# IB DP PHYSICS

## Theme B: The Particulate Nature of Matter

## B.5 Current and Circuits

Revision Notes · Standard and Higher Level

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Original notes prepared for the IB Diploma Programme Physics course (first assessment 2025)

# What the syllabus requires

B.5 is common to SL and HL. By the end of the topic you should be able to work confidently with each of the following. Use this list as a final checklist before the exam.

| Understanding | You should be able to... |
|---|---|
| Electric current | Use $I = \Delta q / \Delta t$; describe direct current (d.c.) and the free-electron model of conduction in metals. |
| Potential difference | Use $V = W / q$ as work done per unit charge moving between two points. |
| Resistance and Ohm's law | Use $R = V / I$; sketch and interpret I-V characteristics of ohmic and non-ohmic components. |
| Resistivity | Use $\rho = RA / L$ and explain how resistance depends on the dimensions and material of a conductor. |
| Electrical power | Use $P = IV = I^2R = V^2 / R$ and choose the correct form for the situation. |
| Circuit laws | Apply conservation of charge and energy to series, parallel and mixed resistor networks, and to combinations of cells. |
| Meters and dividers | Treat ideal ammeters (zero resistance) and ideal voltmeters (infinite resistance); analyse potential-divider and sensing circuits with thermistors and LDRs. |
| emf and internal resistance | Use $\varepsilon = I(R + r)$; interpret terminal p.d. and the V-I graph of a real cell. |

> **Exam note:** B.5 sits in Theme B because a current is a flow of charged particles: the microscopic (particle) picture explains the macroscopic rules. Expect links to B.1 (heating water electrically) and to Theme D (fields doing work on charges).

# 1. Charge and electric current

An **electric current** is a flow of charged particles. The current through a surface is defined as the rate of flow of charge:

$$\boldsymbol{I = \Delta q / \Delta t}$$

The SI unit of current, the **ampere (A)**, is a base unit; the unit of charge, the **coulomb (C)**, is defined from it: 1 C is the charge passing a point in 1 s when the current is 1 A, so 1 C = 1 A s. All charge comes in multiples of the elementary charge $e = 1.60 \times 10^{-19}$ C (data booklet).

In a **direct current (d.c.)** circuit the charge carriers drift in one direction only, driven by a cell or battery. (Alternating current, in which the direction reverses periodically, is not required in B.5.)

## Conventional current and electron flow

**Conventional current** is taken to flow from the positive terminal of a cell, around the external circuit, to the negative terminal - the direction in which positive charge would move. In a metal the actual carriers are **negatively charged electrons**, which drift the opposite way. All circuit rules and diagrams use conventional current; only mention electron flow when a question explicitly asks about the microscopic picture.

### The free-electron model of a metal

A metal consists of a lattice of fixed positive ions permeated by a 'gas' of delocalised free electrons (roughly one per atom). With no p.d. applied, the electrons move rapidly (about $10^5$-$10^6$ m $s^{-1}$) but randomly, so there is no net flow. Applying a p.d. sets up an electric field in the wire that superimposes a slow **drift velocity** - typically well below a millimetre per second - on the random motion. Collisions between drifting electrons and vibrating lattice ions transfer energy to the lattice: this is the microscopic origin of resistance and of resistive heating.

> **Why do lights come on instantly?** The electric field that pushes the electrons is established around the circuit at close to the speed of light, so electrons *everywhere* in the circuit start drifting almost simultaneously. You do not wait for one electron to travel from the switch to the lamp - the drift itself would take hours.

**Worked example 1 - counting electrons**

The current in a lamp filament is 0.25 A. Calculate (a) the charge passing through the lamp in 2.0 minutes, (b) the number of electrons this represents.

(a) $\Delta q = I\Delta t = 0.25 \times 120 =$ **30 C**.

(b) $N = \Delta q / e = 30 / (1.60 \times 10^{-19}) =$ $\mathbf{1.9 \times 10^{20}}$ **electrons**.

Sanity check: enormous numbers of electrons are involved even for small everyday currents - each carries a tiny charge.

## 2. Potential difference and emf

The **potential difference (p.d.)** between two points is the work done per unit charge as charge moves between them:

$$V = W / q$$

Its unit is the **volt**: 1 V = 1 J $C^{-1}$. A p.d. of 6 V across a component means every coulomb of charge passing through it transfers 6 J of energy to that component (as heat, light, mechanical work, and so on).

### The energy picture of a circuit

Think of charge as an energy courier. Inside the source, each coulomb is given energy (chemical energy in a cell is converted to electrical potential energy); as the charge passes through the components of the external circuit it delivers that energy. The charge itself is never used up - it returns to the source to be 'recharged' with energy. Currents are the same on both sides of a lamp; it is *energy*, not charge, that the lamp removes.

### Electromotive force (emf)

The **emf** ε of a source is the total work done per unit charge in driving charge once around a complete circuit - equivalently, the energy converted from other forms to electrical energy per unit charge inside the source:

$$\varepsilon = W / q$$

Despite its historical name, emf is **not a force**; like p.d. it is measured in volts. The distinction between emf and terminal p.d. matters once the source has internal resistance (Section 9).

**Worked example 2 - energy carried by charge**

A battery of emf 9.0 V drives a current of 0.40 A through a small motor for 30 s. Calculate (a) the charge that flows, (b) the total energy transferred by the battery.

(a) $q = I\,t = 0.40 \times 30 =$ **12 C**.

(b) $W = q\varepsilon = 12 \times 9.0 =$ **108 J** (about 110 J to 2 s.f.).

# 3. Resistance and I-V characteristics

The **resistance** of a component is defined as the ratio of the p.d. across it to the current through it:

$$R = V / I$$

The unit is the **ohm**: $1\ \Omega = 1\ \mathrm{V\ A^{-1}}$. This definition applies to *every* component at *every* point of its characteristic - it is not restricted to ohmic conductors.

## Ohm's law - the special case

**Ohm's law** states that for a metallic conductor **at constant temperature**, the current is directly proportional to the p.d. across it ($I \propto V$, so $R$ is constant). Components that obey it are called **ohmic**; their I-V characteristic is a straight line through the origin. Ohm's law is an experimental result about particular materials, not a universal law - the definition $R = V/I$ always holds, but constancy of $R$ does not.

## I-V characteristics you must describe

| Component | Shape of I-V graph | Physical explanation |
|---|---|---|
| Ohmic (metal) resistor | Straight line through the origin, same in both directions. | Temperature (and hence lattice vibration) essentially constant, so R is constant. |
| Filament lamp | Curve through the origin whose gradient decreases as V grows (current levels off); symmetric for reversed p.d. | Larger currents heat the filament; the ions vibrate more, electrons collide more often, so R increases with temperature. |
| Diode | Almost zero current for reverse bias and for small forward p.d.; beyond about 0.7 V current rises very steeply. | Conducts in one direction only: resistance is very high below the threshold and very low above it. |
| NTC thermistor | R decreases as temperature increases. | Higher temperature frees more charge carriers in the semiconductor; this outweighs increased lattice vibration. |
| LDR | R decreases as light intensity increases. | Photons free additional charge carriers in the semiconductor. |

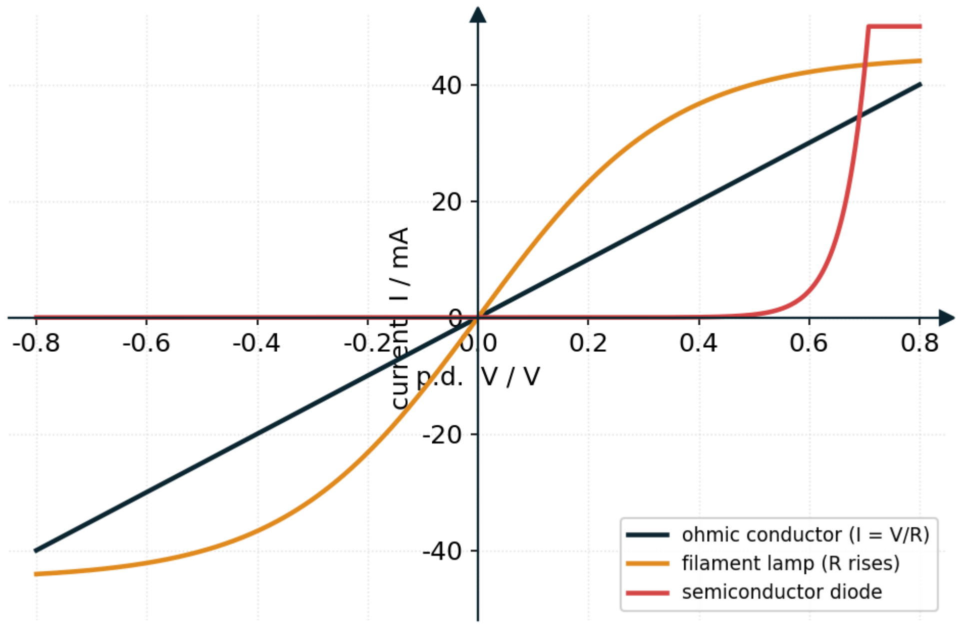


*Figure 1. I–V characteristics on common axes: an ohmic conductor is a straight line through the origin (gradient 1/R); the filament lamp bends over as its resistance rises with temperature; the diode passes almost no current until the forward p.d. reaches about 0.7 V, then rises steeply.*

**Graph-drawing tip:** Check the axes before sketching. On an I-V graph a filament lamp bends *towards* the V-axis (current levels off); on a V-I graph the same physics bends the curve towards the I-axis... wait - towards the *V* direction. Safest rule: for a filament lamp, resistance (the ratio V/I) always increases as the current increases.

**Worked example 3 - a lamp is not ohmic**

A filament lamp passes 0.40 A when the p.d. across it is 1.0 V, and 2.0 A when the p.d. is 12 V. Show that the lamp does not obey Ohm's law, and find its resistance in each case.

At 1.0 V: $R$ = 1.0 / 0.40 = **2.5** Ω.

At 12 V: $R$ = 12 / 2.0 = **6.0** Ω.

The ratio V/I is not constant, so I is not proportional to V: the lamp is non-ohmic. The resistance rises because the filament runs much hotter at the higher current.

# 4. Resistivity

Resistance depends on the geometry of a sample as well as on its material. Experiment shows that for a uniform conductor of length $L$ and cross-sectional area $A$:

$$R = \rho L / A \text{ or equivalently } \rho = RA / L$$

The constant ρ is the **resistivity** of the material, with unit Ω **m** (ohm metre - not ohm per metre). It is a property of the material alone (at a given temperature), whereas resistance is a property of a particular specimen.

- Doubling the length doubles the resistance (twice as many collisions along the path): $R \propto L$.

- Doubling the area halves the resistance (more parallel paths for the current): $R \propto 1/A$.
- For a circular wire of diameter $d$, $A = \pi d^2/4$, so halving the diameter multiplies the resistance by four.

### Typical values (orders of magnitude at room temperature)

| Material | Class | Resistivity / Ω m | Notes |
|---|---|---|---|
| Copper | Metal (good conductor) | $1.7 \times 10^{-8}$ | Connecting wires - resistance usually negligible. |
| Nichrome | Alloy | about $1.1 \times 10^{-6}$ | High for a metal and stable when hot: heating elements. |
| Silicon (pure) | Semiconductor | about $2 \times 10^{3}$ | Between metals and insulators; falls steeply as temperature rises. |
| Glass | Insulator | about $10^{10}$-$10^{14}$ | Practically no free charge carriers. |

The spread from copper to glass covers more than 20 orders of magnitude - one of the largest ranges of any physical property.

**Worked example 4 - designing a heating element**

A heating element is made from nichrome wire ($\rho = 1.1 \times 10^{-6}$ Ω m) of length 0.80 m and diameter 0.40 mm. (a) Calculate its resistance. (b) What length of copper wire ($\rho = 1.7 \times 10^{-8}$ Ω m) of the same diameter would have a resistance of 1.0 Ω?

(a) $A = \pi d^2/4 = \pi \times (0.40 \times 10^{-3})^2 / 4 = 1.26 \times 10^{-7}$ m$^2$.

$R = \rho L / A = (1.1 \times 10^{-6} \times 0.80) / (1.26 \times 10^{-7}) =$ **7.0** Ω.

(b) Rearranging: $L = RA / \rho = (1.0 \times 1.26 \times 10^{-7}) / (1.7 \times 10^{-8}) =$ **7.4 m**.

Moral: you need metres of copper to make even one ohm - which is why we treat connecting leads as resistance-free.

**Unit trap:** Convert diameters in mm to metres *before* squaring, and do not confuse diameter with radius. A factor-of-4 error in A is the single most common resistivity mistake.

# 5. Electrical power and energy

A component with p.d. $V$ across it and current $I$ through it converts electrical energy at the rate

$$P = IV = I^2R = V^2 / R$$

The second and third forms follow by substituting $V = IR$. The energy transferred in time $t$ is $E = Pt$.

### Choosing the right form

| Situation | Best form | Why |
|---|---|---|
| You know the current through the component | $P = I^2R$ | Series circuits: the current is common to all components. |

| Situation | Best form | Why |
|---|---|---|
| You know the p.d. across the component | $P = V^2 / R$ | Parallel branches: the p.d. is common to all branches. |
| You know both V and I | $P = IV$ | No resistance needed - works even for non-ohmic devices. |

## The kilowatt hour

Electricity meters record energy in **kilowatt hours**: 1 kWh is the energy transferred by a 1 kW device in 1 hour, so 1 kWh = 1000 W × 3600 s = $3.6 \times 10^6$ J = 3.6 MJ. It is a unit of *energy*, not power.

## Heating applications

Kettles, toasters, immersion heaters and fuses all exploit resistive (Joule) heating: drifting electrons transfer kinetic energy to the lattice in collisions, raising its temperature. Combining with Theme B.1, the energy balance for an electric heater warming a liquid is $Pt = mc\Delta T$ (assuming negligible losses).

**Worked example 5 - the electric kettle**

A kettle is rated 2.2 kW at 230 V. Calculate (a) the current it draws, (b) the resistance of its element, (c) the time to heat 0.50 kg of water by 80 K (c = 4200 J $kg^{-1}$ $K^{-1}$, no losses).

(a) $I = P / V = 2200 / 230 =$ **9.6 A**.

(b) $R = V^2 / P = 230^2 / 2200 =$ **24 Ω**.

(c) Energy needed: $mc\Delta T = 0.50 \times 4200 \times 80 = 1.68 \times 10^5$ J. Time: $t = E / P = 1.68 \times 10^5 / 2200 =$ **76 s**.

# 6. Circuit rules: series, parallel and mixed networks

All circuit analysis rests on two conservation laws.

| Law | Circuit statement | Consequence |
|---|---|---|
| Conservation of charge (junction rule) | The total current into a junction equals the total current out: charge cannot pile up or vanish. | Current is the same everywhere in a series loop; branch currents in parallel add to the supply current. |
| Conservation of energy (loop rule) | Around any closed loop, the sum of the emfs equals the sum of the p.d.s across components. | Series p.d.s add up to the supply p.d.; parallel branches all have the same p.d. |

## Resistors in series

The same current $I$ flows through each resistor, and the p.d.s add: $V = V_1 + V_2 + ... = IR_1 + IR_2 + ...$ Dividing by $I$:

$$\boldsymbol{R_s = R_1 + R_2 + R_3 + ...}$$

## Resistors in parallel

Each resistor has the same p.d. $V$, and the currents add: $I = I_1 + I_2 + ... = V/R_1 + V/R_2 + ...$ Dividing by $V$:

$$1/R_p = 1/R_1 + 1/R_2 + 1/R_3 + ...$$

Useful facts about the parallel combination: it is always **smaller than the smallest** individual resistance; two equal resistors in parallel give half of one of them; and for exactly two resistors the 'product over sum' shortcut $R_p = R_1R_2 / (R_1 + R_2)$ is quickest.

> **Classic error:** $1/R_p = 1/6 + 1/3$ gives $1/R_p = 1/2$, so $R_p = 2\ \Omega$ - do not forget the final reciprocal. An answer larger than the smallest branch resistance is always wrong.

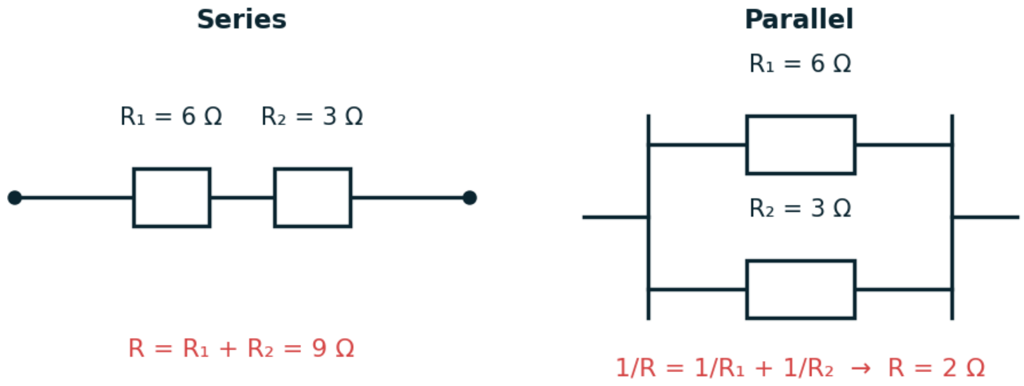


*Figure 2. Series vs parallel. In series $R = R_1 + R_2$; in parallel $1/R = 1/R_1 + 1/R_2$. With $R_1 = 6\ \Omega$ and $R_2 = 3\ \Omega$, the series total is 9 Ω and the parallel total is 2 Ω (smaller than the smallest branch).*

## Strategy for mixed networks

1. Redraw the circuit, collapsing obvious series or parallel clusters into single equivalent resistors, repeating until one resistance remains. 2. Find the supply current from the total resistance. 3. Work back outwards: series elements share the current (find their p.d.s); parallel elements share the p.d. (find their currents). 4. Check: branch currents must satisfy the junction rule, and p.d.s around each loop must sum to the emf.

**Worked example 6 - a three-resistor network**

A 12 V battery of negligible internal resistance is connected to a 4.0 Ω resistor in series with a parallel pair: 6.0 Ω and 3.0 Ω. Find the current in and p.d. across every resistor, and the power delivered by the battery.

Step 1 - collapse: parallel pair: $R_p = (6.0 \times 3.0)/(6.0 + 3.0) = 2.0\ \Omega$. Total: $R = 4.0 + 2.0 = 6.0\ \Omega$.

Step 2 - supply current: $I = 12 / 6.0 =$ **2.0 A** (this flows through the 4.0 Ω).

Step 3 - p.d.s: $V_4 = 2.0 \times 4.0 =$ **8.0 V**; the parallel pair takes the remaining $V_p = 12 - 8.0 =$ **4.0 V**.

Branch currents: $I_6 = 4.0/6.0 =$ **0.67 A**; $I_3 = 4.0/3.0 =$ **1.33 A**. Check: 0.67 + 1.33 = 2.0 A - junction rule satisfied.

Power: $P = \varepsilon I = 12 \times 2.0 =$ **24 W**. (Check: $2.0^2 \times 4.0 + 4.0^2/6.0 + 4.0^2/3.0 = 16 + 2.7 + 5.3 = 24$ W.)

## Cells in series and parallel

Identical cells in **series** add their emfs (and their internal resistances). Identical cells in **parallel** give the same emf as one cell but share the current, so the effective internal resistance is reduced (halved for two cells) and the battery lasts longer.

## 7. Measuring current and potential difference

| Meter | How connected | Ideal property | Why |
|---|---|---|---|
| Ammeter | In **series** with the component, so the full current passes through it. | **Zero** resistance | It must not reduce the current it is trying to measure. |
| Voltmeter | In **parallel** with (across) the component. | **Infinite** resistance | It must not draw ('steal') any current from the circuit. |

Real meters fall short of these ideals: a real ammeter has a small but non-zero resistance (its reading is correct, but inserting it slightly reduces the circuit current), and a real voltmeter has a large but finite resistance (it diverts some current, lowering the p.d. it is measuring). Digital multimeters come close to ideal; moving-coil voltmeters can disturb high-resistance circuits badly.

**Worked example 7 - the loading effect of a voltmeter**

Two 10 kΩ resistors are connected in series across an ideal 12.0 V supply. A voltmeter of resistance 10 kΩ is connected across the lower resistor. (a) What p.d. is it trying to measure? (b) What does it actually read?

(a) Without the meter the two equal resistors split the supply equally: **6.0 V**.

(b) Meter + lower resistor in parallel: (10 × 10)/(10 + 10) = 5.0 kΩ. Circuit is now 10 kΩ in series with 5.0 kΩ, so the lower section takes 12.0 × 5.0/15 = **4.0 V**.

The act of measuring changed the answer by 33%. The higher the circuit resistances, the worse the loading - a key evaluation point in practical write-ups.

**Spot the wiring error:** An ammeter connected in parallel (almost zero resistance) short-circuits the component and may blow its fuse; a voltmeter connected in series (huge resistance) reduces the current almost to zero, and the circuit simply stops working.

## 8. Potential dividers and sensing circuits

A **potential divider** is two components in series across a supply; the p.d. divides between them in proportion to their resistances (same current, so $V \propto R$). The output across $R_2$ is:

$$V_{out} = V_{in} \times R_2 / (R_1 + R_2)$$

Quick check: with $R_1$ = 400 Ω and $R_2$ = 800 Ω across 6.0 V, the output across $R_2$ is 6.0 × 800/1200 = 4.0 V and the remaining 2.0 V appears across $R_1$.

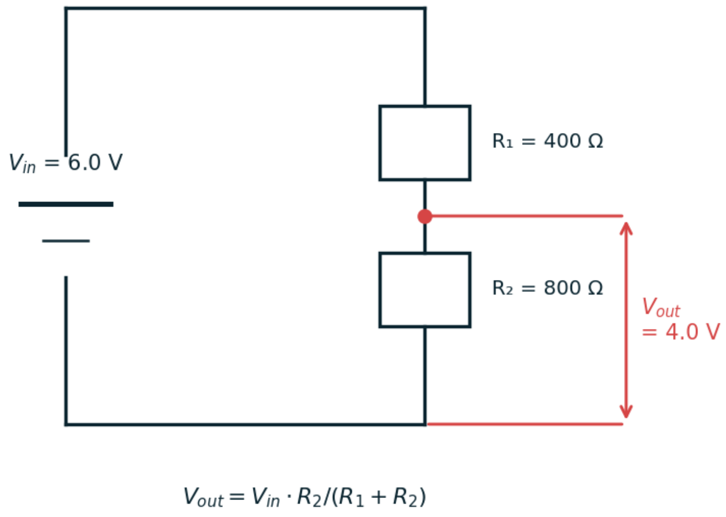


*Figure 3. Potential divider: $V_{out} = V_{in} \cdot R_2/(R_1+R_2)$. With $V_{in}$ = 6.0 V, $R_1$ = 400 Ω and $R_2$ = 800 Ω, the output across $R_2$ is 4.0 V.*

## Sensing circuits

Replace one resistor with a sensor whose resistance responds to the environment and the output voltage becomes a signal:

- **NTC thermistor**: resistance falls as temperature rises - temperature sensing (thermostats, fire alarms).
- **LDR**: resistance falls as light intensity rises - light sensing (automatic street lights).

Reasoning template (learn it): as light intensity increases, the LDR's resistance decreases, so the LDR takes a *smaller share* of the supply p.d. and the p.d. across the fixed resistor increases. Swapping the two components, or taking the output across the other component, inverts the behaviour - so read the circuit carefully before answering.

**Worked example 8 - a light-activated switch**

A 12 V supply feeds a divider consisting of a 2.0 kΩ fixed resistor in series with an LDR. The output, taken across the LDR, switches a lamp on when it exceeds 6.0 V. The LDR's resistance is 10 kΩ in darkness and 0.50 kΩ in bright light. Find the output in each case and explain the circuit's action.

Darkness: $V_{out}$ = 12 × 10/(2.0 + 10) = **10 V** - above the 6.0 V threshold, so the lamp is ON at night.

Bright light: $V_{out}$ = 12 × 0.50/(2.0 + 0.50) = **2.4 V** - below threshold, lamp OFF by day.

The divider converts a resistance change into a voltage change: this is the standard input stage of almost every electronic sensor.

## Variable resistors: rheostat vs potentiometer

A variable resistor wired with **two terminals** acts as a **rheostat**: it controls the current, but the p.d. across the load can never be reduced to zero. Wired with **all three terminals** as a potential divider (potentiometer), the sliding contact taps off any output from 0 up to the full supply p.d. - the arrangement used to obtain the widest possible range of readings in practical work.

# 9. emf and internal resistance

A real cell has **internal resistance** *r*: its own chemicals and electrodes resist the flow of charge. Model a real cell as an ideal emf ε in series with a small resistor *r* (both inside the casing). Energy conservation around the circuit with external load *R* gives:

$$\varepsilon = I(R + r) = IR + Ir$$

The **terminal potential difference** - what a voltmeter across the cell actually reads while current flows - is:

$$V = \varepsilon - Ir$$

The 'lost volts' *Ir* represent energy dissipated per coulomb *inside* the cell. When no current flows (open circuit, or an ideal voltmeter alone), *V* = ε: this is how emf is measured directly.

## The V-I graph of a real cell

Plot terminal p.d. *V* against current *I* (vary *I* with a rheostat). Since *V* = ε - *rI*, the graph is a straight line with:

- **y-intercept** = ε (terminal p.d. at zero current),
- **gradient** = -*r* (the steeper the line, the larger the internal resistance),
- x-intercept = ε/*r*, the maximum (short-circuit) current the cell can supply.

## Power to the load

The load receives $P = I^2R$ while $I^2r$ is wasted internally. As *R* is reduced the current rises but a growing fraction of the energy is lost inside the cell; as *R* is increased the current falls towards zero. Between the extremes the power delivered to the load passes through a **maximum, which occurs when *R* = *r*** (qualitative knowledge of this result is expected). Efficiency, by contrast, keeps rising with *R*: efficiency = *R*/(*R*+*r*).

**Worked example 9 - a battery under load**

A battery of emf 9.0 V and internal resistance 0.50 Ω is connected to a 4.0 Ω load. Find (a) the current, (b) the terminal p.d., (c) the power delivered to the load and the power wasted internally, (d) the efficiency.

(a) *I* = ε / (*R* + *r*) = 9.0 / 4.5 = **2.0 A**.

(b) *V* = ε - *Ir* = 9.0 - 2.0 × 0.50 = **8.0 V** (check: *IR* = 2.0 × 4.0 = 8.0 V).

(c) Load: $I^2R$ = 4.0 × 4.0 = **16 W**; internal: $I^2r$ = 4.0 × 0.50 = **2.0 W**.

(d) Efficiency = 16 / 18 = **0.89 (89%)** - equivalently R/(R+r) = 4.0/4.5.

## Measuring ε and r experimentally

Connect the cell to a variable load with an ammeter in series and a voltmeter across the terminals. Adjust the load to record several (*I*, *V*) pairs, keeping currents small and switching off between readings so the cell does not heat up (its internal resistance would change). Plot *V* against *I* and draw the best straight line:

intercept gives ε, magnitude of gradient gives *r*. Repeating and averaging reduces random error from contact resistance.

> **Why car lights dim:** When the starter motor draws a very large current, the lost volts $Ir$ become significant, the terminal p.d. of the 12 V battery drops, and the headlamps - connected across the terminals - momentarily dim. One sentence, three marks.

## 10. Common pitfalls

- Saying current is 'used up' by components. Current is the same on both sides of a lamp; energy is transferred, charge is conserved.
- Forgetting the final reciprocal in parallel-resistor calculations, or using the product-over-sum shortcut for three resistors (it works only for two).
- Applying $V^2/R$ with the supply voltage when the component does not have the full supply p.d. across it. Identify the p.d. across *that* component first.
- Quoting Ohm's law as if it applied to all components. R = V/I is a definition; proportionality holds only for ohmic conductors at constant temperature.
- Mixing up series and parallel meter connections, or ignoring the loading effect of a real voltmeter in high-resistance circuits.
- Sign slips with internal resistance: terminal p.d. is $\varepsilon - Ir$ when the cell discharges. Writing $\varepsilon + Ir$ gains no marks (that case is for charging a cell).
- Using $\text{mm}^2$ in resistivity calculations without converting to $\text{m}^2$ (factor $10^{-6}$), or halving the diameter incorrectly.
- Treating the kilowatt hour as a unit of power. It is energy: 1 kWh = 3.6 MJ.

## 11. Quick reference

| Result | Statement |
|---|---|
| Current | $I = \Delta q / \Delta t$; 1 A = 1 C $\text{s}^{-1}$ |
| Potential difference | $V = W / q$; 1 V = 1 J $\text{C}^{-1}$ |
| Resistance | $R = V / I$; ohmic conductor: R constant (constant temperature) |
| Resistivity | $\rho = RA / L$; unit Ω m |
| Power / energy | $P = IV = I^2R = V^2/R$; $E = Pt$; 1 kWh = $3.6 \times 10^6$ J |
| Series | same I; $R_s = R_1 + R_2 + ...$ |
| Parallel | same V; $1/R_p = 1/R_1 + 1/R_2 + ...$ |
| Potential divider | $V_{out} = V_{in}R_2/(R_1+R_2)$ |
| Real cell | $\varepsilon = I(R + r)$; terminal p.d. $V = \varepsilon - Ir$; V-I graph: intercept ε, gradient $-r$ |
| Meters | ideal ammeter: series, zero resistance; ideal voltmeter: parallel, infinite resistance |

# 12. Test yourself

Attempt these without notes; full answers below.

1. A current of 3.0 A flows for 10 minutes. Find the charge transferred and the number of electrons that pass.

2. A 2.0 m wire of diameter 0.30 mm has resistance 5.6 Ω. Calculate the resistivity of its material.

3. A wire is replaced by one of the same material with twice the length and half the diameter. By what factor does its resistance change?

4. Resistors of 2.0 Ω, 3.0 Ω and 6.0 Ω are connected in parallel, and the combination is in series with a 3.0 Ω resistor across an 8.0 V ideal supply. Find the supply current.

5. A 2.0 kW heater runs for 3.5 hours. How much energy is used in kWh and in joules, and what does it cost at $0.25 per kWh?

6. A divider consists of a 300 Ω resistor and a 600 Ω resistor across 9.0 V, output across the 600 Ω. Find the output. The 300 Ω resistor is now replaced by an NTC thermistor; state and explain what happens to the output as the temperature rises.

7. A cell of emf 1.5 V drives a current of 0.50 A through a 2.5 Ω resistor. Find the internal resistance and the terminal p.d.

8. The V-I graph for a battery is a straight line from (0, 6.0 V) to (4.0 A, 4.0 V). State the emf and internal resistance.

9. A lamp is rated '12 V, 36 W'. Find its working current and working resistance, and explain why an ohmmeter across the cold lamp reads much less than this resistance.

10. Explain why an ammeter must have very low resistance and a voltmeter very high resistance, referring to how each is connected.

## Answers

1. $q = It = 3.0 \times 600 = 1800$ C. $N = 1800 / (1.60 \times 10^{-19}) = 1.1 \times 10^{22}$ electrons.

2. $A = \pi d^2/4 = \pi \times (0.30 \times 10^{-3})^2/4 = 7.1 \times 10^{-8}$ m$^2$. $\rho = RA/L = 5.6 \times 7.1 \times 10^{-8} / 2.0 = 2.0 \times 10^{-7}$ Ω m (nichrome-like alloy).

3. $R \propto L/d^2$: doubling L gives ×2; halving d gives ×4. New resistance = **8 times** the original.

4. $1/R_p = 1/2 + 1/3 + 1/6 = 1$, so $R_p = 1.0$ Ω. Total R = 1.0 + 3.0 = 4.0 Ω. I = 8.0 / 4.0 = 2.0 A.

5. E = 2.0 kW × 3.5 h = 7.0 kWh = $7.0 \times 3.6 \times 10^6 = 2.5 \times 10^7$ J. Cost = 7.0 × $0.25 = $1.75.

6. $V_{out} = 9.0 \times 600/900 = 6.0$ V. As temperature rises the thermistor's resistance falls, so it takes a smaller share of the 9.0 V and the output across the 600 Ω resistor **rises** towards 9.0 V.

7. $\varepsilon = I(R + r)$: 1.5 = 0.50(2.5 + r), so 2.5 + r = 3.0 and r = 0.50 Ω. Terminal p.d. V = IR = 0.50 × 2.5 = 1.25 V (check: 1.5 - 0.50 × 0.50 = 1.25 V).

8. Intercept: $\varepsilon$ = 6.0 V. Gradient = (4.0 - 6.0)/(4.0 - 0) = -0.50, so r = 0.50 Ω.

9. I = P/V = 36/12 = 3.0 A; R = V/I = 12/3.0 = 4.0 Ω. Cold, the filament's temperature is far lower: lattice ions vibrate less, electrons collide less often, so the resistance is several times smaller than the working value.

10. The ammeter is in series, so its resistance adds to the circuit's: unless it is very small, inserting the meter would reduce the current being measured. The voltmeter is in parallel, so it provides an alternative

path: unless its resistance is very large it diverts current, lowering the p.d. it is measuring.