IB DP PHYSICS

Theme C: Wave Behaviour

C.3 Wave Phenomena

Revision Notes · Standard and Higher Level

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Original notes prepared for the IB Diploma Programme Physics course (first assessment 2025)

What the syllabus requires

C.3 is about what waves do when they meet boundaries, obstacles and each other. Use this list as a final checklist before the exam. Sections flagged (HL) are Higher Level only.

Understanding	You should be able to...
Reflection	State and apply the law of reflection; describe reflection of pulses at fixed and free ends of strings, including the phase change.
Refraction	Explain refraction as a consequence of a change in wave speed; apply Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$.
Total internal reflection	Calculate critical angles with $\sin c = n_2/n_1$ and explain applications such as optical fibres.
Diffraction	Describe the spreading of waves at apertures and edges, and state when the effect is greatest (aperture ≈ λ).
Superposition and interference	Apply the superposition principle; state the coherence condition; use path-difference conditions $n\lambda$ and $(n + ½)\lambda$.
Double-slit interference	Describe Young's experiment and apply $s = \lambda D/d$ to fringe patterns.
(HL) Single-slit diffraction	Sketch the intensity pattern; use $\theta = \lambda/b$ for the first minimum; explain the modulation of double-slit fringes.
(HL) Diffraction gratings	Explain why more slits give sharper maxima; apply $n\lambda = d \sin\theta$; find the maximum order; describe spectroscopy.
(HL) Interference intensity	Use $I \propto A^2$ to compare intensities at maxima and minima.

> **Exam note:** SL candidates need sections 1-6 of these notes. HL candidates need everything, and should expect C.3 to be combined with C.1 (simple harmonic motion), C.2 (wave basics) and C.4 (standing waves) in Paper 2.

1. Reflection

When a wave meets a boundary it cannot cross, it is reflected. The rule is the same for every kind of wave - water, sound, light, pulses on a rope:

angle of incidence = angle of reflection ($\theta_i = \theta_r$)

Both angles are measured **from the normal** - the line perpendicular to the surface at the point of incidence - and the incident ray, reflected ray and normal all lie in the same plane. Reflection changes the direction of travel but leaves the speed, frequency and wavelength unchanged, because the wave stays in the same medium.

Wavefronts and rays

A **wavefront** joins neighbouring points that oscillate in phase (for example, all the crests); a **ray** is an arrow drawn perpendicular to the wavefronts showing the direction of energy travel. The two pictures are equivalent: plane wavefronts hitting a flat barrier at some angle leave the barrier at the same angle on the other side of the normal, exactly as the ray diagram predicts. In exam sketches, always keep wavefronts perpendicular to rays and keep the wavefront spacing (the wavelength) constant on reflection.

Echoes

An echo is simply the reflection of sound from a large surface. Because the sound travels to the reflector *and back*, the distance to the reflector is $d = v\,t / 2$. Sonar, ultrasound imaging and radar all use this round-trip timing idea.

Worked example 1 - echo timing

A student stands facing a large cliff, claps once, and hears the echo 0.60 s later. Taking the speed of sound as 340 m s^{-1}, find the distance to the cliff.

The sound covers the distance twice (there and back) in 0.60 s.

$d = v\,t / 2 = (340 \times 0.60) / 2 =$ **102 m ≈ $\mathbf{1.0 \times 10^2}$ m**.

Forgetting the factor of 2 is the classic error - it doubles the answer.

Reflection of pulses: fixed and free ends

Send a pulse along a rope and watch what returns:

- **Fixed end** (rope tied to a wall): the reflected pulse is **inverted** - a crest returns as a trough. There is a phase change of π rad (half a wavelength). Physically, the wall pulls back on the rope with an equal and opposite force (Newton's third law), flipping the pulse.
- **Free end** (rope ending in a light ring on a pole): the reflected pulse comes back **upright** - no phase change. The free end overshoots to double amplitude as the pulse arrives.

> **Why it matters:** The π phase change at a fixed (denser) boundary explains the node at the closed end of a string in C.4 standing waves, and the same idea reappears for light reflecting off a denser medium in thin-film interference contexts.

2. Refraction and Snell's law

Refraction is the change of direction of a wave when it crosses a boundary into a medium where its **speed is different**. The speed change is the cause; the bending is the effect. The frequency cannot change at a boundary (the media are driven at the rate the wave arrives), so from $v = f\lambda$ the wavelength changes in the same ratio as the speed.

Picture plane wavefronts arriving obliquely at the boundary between deep and shallow water in a ripple tank. The end of each wavefront that enters the shallow (slower) region first is slowed down first, so the wavefront pivots - like a marching band wheeling as one end steps onto muddy ground. The wavefronts end up closer together (shorter λ) and turned toward the normal.

- Entering a **slower** medium (higher n): the ray bends **toward** the normal.
- Entering a **faster** medium (lower n): the ray bends **away from** the normal.
- Along the normal ($\theta_1 = 0$): the wave slows or speeds up but does not bend.

Refractive index and Snell's law

The **refractive index** of a medium compares the wave speed in the medium with the speed in a reference medium (for light, vacuum):

$$n = c / v$$

Because $v \le c$, we have $n \ge 1$; a larger n means a slower, optically denser medium. Snell's law connects the angles (measured from the normal) on the two sides of the boundary:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \text{ or equivalently } \sin\theta_1 / \sin\theta_2 = v_1 / v_2 = \lambda_1 / \lambda_2$$

The second form is the more general one: it applies to *any* wave, including sound and water waves, where 'refractive index' is not usually quoted.

Medium (for visible light)	Refractive index n	Speed of light v = c/n
Vacuum	1 (exactly)	3.00×10^8 m s^{-1}
Air	1.0003 ≈ 1.00	3.00×10^8 m s^{-1}
Water	1.33	2.26×10^8 m s^{-1}
Crown glass	≈ 1.5	2.0×10^8 m s^{-1}
Diamond	2.42	1.24×10^8 m s^{-1}

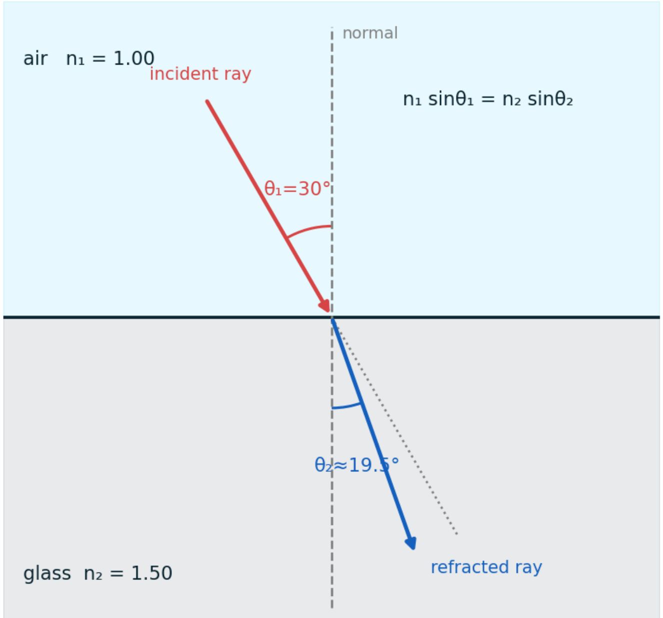

Figure 1. Refraction from air (n_1 = 1.00) into glass (n_2 = 1.50): the ray bends toward the normal, and the dotted line shows the undeviated path. Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$ with θ_1 = 30° gives θ_2 = arcsin(sin 30° / 1.5) ≈ 19.5°.

Worked example 2 - light entering glass

A ray of light in air strikes a glass block ($n = 1.52$) at 40° to the normal. Find (a) the angle of refraction, (b) the speed of light in the glass.

(a) $n_1 \sin \theta_1 = n_2 \sin \theta_2$: $1.00 \times \sin 40° = 1.52 \times \sin \theta_2$

$\sin \theta_2 = 0.643 / 1.52 = 0.423$, so $\theta_2 =$ **25°**. The ray bends toward the normal, as expected for a slower medium.

(b) $v = c / n = 3.00 \times 10^8 / 1.52 =$ $\mathbf{1.97 \times 10^8\ m\ s^{-1}}$.

The frequency is unchanged; the wavelength inside the glass shrinks to $\lambda/1.52$.

Wavelength and frequency in a medium

Two statements are worth memorising because they anchor every refraction argument: the **frequency never changes** at a boundary (each wavefront arriving must produce one wavefront leaving), while the **speed and wavelength change in the same ratio**, since $v = f\lambda$. In a medium of refractive index n, light therefore has speed $v = c/n$ and wavelength $\lambda_{medium} = \lambda_{vacuum}/n$.

Worked example 3 - sodium light entering water

Sodium light has wavelength 589 nm in air. It enters water ($n = 1.33$). Find (a) its frequency, (b) its wavelength in the water.

(a) $f = c/\lambda = 3.00 \times 10^8 / 589 \times 10^{-9} =$ $\mathbf{5.09 \times 10^{14}\ Hz}$ - and it keeps this frequency in the water.

(b) $\lambda_{water} = \lambda/n = 589 / 1.33 =$ **443 nm**.

Check: $v = f\lambda_{water} = 5.09 \times 10^{14} \times 443 \times 10^{-9} = 2.26 \times 10^8\ m\ s^{-1} = c/1.33$, as required.

> **Exam technique:** Angles in Snell's law are always measured from the normal, never from the surface. If a question quotes the angle to the surface, subtract from 90° before doing anything else.

3. Total internal reflection and optical fibres

When light travels from a denser medium toward a less dense one ($n_1 > n_2$), it bends away from the normal, so the refracted angle is larger than the incident angle. As the incident angle grows, the refracted ray flattens toward the boundary. At the **critical angle** c the refracted ray grazes along the boundary ($\theta_2 = 90°$). Putting $\theta_2 = 90°$ into Snell's law:

$$\sin c = n_2 / n_1 \quad \text{(for a boundary with air: } \sin c = 1/n\text{)}$$

For incidence **beyond** the critical angle no refracted ray is possible: 100% of the light is reflected back into the dense medium, obeying the ordinary law of reflection. This is **total internal reflection** (TIR). Two conditions must both hold:

- the light must be travelling from higher n toward lower n, and

- the angle of incidence must exceed the critical angle: θ > c.

Worked example 4 - critical angles

Calculate the critical angle for (a) water (n = 1.33) and (b) glass (n = 1.50), each against air.

(a) sin c = 1/1.33 = 0.752, so c = **48.8°**.

(b) sin c = 1/1.50 = 0.667, so c = **41.8°**.

Because 45° > 41.8°, a 45°-45°-90° glass prism totally internally reflects light hitting its hypotenuse - the principle behind prism binoculars and bicycle reflectors.

Optical fibres

An optical fibre is a hair-thin thread with a glass **core** of high refractive index surrounded by **cladding** of slightly lower refractive index. Light entering the core at a shallow angle strikes the core-cladding boundary beyond the critical angle and is totally internally reflected, again and again, zig-zagging along the fibre with almost no loss even around gentle bends. Digital data travels as pulses of infrared laser light.

Advantages over copper cable: enormous bandwidth (many signals multiplexed), low attenuation (fewer repeater amplifiers), no electromagnetic interference or crosstalk, hard to tap, thin and light. The cladding also protects the reflecting surface and stops light leaking between adjacent fibres in a bundle - the same idea is used in medical endoscopes to see inside the body.

Worked example 5 - critical angle in a fibre

An optical fibre has a core of refractive index 1.50 and cladding of refractive index 1.40. Find the critical angle at the core-cladding boundary.

sin c = n_2 / n_1 = 1.40 / 1.50 = 0.933, so c = **69.0°**.

Only rays hitting the wall at more than 69° from the normal (i.e. within about 21° of the fibre axis) are guided - which is why light must be launched nearly parallel to the axis.

Common misconception: TIR cannot happen going from air into glass. Going into a denser medium the ray bends toward the normal, so a refracted ray always exists. Check the direction of travel before using sin c = n_2/n_1.

4. Diffraction

Diffraction is the **spreading of a wave** when it passes through an aperture or past the edge of an obstacle. The wave bends into the geometric shadow region where a stream of particles could never go. Like reflection, diffraction changes only the direction of travel: the speed, frequency and wavelength are all **unchanged** (same medium).

The amount of spreading depends on how the aperture width b compares with the wavelength:

Aperture vs wavelength	What emerges
b much larger than λ	Almost straight-through beam; slight curving only at the edges of the wavefronts.
b comparable to λ (b ≈ λ)	Strong spreading - maximum diffraction; wavefronts emerge as near-semicircles.
b smaller than λ	The gap acts like a point source; circular wavefronts spread into the whole region beyond, but less energy gets through.

Everyday examples

- You can hear a conversation through an open door without seeing the speakers: audible sound has λ ≈ 0.1-10 m, comparable to the doorway, so sound diffracts strongly around it - but light, with $\lambda \approx 5 \times 10^{-7}$ m, passes through in effectively straight lines.
- Long-wavelength radio waves (kilometres) diffract over hills and around buildings; short-wavelength TV and satellite signals need line of sight.
- Water waves entering a harbour mouth spread out in semicircular arcs when the entrance is about one wavelength wide.

Why waves spread: Huygens' picture

A helpful model (and the one HL uses for the single slit in section 7) is Huygens' principle: **every point on a wavefront acts as a source of secondary wavelets**, and the new wavefront is the envelope of those wavelets. A wide gap transmits many wavelets whose envelope is nearly flat, so the beam continues straight; a gap about one wavelength wide transmits essentially a single wavelet, which spreads as a semicircle.

Reflection, refraction and diffraction compared

Phenomenon	Speed	Wavelength	Frequency	Direction
Reflection	unchanged	unchanged	unchanged	reversed about the normal ($\theta_i = \theta_r$)
Refraction	**changes**	**changes** (same ratio as speed)	unchanged	bends toward/away from normal (unless along the normal)
Diffraction	unchanged	unchanged	unchanged	spreads into the shadow region

> **Exam technique:** When sketching diffraction at a gap, keep the wavelength (spacing of wavefronts) the same before and after the gap - changing it is one of the most common ways to lose the mark.

5. Superposition and interference

The principle of superposition

When two or more waves meet at a point, the resultant displacement is the **vector sum** of the individual displacements. The waves then pass through each other completely unchanged. Displacements in the same direction reinforce; opposite displacements cancel.

Interference and coherence

Interference is the stable pattern of reinforcement and cancellation produced when waves from two sources overlap. A *stable* pattern needs **coherent** sources: sources with the same frequency and a **constant phase difference**. (Incoherent sources still superpose, but the pattern shifts randomly billions of times per second and averages away.) In practice coherence is achieved by driving two sources from one oscillator, or by splitting one wavefront in two, as Young did.

Whether two coherent waves reinforce or cancel at a point depends on the **path difference** - the difference between the distances from the two sources to that point (for sources oscillating in phase):

Constructive: path difference = $n\lambda$ Destructive: path difference = $(n + ½)\lambda$ $n = 0, 1, 2, ...$

A whole number of wavelengths means the waves arrive crest-on-crest (in phase, phase difference 0, 2π, 4π...); an extra half wavelength means crest-on-trough (antiphase, phase difference π).

Path difference	Phase difference	Result at that point
$0, \lambda, 2\lambda, ... = n\lambda$	$0, 2\pi, 4\pi, ...$ rad	Constructive: maximum, double amplitude
$½\lambda, 1½\lambda, ... = (n + ½)\lambda$	$\pi, 3\pi, ...$ rad	Destructive: minimum, zero amplitude (equal sources)
anything in between	between 0 and π	Intermediate amplitude

Two-source patterns: water and sound

Two dippers in a ripple tank, driven by the same beam, send out circular waves that overlap in a fan of alternating **maxima** (lines of double- amplitude water, called antinodal lines) and **minima** (calm nodal lines). The central line, equidistant from both sources, is always a maximum (path difference zero). Exactly the same experiment works with two loudspeakers fed by one signal generator: walking across the room you hear loud-quiet-loud alternation.

Worked example 6 - two loudspeakers

Two loudspeakers connected in phase to the same 680 Hz signal generator face a listener. The listener stands 3.75 m from one speaker and 2.50 m from the other. The speed of sound is 340 m s^{-1}. Is the sound at this point loud or quiet?

$\lambda = v / f = 340 / 680 = 0.500$ m.

Path difference = 3.75 - 2.50 = 1.25 m = 1.25 / 0.500 = 2.5 wavelengths = $(2 + ½)\lambda$.

A half-integer number of wavelengths means **destructive interference - a quiet point** (in a real room, only nearly silent, because the amplitudes from the two speakers are not exactly equal there and reflections intrude).

Link: Superposition is the master idea of Theme C: interference (this topic), beats and standing waves (C.4) are all superposition in different geometries.

6. Young's double-slit experiment

In 1801 Thomas Young produced interference with light - the decisive evidence that light is a wave. Monochromatic light falls on two very narrow slits, separated by a small distance d. Each slit diffracts the light into the region beyond, and because both slits are fed by the *same* incident wavefront, they behave as two coherent sources. On a screen a distance D away, the overlapping light forms **fringes**: evenly spaced bright and dark bands.

A bright fringe forms where the path difference from the two slits is $n\lambda$; a dark fringe where it is $(n + \frac{1}{2})\lambda$. Since the slit separation is tiny compared with the screen distance ($d \ll D$), the geometry gives a simple result for the spacing s between the centres of adjacent bright fringes (or adjacent dark fringes - it is the same):

$$s = \lambda D / d$$

Symbol	Meaning	Typical size (light)
s	fringe spacing: centre of one bright fringe to the next	≈ 1 mm
λ	wavelength of the light	$4\text{-}7 \times 10^{-7}$ m
D	distance from slits to screen	≈ 1-3 m
d	separation of the two slit centres (NOT the slit width)	≈ 0.1-1 mm

Where the formula comes from

For a point on the screen at distance x from the centre, the path difference between the two slits is approximately $d \sin\theta$, and for small angles $\sin\theta \approx \tan\theta = x/D$. Bright fringes need path difference $n\lambda$, so $x_n = n\lambda D/d$: the maxima are **equally spaced**, one fringe every $\lambda D/d$. The same reasoning with $(n + \frac{1}{2})\lambda$ puts the dark fringes exactly halfway between the bright ones.

Worked example 7 - laser fringes

A laser of wavelength 633 nm illuminates two slits 0.25 mm apart. Fringes are observed on a screen 2.0 m away. Find the fringe spacing.

$s = \lambda D / d = (633 \times 10^{-9} \times 2.0) / (0.25 \times 10^{-3})$

$s =$ **5.1×10^{-3} m ≈ 5 mm** - comfortably visible by eye, which is why laser double-slit demos work so well.

Worked example 8 - measuring a wavelength

In a double-slit experiment with d = 0.40 mm and D = 1.5 m, a student measures the distance across 10 fringe spacings (from fringe 0 to fringe 10) as 18.0 mm. Find the wavelength of the light.

Measuring across many fringes reduces the percentage uncertainty: s = 18.0 / 10 = 1.80 mm.

$\lambda = s\,d / D = (1.80 \times 10^{-3} \times 0.40 \times 10^{-3}) / 1.5 =$ **4.8×10^{-7} m = 480 nm** (blue-green light).

Worked example 9 - bright or dark?

Light of wavelength 600 nm passes through slits of separation 0.40 mm; the screen is 2.0 m away. What is observed at a point P on the screen 7.5 mm from the centre of the pattern?

Fringe spacing: $s = \lambda D/d = (600 \times 10^{-9} \times 2.0)/(0.40 \times 10^{-3}) = 3.0$ mm.

P lies at 7.5 / 3.0 = 2.5 fringe spacings, so the path difference there is $2.5\lambda = (2 + ½)\lambda$.

Half-integer path difference means **a dark fringe** - the third minimum out from the central maximum.

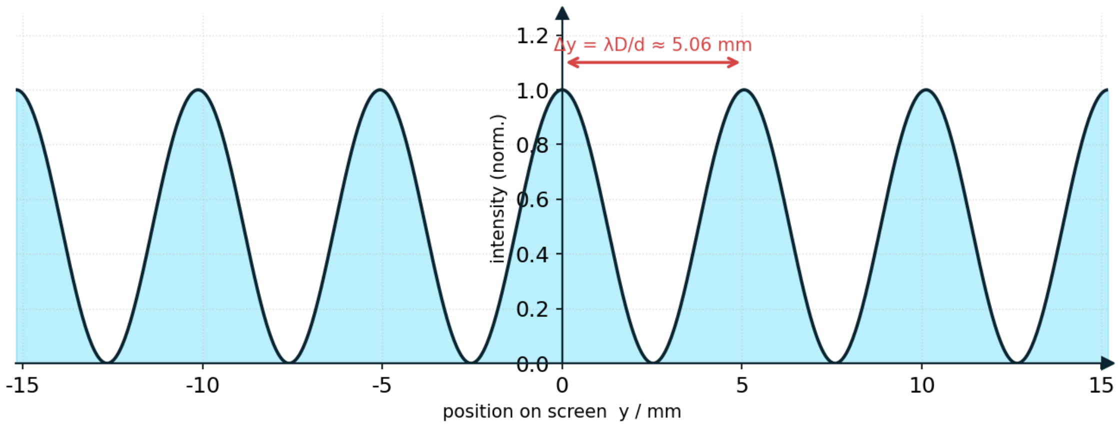

Figure 2. Young's double-slit intensity: evenly spaced bright $\cos^2$ fringes with spacing $\Delta y = \lambda D/d$. For λ = 633 nm, D = 2.0 m and d = 0.25 mm the spacing is ≈ 5.06 mm.

What happens when you change λ, D or d?

Change	Effect on fringe spacing $s = \lambda D/d$
Increase the wavelength (red instead of blue)	s increases - red fringes are wider apart than blue.
Move the screen further away (larger D)	s increases, but the fringes become dimmer.
Move the slits closer together (smaller d)	s increases - narrower slit separation, wider pattern.
Use water instead of air between slits and screen	λ is shorter in water, so s decreases.
Cover one slit	Fringes vanish - only the broad single-slit diffraction patch remains.
Widen both slits (same separation)	Brighter fringes, but fewer visible (see HL section 7).

White-light fringes

With white light, every wavelength makes its own fringe pattern with its own spacing. All colours have a maximum at the centre (path difference zero), so the **central fringe is white**. Either side, the fringes become coloured: blue (shortest λ, smallest s) appears on the inner edge of each fringe and red on the outer edge. After a few fringes the overlapping colours wash out into general illumination - so only a handful of white-light fringes are visible.

> **Common pitfall:** In $s = \lambda D/d$ the two distances on top and bottom are easy to swap. Remember d is the *tiny* one (slit separation, under a millimetre) and D is the *big* one (metres to the screen). A fringe spacing that comes out bigger than the screen is a sure sign they were swapped.

7. (HL) Single-slit diffraction

A single slit of finite width b does not produce uniform illumination: it produces a **diffraction pattern**. Every point across the width of the slit acts as a source of secondary wavelets (Huygens' idea), and these wavelets interfere with each other.

The intensity pattern

- A broad, bright **central maximum** centred on the straight-through direction.
- Dark minima either side, the first at angle $\theta = \lambda/b$.
- Weak **secondary maxima** beyond, roughly half the width of the central maximum and much fainter (the first secondary maximum has only about 5% of the central intensity; the next about 2%).

The first minimum occurs where wavelets from the top half of the slit cancel pairwise with wavelets from the bottom half - each pair differing in path by $\lambda/2$. For the small angles typical of light this gives:

$$\textbf{first minimum: } \theta = \lambda / b \ (\theta \text{ in radians; } b = \text{slit width})$$

The central maximum therefore has angular width $2\lambda/b$ - twice the width of every other bright band. Two consequences follow directly from $\theta = \lambda/b$:

- **Narrower slit** (smaller b): wider pattern, dimmer overall - less energy gets through.
- **Longer wavelength** (larger λ): wider pattern - red light spreads more than blue.

Worked example 10 - width of the central maximum

Light of wavelength 550 nm passes through a slit of width 0.10 mm and falls on a screen 2.0 m away. Find (a) the angle of the first minimum, (b) the linear width of the central maximum on the screen.

(a) $\theta = \lambda/b = (550 \times 10^{-9}) / (0.10 \times 10^{-3}) =$ **5.5×10^{-3} rad** (about 0.32°).

(b) Each first minimum sits a distance $D\theta = 2.0 \times 5.5 \times 10^{-3} = 11$ mm from the centre, so the central maximum is $2D\theta =$ **22 mm** wide.

Note the answer to (a) is in radians directly - no conversion needed for the small-angle formula, but convert to degrees only if asked.

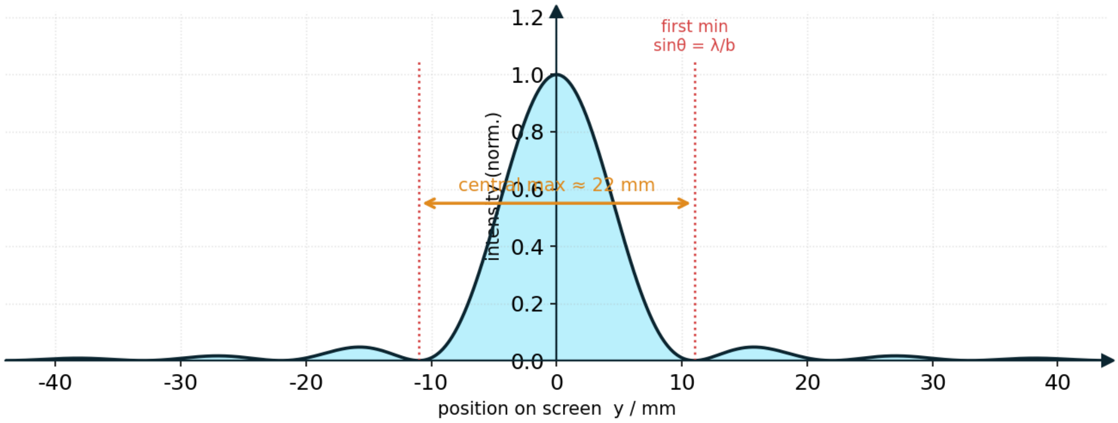

Figure 3. Single-slit diffraction: a sinc^2 envelope with a broad central maximum whose first minima lie where sin θ = λ/b. For λ = 550 nm, b = 0.10 mm and D = 2.0 m the first minima fall ≈ 11 mm from the centre, so the central maximum is ≈ 22 mm wide.

How the single slit modulates the double-slit pattern

Real double slits have finite width, so two effects act at once: the **interference** of light from the two slits (fine fringes, spacing set by the separation d) and the **diffraction** of light at each slit (broad envelope, width set by the slit width b). The result is the fine Young fringes drawn *inside* the single-slit intensity envelope: fringes near the centre are bright, those further out fade, and any fringe that lands exactly on a diffraction minimum is **missing**. Since d > b always, the fringes are always finer than the envelope. For example, if d = 3b, every third interference maximum is suppressed.

> **Sketching tip:** Draw the broad single-slit envelope first (central hump twice the width of the side humps), then fill it with evenly spaced interference fringes whose heights follow the envelope. Examiners look for: equal fringe spacing, envelope symmetry, and weak secondary humps.

8. (HL) Multiple slits and diffraction gratings

From 2 slits to N slits

Keep the slit separation d fixed and add more slits. The condition for a **principal maximum** is unchanged - all slits in phase - so the bright fringes stay in the *same positions*. But between them, N slits give N - 2 tiny secondary maxima and N - 1 minima, and as N grows these mop up the light between principal maxima. The principal maxima become:

- **sharper** - their width is proportional to 1/N, and
- **brighter** - peak intensity proportional to N^2 (see section 9).

A **diffraction grating** takes this to the extreme: hundreds of slits ('lines') per millimetre, N in the tens of thousands, giving needle-sharp bright lines on a dark background - ideal for precise wavelength measurement.

The grating equation

Light from neighbouring slits travels an extra path d sin θ to reach a distant point at angle θ from the straight-through direction. All N slits reinforce when this equals a whole number of wavelengths:

$$n\lambda = d \sin\theta \quad n = 0, 1, 2, ... \text{ (order of the maximum)}$$

Here d is the **grating spacing**: for a grating ruled with k lines per metre, $d = 1/k$. The $n = 0$ maximum is straight through for every wavelength; higher orders appear symmetrically either side. Note that grating angles are usually large, so the small-angle approximation must NOT be used - work with $\sin\theta$ itself.

Maximum order

Since $\sin\theta$ cannot exceed 1, the highest visible order is the largest integer n satisfying $n \le d/\lambda$. Beyond that, the geometry simply cannot supply the path difference.

Worked example 11 - sodium light on a grating

Sodium light ($\lambda = 589$ nm) falls normally on a grating with 600 lines per millimetre. Find (a) the angle of the first-order maximum, (b) the angle of the second-order maximum, (c) the total number of orders visible.

$d = 1 / 600$ mm $= 1.667 \times 10^{-6}$ m.

(a) $\sin\theta_1 = \lambda/d = 589 \times 10^{-9} / 1.667 \times 10^{-6} = 0.353$, so $\theta_1 =$ **20.7°**.

(b) $\sin\theta_2 = 2 \times 0.353 = 0.707$, so $\theta_2 =$ **45.0°**.

(c) $d/\lambda = 2.83$, so $n_{max} = 2$: orders $n = 0, 1, 2$ on each side - **5 maxima** in total (the pattern is symmetric about $n = 0$).

Spectroscopy

Because the angle of each maximum depends on λ, a grating spreads light into its component wavelengths: each order $n > 0$ is a complete **spectrum**, with red deviated more than violet (opposite to a prism). Measuring θ for a spectral line gives λ directly from $n\lambda = d \sin\theta$. This is how the compositions of stars are read from their emission and absorption lines, how gas discharge lamps are identified, and how instruments from CD-player pickups to astronomical spectrographs select wavelengths. Higher orders are more spread out (better resolution), but dimmer, and adjacent orders may overlap: the red end of the second order can overlap the violet end of the third.

Worked example 12 - do the orders overlap?

White light (400-700 nm) falls normally on a grating with 400 lines per millimetre. Show that the second-order and third-order spectra overlap.

$d = 1/400$ mm $= 2.50 \times 10^{-6}$ m.

Red end of 2nd order: $\sin\theta = 2 \times 700 \times 10^{-9} / 2.50 \times 10^{-6} = 0.560$, $\theta = 34.1°$.

Violet end of 3rd order: $\sin\theta = 3 \times 400 \times 10^{-9} / 2.50 \times 10^{-6} = 0.480$, $\theta = 28.7°$.

The violet of the 3rd order (28.7°) appears at a **smaller** angle than the red of the 2nd order (34.1°), so the two spectra overlap between these angles.

> **Check yourself:** A grating question that yields $\sin\theta > 1$ means that order does not exist - state this rather than forcing a calculator answer. It is a favourite way of testing understanding of n_{max}.

9. (HL) Intensity in interference patterns

The intensity of a wave (power per unit area, W m^{-2}) is proportional to the **square of the amplitude**:

$$I \propto A^2$$

Apply this to two coherent sources, each alone producing amplitude A and intensity I_0 at the screen:

Location	Resultant amplitude	Intensity
Constructive maximum (path difference nλ)	A + A = 2A	$(2A)^2 \propto$ **4 I_0**
Destructive minimum (path difference (n + ½)λ)	A - A = 0	**0**
Average over the pattern	-	2 I_0 (energy is conserved)

The maxima are four times - not twice - as intense as a single source. No energy is destroyed at the dark fringes; it is **redistributed** to the bright ones, so the average across the pattern is exactly the $2I_0$ that two independent sources would give.

With N slits, the amplitudes of all N waves add in phase at a principal maximum, giving amplitude NA and intensity N^2I_0. Since total energy scales only as N, the maxima must simultaneously become narrower (width $\propto$ 1/N) - which is precisely why a grating gives sharp, bright lines.

If the two amplitudes are unequal ($A_1 > A_2$), the minima are no longer completely dark: the resultant amplitude there is $A_1 - A_2$, so $I_{min} \propto (A_1 - A_2)^2 > 0$. This is why real fringe patterns lose contrast away from the centre.

> **Exam technique:** Amplitudes add; intensities do not. Always convert intensity to amplitude (square root), add or subtract the amplitudes, then square to get back to intensity.

10. Common pitfalls

- Measuring angles from the surface instead of the normal in reflection and Snell's law.
- Saying frequency changes on refraction - it never does; speed and wavelength change together.
- Applying total internal reflection when light travels into a denser medium - TIR needs $n_1 > n_2$ AND $\theta > c$.
- Mixing up the symbols in $s = \lambda D/d$: d is the slit separation (small), D the screen distance (large), s the fringe spacing - and confusing d with the slit width b of single-slit work.
- Treating $\theta = \lambda/b$ as if it gave degrees - it gives radians, and it is a small-angle formula; the grating equation $n\lambda = d \sin\theta$ is exact and must be used at large angles.
- Forgetting the phase change of π when a pulse reflects from a fixed end (none at a free end).
- Claiming maxima with two sources are twice as intense as one source - amplitude doubles, so intensity quadruples.
- Converting lines per mm to grating spacing incorrectly: 600 lines per mm means d = (1/600) mm = 1.67×10^{-6} m, not 600×10^{-3} m.

- Forgetting the factor of 2 in echo problems - the sound travels there and back.

11. Quick reference

Result	Statement
Law of reflection	$\theta_i = \theta_r$, measured from the normal
Pulse reflection	fixed end: inverted (π phase change); free end: upright (no phase change)
Refractive index	$n = c/v$; $\sin\theta_1/\sin\theta_2 = v_1/v_2 = \lambda_1/\lambda_2$
Snell's law	$n_1 \sin\theta_1 = n_2 \sin\theta_2$
Critical angle	$\sin c = n_2/n_1$ (= 1/n against air); TIR when θ > c, dense to less dense only
Diffraction	spreading at gaps and edges; greatest when aperture ≈ λ
Interference conditions	constructive: path difference = nλ; destructive: (n + ½)λ; sources must be coherent
Double slit	s = λD/d (d = slit separation)
(HL) Single slit	first minimum at θ = λ/b (radians); central max has angular width 2λ/b
(HL) Grating	nλ = d sin θ; d = 1/(lines per metre); n_{max} = largest integer ≤ d/λ
(HL) Intensity	$I \propto A^2$; two equal coherent sources: maxima $4I_0$, minima 0; N slits: N^2I_0

12. Test yourself

Attempt these without notes; full answers below. Questions 7-9 are HL only.

1. Light strikes the surface of a glass block (n = 1.50) at 55° to the normal. Find the angle of refraction, and state which way the ray bends.

2. The refractive index of diamond is 2.42. Calculate the speed of light in diamond and its critical angle against air.

3. A pulse travels along a rope toward a wall to which the rope is firmly tied. Describe the reflected pulse, and contrast it with reflection from a free end.

4. Two loudspeakers in phase emit sound of frequency 850 Hz (speed of sound 340 m s^{-1}). At a point where the path difference is 0.60 m, is the interference constructive or destructive?

5. In a double-slit experiment, light of wavelength 486 nm passes through slits 0.30 mm apart onto a screen 2.4 m away. Find the fringe spacing.

6. Double-slit fringes of spacing 2.1 mm are formed with d = 0.50 mm and D = 1.8 m. Find the wavelength. Describe the pattern if the source were white light instead.

7. (HL) Light of wavelength 640 nm passes through a single slit of width 0.080 mm. Find the angular position of the first minimum and the width of the central maximum on a screen 1.5 m away.

8. (HL) A grating has 300 lines per millimetre. For light of wavelength 650 nm find the angle of the first-order maximum and the highest order observable.

9. (HL) Each of two coherent sources alone produces intensity I_0 at a point P. What is the intensity at P if the waves arrive (a) in phase, (b) in antiphase, (c) if one source's amplitude is doubled and they arrive in

phase?

10. Explain why you can hear someone speaking around an open doorway but cannot see them.

Answers

1. $\sin\theta_2 = \sin 55° / 1.50 = 0.819 / 1.50 = 0.546$, so $\theta_2 = 33°$. The ray bends toward the normal (air to glass = slower medium).

2. $v = c/n = 3.00 \times 10^8 / 2.42 = 1.24 \times 10^8$ m s^{-1}. $\sin c = 1/2.42 = 0.413$, so $c = 24.4°$. This tiny critical angle traps light inside a cut diamond, causing its sparkle.

3. Fixed end: the pulse returns inverted (crest becomes trough) - a phase change of π rad - with the same speed, length and (ideally) amplitude. Free end: the pulse returns upright with no phase change.

4. $\lambda = 340/850 = 0.40$ m. Path difference = 0.60 m = $1.5\lambda = (1 + ½)\lambda$: **destructive** - a quiet point.

5. $s = \lambda D/d = (486 \times 10^{-9} \times 2.4)/(0.30 \times 10^{-3}) = 3.9 \times 10^{-3}$ m ≈ 3.9 mm.

6. $\lambda = sd/D = (2.1 \times 10^{-3} \times 0.50 \times 10^{-3})/1.8 = 5.8 \times 10^{-7}$ m ≈ 580 nm. With white light: a white central fringe, then coloured fringes with blue edges nearer the centre and red edges further out, fading into overlap after a few fringes.

7. $\theta = \lambda/b = 640 \times 10^{-9} / 8.0 \times 10^{-5} = 8.0 \times 10^{-3}$ rad. Central maximum width = $2D\theta = 2 \times 1.5 \times 8.0 \times 10^{-3} = 2.4 \times 10^{-2}$ m = 24 mm.

8. $d = 1/300$ mm $= 3.33 \times 10^{-6}$ m. $\sin\theta_1 = \lambda/d = 650 \times 10^{-9}/3.33 \times 10^{-6} = 0.195$, so $\theta_1 = 11.2°$. $d/\lambda = 5.13$, so the highest order is $n = 5$.

9. (a) Amplitudes add: 2A, so $I = 4I_0$. (b) Amplitudes cancel: $I = 0$. (c) Amplitudes A + 2A = 3A, so $I = 9I_0$.

10. Audible sound has wavelengths (roughly 0.1-10 m) comparable to the width of a doorway, so it diffracts strongly and spreads around the opening. Visible light has a wavelength about a factor of 10^6 smaller than the doorway, so its diffraction is negligible and it travels in effectively straight lines - no line of sight, no image.