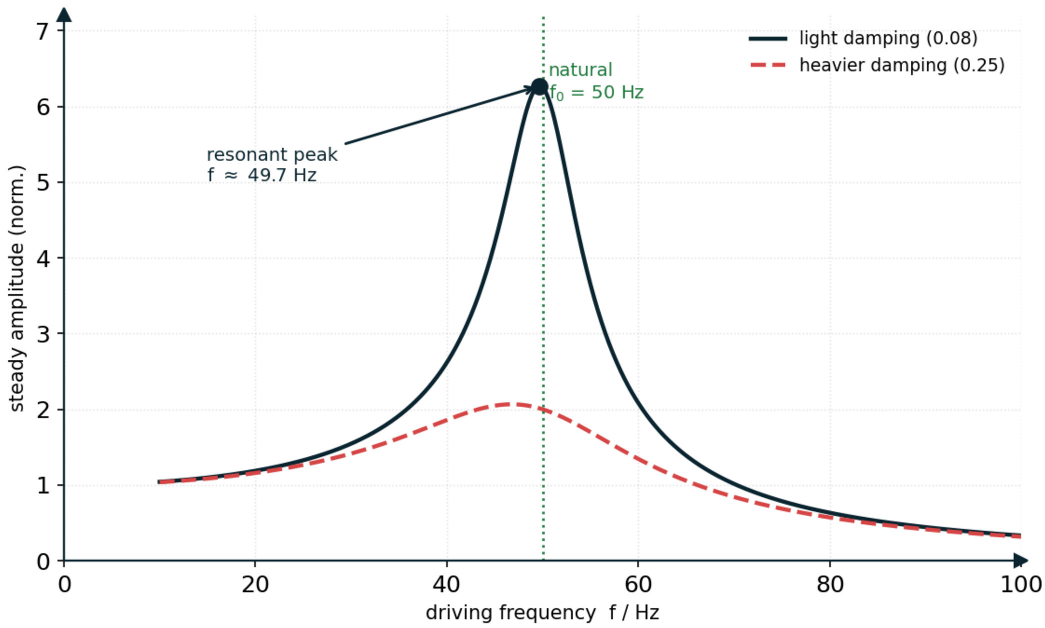

Figure 3. Resonance response: steady amplitude versus driving frequency for a system with natural frequency f_0 = 50 Hz. Lighter damping gives a taller, narrower peak; heavier damping a lower, broader one. The peak sits just below f_0 (at f ≈ 49.7 Hz for the light curve).

Effect of damping on the curve

Increasing the damping makes the peak **lower** and **broader**, and shifts the peak to a **slightly lower frequency** than f_0. With very light damping the peak is tall and narrow - the response is dramatic but only over a narrow band of driving frequencies.

Type of damping	Behaviour	Example
Light (under-damping)	Oscillates with gradually (approximately exponentially) decaying amplitude; period almost unchanged	Pendulum in air; guitar string ringing on
Critical damping	Returns to equilibrium in the shortest possible time with no oscillation	Car suspension; analogue meter needles; some door closers
Heavy (over-damping)	Returns to equilibrium slowly, without oscillating	Oscillation in thick oil; heavily damped door closer

Resonance in the real world

Situation	What resonates	Useful or harmful?
Musical instruments	Strings and air columns driven at their natural frequencies; the body of the instrument amplifies them	Useful - it is how the sound is produced
Microwave oven	The 2.45 GHz field drives rotation of polar water molecules, transferring energy to the food	Useful
MRI scanner	Nuclear magnetic resonance: nuclei absorb radio-frequency energy at their precession frequency	Useful - medical imaging
Wine glass and sound	A loud tone at the glass's natural frequency can build a large enough amplitude to shatter it	Demonstration of harmful resonance
Bridges and buildings	Wind, marching feet or earthquakes can drive structures near a natural frequency (e.g. the Millennium Bridge's lateral wobble, 2000)	Harmful - engineers add damping and change f_0
Vehicle parts	Mirrors, panels and aerials buzz at certain engine speeds	Harmful - designed out with damping

> **Explain-question checklist:** A full “explain resonance” answer states: (1) the system has a natural frequency; (2) it is driven by a periodic force; (3) when driver frequency = natural frequency, energy transfer to the system is maximum; (4) so amplitude becomes large / maximum.

8. Energy in damped oscillations

An undamped oscillator swaps energy back and forth between kinetic and potential forms with the total constant. Damping (friction, air resistance) converts that energy irreversibly to internal (thermal) energy, so the total mechanical energy falls with time.

- With light damping, the amplitude decays in an approximately **exponential** way: it falls by the same **fraction** in each successive cycle.
- Energy is proportional to amplitude squared ($E \propto A^2$), so the energy decays by a constant fraction per cycle too - and faster than the amplitude. If A halves, E falls to one quarter.
- The oscillation frequency is very slightly lowered by damping, but for light damping this shift is negligible.

- In a driven system at steady state, the driver supplies energy at exactly the rate damping dissipates it - which is why the amplitude is steady, and why weaker damping allows a larger resonant amplitude.

Worked example 8 - energy decay

A lightly damped pendulum has amplitude 40 mm. After 10 complete cycles the amplitude is 20 mm. (a) What fraction of the initial energy remains? (b) Estimate the amplitude after 20 cycles.

(a) $E \propto A^2$, so $E/E_0 = (20/40)^2$ = **0.25 - one quarter remains** (75% has been dissipated as thermal energy).

(b) Exponential-style decay loses the same fraction in equal numbers of cycles: the amplitude halves every 10 cycles, so after 20 cycles $A \approx$ **10 mm**.

9. Common pitfalls

- Using $f_n = nv/4L$ with **even** n for a closed pipe - closed pipes have odd harmonics only. Count n = 1, 3, 5, ...
- Writing the node spacing as λ instead of λ/2, or the node-to-antinode distance as λ/2 instead of λ/4.
- Confusing the number of loops (= harmonic number n for a string) with the number of nodes (n + 1) or antinodes (n).
- Claiming a standing wave transfers energy along the medium - it stores energy; there is no net transfer.
- Saying all points of a standing wave have the same amplitude. Amplitude depends on position (zero at nodes, 2A at antinodes); it is the **frequency** that is common to all points.
- Using the speed of sound for waves on a string, or the string-wave speed for the sound the string produces in air. The frequency is the same in both; the wavelengths differ.
- Forgetting that a displacement node in a sound wave is a pressure antinode (the closed end of a pipe is where pressure varies most).
- Saying the resonance peak of a damped system is exactly at f_0 - damping lowers and broadens the peak and shifts it slightly below f_0.

10. Quick reference

Result	Statement
Formation	Two identical waves travelling in opposite directions superpose (wave + its reflection)
Node spacing	adjacent nodes (or antinodes): λ/2; node to nearest antinode: λ/4
Phase rule	same loop: in phase; opposite sides of a node: antiphase (180°)
String (fixed-fixed) / open-open pipe	$\lambda_n = 2L/n$, $f_n = nv/2L = nf_1$, n = 1, 2, 3, ...
Pipe closed at one end	$\lambda_n = 4L/n$, $f_n = nv/4L = nf_1$, n = 1, 3, 5, ... only
Resonance tube	$v = 2f(L_2 - L_1)$; end correction $e = \lambda/4 - L_1$
Resonance	maximum amplitude when driver frequency = natural frequency f_0

Result	Statement
More damping	resonance peak lower, broader, at slightly lower frequency; light damping: A decays exponentially, $E \propto A^2$

11. Test yourself

Attempt these without notes; full answers below.

1. State the conditions needed for two waves to form a standing wave, and explain why a reflected wave usually satisfies them.

2. A string 0.90 m long, fixed at both ends, has a fundamental frequency of 150 Hz. Find the speed of waves on the string and the frequency of the fourth harmonic.

3. In a standing sound wave in a pipe, adjacent nodes are 0.25 m apart. Taking the speed of sound as 340 m s^{-1}, find the frequency.

4. An organ pipe open at both ends sounds a fundamental of 264 Hz (speed of sound 340 m s^{-1}). Find its length.

5. What length of pipe closed at one end would give the same fundamental of 264 Hz? Comment on the comparison.

6. Two points on a vibrating string lie on opposite sides of a node; two other points lie within the same loop. State the phase difference for each pair.

7. A tuning fork of frequency 480 Hz gives resonances of a closed tube at column lengths 17.2 cm and 52.6 cm. Find the speed of sound and the end correction.

8. Explain why a pipe closed at one end cannot produce the second harmonic.

9. The tension in a string is quadrupled without changing its length or mass per unit length. What happens to the fundamental frequency? Justify your answer.

10. Sketch (or describe) resonance curves for the same oscillator with light and with heavier damping, and identify three differences between them.

Answers

1. Same frequency and wavelength, similar amplitude, travelling in opposite directions through the same region. A reflected wave automatically has the same frequency and (nearly) the same amplitude as the incident wave and travels back through it - so incident + reflected waves superpose to form the standing wave.

2. $\lambda_1 = 2L = 1.80$ m, so $v = 150 \times 1.80 = 270$ m s^{-1}. Harmonics of a string are all integers: $f_4 = 4 \times 150 = 600$ Hz.

3. Node spacing = $\lambda/2$, so $\lambda = 0.50$ m and $f = v/\lambda = 340 / 0.50 = 680$ Hz.

4. $\lambda_1 = v/f = 340 / 264 = 1.29$ m; $L = \lambda_1/2 = 0.64$ m.

5. Closed pipe: $L = \lambda_1/4 = 1.29 / 4 = 0.32$ m - half the length of the open pipe for the same note. (This is why closed organ pipes save space, at the cost of missing even harmonics.)

6. Opposite sides of a node: antiphase, phase difference 180° (half a cycle). Within the same loop: in phase, phase difference zero - even though their amplitudes differ.

7. $L_2 - L_1 = \lambda/2 = 52.6 - 17.2 = 35.4$ cm, so $\lambda = 0.708$ m and $v = 480 \times 0.708 = 340$ m s^{-1}. End correction: $e = \lambda/4 - L_1 = 17.7 - 17.2 = 0.5$ cm.

8. The boundary conditions demand a node at the closed end and an antinode at the open end. A pattern with frequency $2f_1$ (wavelength 2L) would need either nodes or antinodes at both ends, which is impossible here - only patterns fitting an odd number of quarter-wavelengths (odd harmonics) satisfy node-at-one-end, antinode-at-the-other.

9. $v = \sqrt{(T/\mu)}$, so quadrupling T doubles v. Since $f_1 = v/2L$ and L is unchanged, the fundamental frequency doubles (the note rises by an octave).

10. Both curves peak near the natural frequency f_0. With heavier damping the peak is (i) lower - smaller maximum amplitude, (ii) broader - the system responds over a wider range of driving frequencies, and (iii) at a slightly lower frequency than f_0. At very low driving frequencies the two curves converge to roughly the driver amplitude.