MEGA LECTURE

IB DP PHYSICS

Theme B: The Particulate Nature of Matter

B.3 Gas Laws

Revision Notes · Standard and Higher Level

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Original notes prepared for the IB Diploma Programme Physics course (first assessment 2025)

What the syllabus requires

B.3 is common to SL and HL. By the end of this topic you should be able to do everything in the table below. Use it as a final checklist before the exam.

Understanding	You should be able to...
Pressure	Use $P = F / A$; explain gas pressure in terms of molecular collisions with the container walls.
Amount of substance	Convert between number of molecules N, number of moles n and mass, using $n = N / N_A$ and the molar mass.
Empirical gas laws	State and apply Boyle's law, Charles's law and the pressure law; sketch and interpret the corresponding graphs.
Ideal gas equation	Apply $PV = nRT$ and $PV = Nk_BT$, and the combined form $P_1V_1/T_1 = P_2V_2/T_2$.
Kinetic theory	List and justify the assumptions of the ideal gas model; outline how $P = \frac{1}{3}\rho\langle c^2\rangle$ follows from molecular collisions.
Molecular energy	Use $E_k = (3/2)k_BT$ for the average translational kinetic energy of a molecule.
Internal energy	Use $U = (3/2)Nk_BT = (3/2)nRT$ for an ideal monatomic gas.
Limits of the model	State the conditions under which a real gas approximates an ideal gas, and when the model fails.

> **Exam note:** Almost every lost mark in B.3 traces back to one of two slips: using a Celsius temperature inside a gas equation, or mixing units (kPa with m^3, cm^3 with Pa). Convert to kelvin and check units before substituting.

1. Pressure

Pressure is the normal (perpendicular) force exerted per unit area of a surface:

$$P = F / A$$

The SI unit is the **pascal**: 1 Pa = 1 N m^{-2}. It is a small unit — atmospheric pressure at sea level is about 1.0×10^5 Pa (101 kPa, also written 1 atm). Pressure is a scalar: at a point in a gas it acts equally in all directions.

Where gas pressure comes from (microscopic picture)

A gas consists of an enormous number of molecules in rapid, random motion. Each time a molecule strikes a wall of the container it rebounds, so its momentum changes; by Newton's second law the wall exerts a force on the molecule, and by Newton's third law the molecule exerts an equal and opposite force on the wall. Billions of such collisions per second, averaged over the wall's area, produce a steady macroscopic pressure. This single idea explains all three empirical gas laws qualitatively — always answer 'explain' questions in terms of the **rate of collisions** and the **momentum change per collision**.

Worked example 1 — pressure from force and area

(a) A 55 kg person balances briefly on one stiletto heel of area 1.0 cm^2. (b) A 5000 kg elephant stands on four feet, each of area 0.10 m^2. Compare the pressures on the ground (g = 9.8 N kg^{-1}).

(a) $F = 55 \times 9.8 = 539$ N; $A = 1.0 \times 10^{-4}$ m^2; $P = 539 / 1.0 \times 10^{-4} =$ **5.4×10^6 Pa**.

(b) $F = 5000 \times 9.8 = 4.9 \times 10^4$ N; $A = 0.40$ m^2; $P = 4.9 \times 10^4 / 0.40 =$ **1.2×10^5 Pa**.

The heel exerts roughly 40 times the pressure of the elephant — pressure depends on area, not just force.

2. Counting molecules: the mole

Because a laboratory sample contains of order 10^{23} molecules, we count them in bundles. One **mole** is the amount of substance containing exactly 6.02×10^{23} elementary entities (atoms, molecules, ions). This number is the **Avogadro constant**:

$$N_A = 6.02 \times 10^{23}\ \text{mol}^{-1} \quad n = N / N_A$$

where N is the number of molecules and n the number of moles. The **molar mass** M is the mass of one mole; numerically it equals the relative molecular mass in grams (careful: in SI calculations convert to kg mol^{-1}). The three ways of measuring 'how much gas' are linked by:

$$\text{mass } m = nM \quad N = nN_A \quad \text{mass of one molecule} = M / N_A$$

Worked example 2 — mass, moles and molecules

A cylinder contains 8.0 g of oxygen gas, O_2 (molar mass 32 g mol^{-1}). Find (a) the number of moles, (b) the number of molecules, (c) the mass of one molecule.

(a) $n = m / M = 8.0 / 32 =$ **0.25 mol**.

(b) $N = nN_A = 0.25 \times 6.02 \times 10^{23} =$ **1.5×10^{23} molecules**.

(c) $M / N_A = 0.032$ kg mol^{-1} / 6.02×10^{23} $mol^{-1} =$ **5.3×10^{-26} kg**.

> **Unit check:** In $PV = nRT$ everything is SI, so molar mass must be in kg mol^{-1} when you compute masses. 32 g mol^{-1} = 0.032 kg mol^{-1}.

3. The empirical gas laws

Historically, three separate laws were found by experiment, each holding one variable fixed for a **fixed mass of gas**. All temperatures must be in **kelvin**: T(K) = t(°C) + 273.

Law	Held constant	Statement	Graph shape
Boyle's law	T, n	$P \propto 1/V$, i.e. PV = constant	P vs V: falling curve (hyperbola); P vs $1/V$: straight line through origin

Law	Held constant	Statement	Graph shape
Charles's law	P, n	$V \propto T$, i.e. V/T = constant	V vs T(K): straight line through origin; vs t(°C): line hitting axis at -273 °C
Pressure law (Gay-Lussac)	V, n	$P \propto T$, i.e. P/T = constant	P vs T(K): straight line through origin

Boyle's law: isotherms

On a P–V diagram, each constant-temperature curve PV = constant is called an **isotherm**. Curves further from the origin correspond to higher temperatures (at any given V, a hotter gas has larger P). Microscopically: halving the volume doubles the number density of molecules, so collisions with the walls occur twice as often and the pressure doubles.

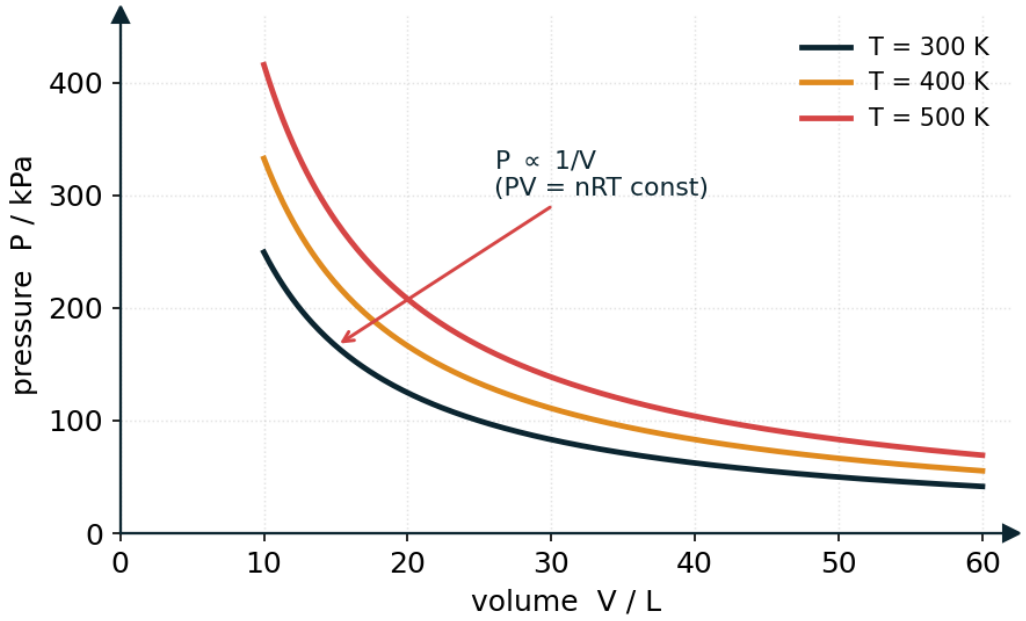

Figure 1. Boyle's law isotherms for 1 mol of ideal gas. Each curve is $P \propto 1/V$, so the product $P{\cdot}V = nRT$ is constant along it (2.49, 3.33 and 4.16 kJ at 300, 400 and 500 K). A hotter gas lies on a higher isotherm.

Charles's law and absolute zero

At constant pressure, the volume of a gas plotted against **Celsius** temperature is a straight line that, extrapolated backwards, cuts the temperature axis at **-273 °C** — the same intercept for every gas. This universal intercept defines **absolute zero**, the natural zero of the kelvin scale, the temperature at which an ideal gas would occupy zero volume (and molecular translational kinetic energy would reach its minimum). The pressure law gives the same intercept when P is extrapolated to zero at constant volume.

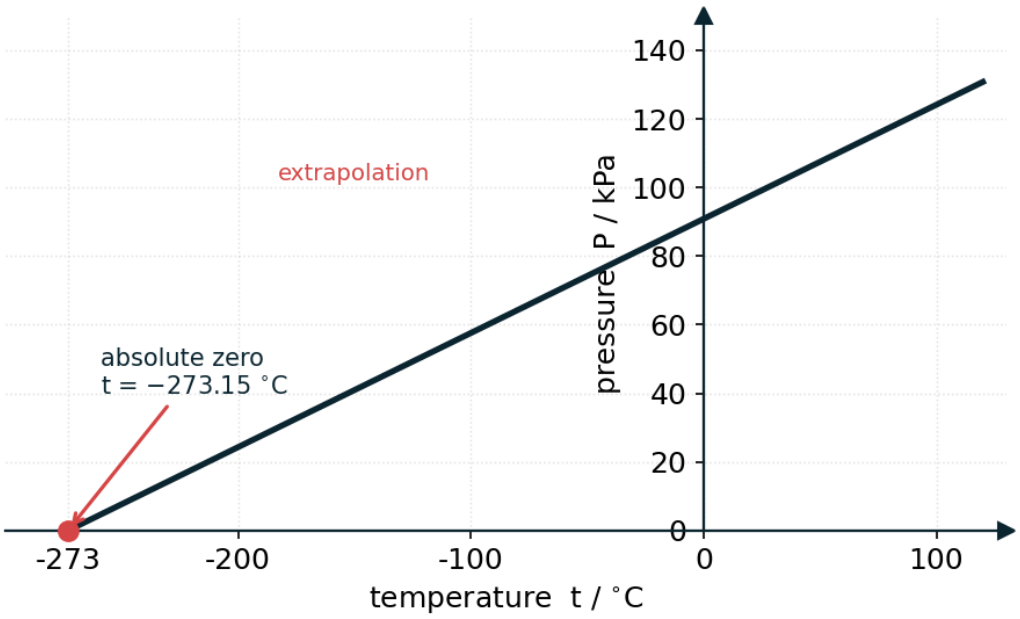

Figure 2. Pressure law (Gay-Lussac) for a fixed volume of gas: $P \propto T$. Extrapolating the straight line back to $P = 0$ meets the temperature axis at absolute zero, $t = -273.15\ °C$ — the same intercept for every gas.

Typical experimental arrangements

- **Boyle:** a fixed mass of dry air trapped in a sealed graduated syringe or tube; pressure changed slowly (to keep T constant) and read from a pressure sensor; plot P against $1/V$ and look for a straight line through the origin.
- **Charles:** a bead of air trapped below a mercury or oil thread in a capillary tube open at the top (constant pressure = atmospheric + thread); tube warmed in a water bath, length of air column recorded against temperature.
- **Pressure law:** a rigid flask of air (constant volume) in a water bath, connected to a pressure gauge; record P as the bath temperature is varied, then extrapolate the P–t line to $P = 0$.

Worked example 3 — Boyle's law (trapped air column)

Air is trapped in a smooth syringe at 100 kPa; the column of gas is 30.0 cm long. The plunger is pushed in slowly until the column is 20.0 cm long, at constant temperature. Find the new pressure.

The cross-sectional area is constant, so $V \propto$ length. Boyle: $P_1V_1 = P_2V_2$.

$P_2 = 100 \times (30.0 / 20.0) =$ **150 kPa**.

'Slowly' matters: a rapid compression would warm the gas, so T would not be constant and Boyle's law would not apply.

4. The ideal gas equation

The three empirical laws combine into a single equation of state. For n moles, or equivalently N molecules:

$$PV = nRT \quad PV = Nk_BT$$

Constant	Value	Meaning
Gas constant R	$8.31\ \text{J K}^{-1}\ \text{mol}^{-1}$	energy per kelvin **per mole**
Boltzmann constant k_B	$1.38 \times 10^{-23}\ \text{J K}^{-1}$	energy per kelvin **per molecule**
Link	$R = N_A k_B$	$6.02 \times 10^{23} \times 1.38 \times 10^{-23} \approx 8.31$

An **ideal gas** is defined as one that obeys $PV = nRT$ exactly at all pressures and temperatures. For a fixed mass of gas (n constant) taken between two states, the equation collapses to the **combined gas law**:

$$\boldsymbol{P_1V_1 / T_1 = P_2V_2 / T_2}$$

In this ratio form, P and V may stay in any consistent units (kPa, cm^3, litres) on both sides — but T must always be in kelvin.

Worked example 4 — tyre pressure on a hot day

A car tyre contains air at an absolute pressure of 320 kPa on a cold morning at 7 °C. After a long drive the air warms to 35 °C. Assuming the tyre volume is unchanged, find the new pressure.

Constant V and n, so P/T = constant. Convert: $T_1 = 280$ K, $T_2 = 308$ K.

$P_2 = 320 \times (308 / 280) =$ **352 kPa ≈ 350 kPa**.

Using 7 and 35 instead of 280 and 308 would give the absurd answer 1600 kPa — the classic Celsius error.

Worked example 5 — how much air is in a room?

A room measures 5.0 m × 4.0 m × 2.5 m. The air is at 101 kPa and 20 °C (molar mass of air ≈ $29\ \text{g mol}^{-1}$). Find (a) the number of moles, (b) the number of molecules, (c) the mass of air.

(a) $V = 50\ \text{m}^3$, $T = 293$ K. $n = PV / RT = (1.01 \times 10^5 \times 50) / (8.31 \times 293) =$ **2.1×10^3 mol**.

(b) $N = nN_A = 2074 \times 6.02 \times 10^{23} \approx$ **1.2×10^{27} molecules**.

(c) $m = nM = 2074 \times 0.029 \approx$ **60 kg** — the air in a room weighs as much as a person.

Worked example 6 — two connected vessels

Vessel A (volume 2.0 L, pressure 300 kPa) is connected through a closed tap to vessel B (3.0 L, 100 kPa) at the same temperature. The tap is opened and the temperature returns to its initial value. Find the final pressure.

Molecules are conserved: $n_{\text{final}} = n_A + n_B$. With T common and using $n = PV/RT$:

$$\boldsymbol{P(V_A + V_B) = P_AV_A + P_BV_B}$$

$P = (300 \times 2.0 + 100 \times 3.0) / 5.0 = 900 / 5.0 =$ **180 kPa**.

Litres are acceptable here because volume units cancel throughout.

5. Kinetic theory: the microscopic model

Kinetic theory derives the gas laws from Newtonian mechanics applied to molecules. The **assumptions of an ideal gas** — learn this list — are:

Assumption	Why it is reasonable / what it buys us
A gas consists of a very large number of identical molecules in random motion.	Large numbers make statistical averages (pressure, temperature) steady and meaningful.
The volume of the molecules themselves is negligible compared with the volume of the container.	True when the gas is dilute; lets us treat molecules as point particles and use the full container volume V.
There are no intermolecular forces except during collisions.	Molecules are far apart on average, so attractions are tiny; this removes potential energy from the model.
All collisions (molecule-molecule and molecule-wall) are perfectly elastic.	Kinetic energy is conserved, so a gas left alone does not spontaneously cool down.
The duration of a collision is negligible compared with the time between collisions.	Molecules spend almost all their time in free flight, so pressure is a smooth average.

Deriving the pressure equation (logic sketch)

Consider one molecule of mass m moving with velocity component c_x perpendicular to a wall of a cubical box of side L:

1. **Momentum change per collision:** the molecule rebounds elastically, so its momentum changes from $+mc_x$ to $-mc_x$: magnitude $\Delta p = 2mc_x$.
2. **Collision rate:** it next hits the same wall after travelling to the far wall and back, a distance $2L$, so the time between hits is $2L/c_x$.
3. **Average force of one molecule:** $F = \Delta p/\Delta t = 2mc_x \div (2L/c_x) = mc_x^2/L$.
4. **Add up all N molecules:** total force $= Nm\langle c_x^2\rangle/L$, where $\langle\ \rangle$ denotes the average over all molecules.
5. **Random directions:** on average $\langle c_x^2\rangle = \langle c_y^2\rangle = \langle c_z^2\rangle$, and they sum to $\langle c^2\rangle$, so $\langle c_x^2\rangle = \langle c^2\rangle/3$.
6. **Divide by wall area** L^2: $P = Nm\langle c^2\rangle / 3L^3 = Nm\langle c^2\rangle / 3V$. Since the density is $\rho = Nm/V$:

$$\boldsymbol{P = (1/3)\ \rho\ \langle c^2\rangle}$$

where $\langle c^2\rangle$ is the **mean square speed** of the molecules. The syllabus asks for the reasoning above qualitatively, not a fully rigorous derivation — but steps 1 (momentum change) and 5 (averaging over three directions gives the factor 1/3) are the two ideas examiners probe.

> **Where temperature enters:** Comparing $PV = (1/3)Nm\langle c^2\rangle$ with $PV = Nk_BT$ forces $(1/2)m\langle c^2\rangle = (3/2)k_BT$. Temperature is a direct measure of average molecular kinetic energy.

6. Molecular kinetic energy and rms speed

The average translational kinetic energy of one molecule of an ideal gas depends **only on the absolute temperature** — not on the type of gas:

$$\boldsymbol{E_k = (1/2)m\langle c^2\rangle = (3/2)k_BT}$$

The square root of the mean square speed is the **root-mean-square (rms) speed**, a representative molecular speed. Rearranging (and using $m = M/N_A$, $R = N_A k_B$):

$$c_{rms} = \sqrt{(3k_B T / m)} = \sqrt{(3RT / M)}$$

- At fixed T: lighter molecules move faster, $c_{rms} \propto 1/\sqrt{M}$.
- For a given gas: $c_{rms} \propto \sqrt{T}$ — quadrupling the kelvin temperature doubles the rms speed.
- In a mixture at one temperature, every species has the **same average kinetic energy** but different rms speeds.

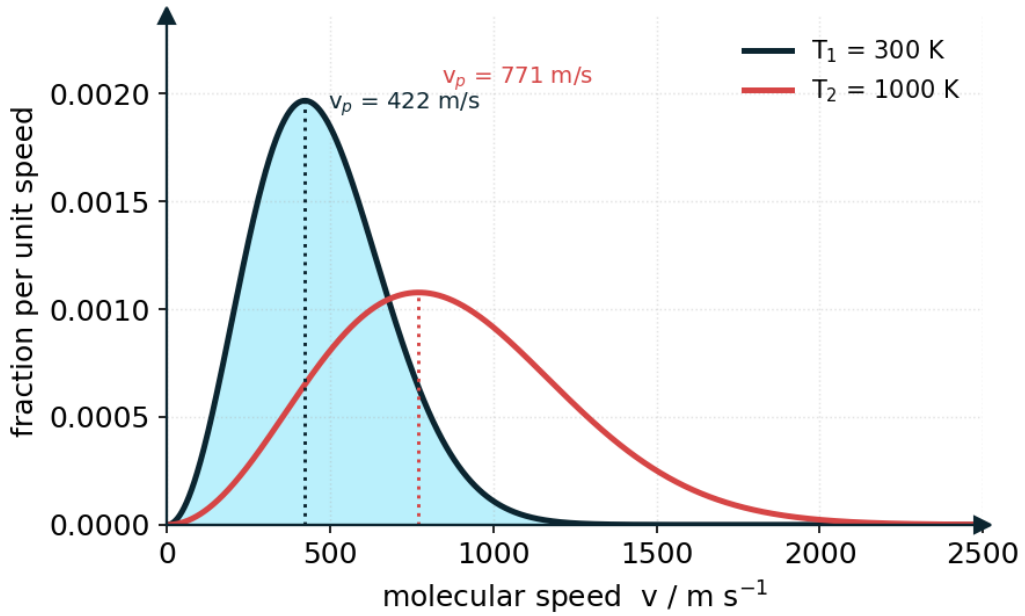

Figure 3. Maxwell-Boltzmann speed distribution for N_2 at 300 K and 1000 K. The most-probable speed $v_p = \sqrt{(2k_B T/m)}$ shifts from 422 to 771 m s^{-1}, and the peak lowers and broadens as T rises (the area under each curve stays 1).

Worked example 7 — hydrogen vs oxygen at 300 K

Compare the average kinetic energy and the rms speeds of H_2 (M = 2.0 g mol^{-1}) and O_2 (M = 32 g mol^{-1}) at 300 K.

Average KE (both gases): $E_k = (3/2) \times 1.38 \times 10^{-23} \times 300 =$ **6.2×10^{-21} J** — identical, because T is the same.

O_2: $c_{rms} = \sqrt{(3 \times 8.31 \times 300 / 0.032)} =$ **480 m s^{-1}**.

H_2: $c_{rms} = \sqrt{(3 \times 8.31 \times 300 / 0.0020)} =$ **1.9×10^3 m s^{-1}** — exactly 4 times faster, since the mass ratio is 16 and speed scales as $1/\sqrt{M}$.

This is why hydrogen escapes Earth's atmosphere far more readily than oxygen.

7. Internal energy of an ideal monatomic gas

The **internal energy** U of a system is the total of the random kinetic energies plus the intermolecular potential energies of its particles. For an **ideal** gas there are no intermolecular forces, so there is **no potential energy term**: the internal energy is purely the sum of the molecular kinetic energies. For a monatomic gas (He, Ne, Ar) each atom carries only translational kinetic energy $(3/2)k_B T$, so multiplying by

the number of atoms:

$$U = (3/2)Nk_BT = (3/2)nRT = (3/2)PV$$

Consequences worth quoting in explanations:

- U depends only on T (and the amount of gas) — an isothermal change of an ideal gas leaves U unchanged.
- Doubling the kelvin temperature doubles the internal energy.
- The last form, $(3/2)PV$, lets you read internal energy straight off a P–V diagram point (used heavily in HL thermodynamics, B.4).

Worked example 8 — internal energy of helium

A cylinder holds 2.0 mol of helium at 300 K. Find (a) the internal energy of the gas, (b) the average kinetic energy per atom, (c) the new internal energy if the gas is heated to 600 K.

(a) $U = (3/2)nRT = 1.5 \times 2.0 \times 8.31 \times 300 = \mathbf{7.5 \times 10^3\ J}$.

(b) $E_k = (3/2)k_BT = 1.5 \times 1.38 \times 10^{-23} \times 300 = \mathbf{6.2 \times 10^{-21}\ J}$.

(c) $U \propto T$, so doubling T doubles U: $\mathbf{1.5 \times 10^4\ J}$.

8. Real gases vs the ideal model

No real gas is ideal, but most gases are excellently approximated by the ideal model under everyday conditions. The model relies on molecules being far apart and interacting only in brief collisions, so it fails when either assumption breaks:

Condition	Which assumption fails	Effect
High pressure / high density	Molecular volume is no longer negligible compared with V	Less free space than the model assumes; measured PV deviates from nRT
Low temperature (near condensation / boiling point)	Intermolecular attractions are no longer negligible — slow molecules linger near each other	Attractions reduce the momentum delivered to the walls; pressure falls below the ideal prediction; eventually the gas liquefies, which the ideal model cannot describe at all

A real gas therefore behaves most ideally at **low pressure** (molecules far apart) and **high temperature** (kinetic energy overwhelms any attraction) — conditions far from liquefaction. This is also why an ideal gas has no intermolecular potential energy: with no intermolecular forces there is nothing to store potential energy, so U is purely kinetic. In a real gas, part of the internal energy is potential, which is why real gases cool when they expand rapidly (work is done against attractions) — an effect absent in the ideal model.

> **Exam phrasing:** 'State two conditions under which a real gas approximates ideal behaviour' — answer: low pressure (or low density) and a temperature well above the boiling point. Then justify each by naming the assumption it protects.

9. Common pitfalls

- **Celsius in a gas equation** — the number one error in B.3. Convert every temperature to kelvin before touching $PV = nRT$, ratios included.
- Mixing unit systems: with $R = 8.31$, pressure must be in Pa and volume in m^3 ($1\ m^3 = 10^3\ L = 10^6\ cm^3$). Ratio problems tolerate any consistent units except temperature.
- Using gauge pressure where absolute pressure is needed — gas laws require absolute pressure (gauge + atmospheric).
- Molar mass in g mol^{-1} substituted into $\sqrt{(3RT/M)}$ — it must be kg mol^{-1}, otherwise speeds come out about 32 times too large.
- Saying temperature measures 'the energy of the gas' — be precise: it measures the **average translational kinetic energy per molecule**.
- Claiming all molecules move at c_{rms} — speeds are distributed; the rms value is a statistical average.
- Applying $U = (3/2)nRT$ to diatomic gases in contexts where rotation matters — the IB formula is stated for **monatomic** ideal gases.
- Forgetting that in a two-vessel or leaking-container problem it is the **number of moles**, not PV/T of one vessel, that is conserved.

10. Quick reference

Result	Statement
Pressure	$P = F / A$ (Pa = N m^{-2}; 1 atm ≈ 1.0×10^5 Pa)
Amount of substance	$n = N / N_A = m / M$; $N_A = 6.02 \times 10^{23}\ mol^{-1}$
Ideal gas equation	$PV = nRT = Nk_BT$; $R = N_Ak_B = 8.31\ J\ K^{-1}\ mol^{-1}$
Combined gas law	$P_1V_1/T_1 = P_2V_2/T_2$ (fixed mass; T in kelvin)
Kinetic theory pressure	$P = (1/3)\rho\langle c^2\rangle$
Average molecular KE	$E_k = (1/2)m\langle c^2\rangle = (3/2)k_BT$; $c_{rms} = \sqrt{(3RT/M)}$
Internal energy (monatomic)	$U = (3/2)Nk_BT = (3/2)nRT = (3/2)PV$
Ideal behaviour	best at low pressure and high temperature, far from liquefaction

11. Test yourself

Attempt these without notes; full answers follow. Take $R = 8.31\ J\ K^{-1}\ mol^{-1}$, $k_B = 1.38 \times 10^{-23}\ J\ K^{-1}$, $N_A = 6.02 \times 10^{23}\ mol^{-1}$.

1. A laboratory is at 25 °C. State this temperature in kelvin, and explain why Celsius temperatures can never be used directly in $PV = nRT$.

2. Find the number of moles and the number of molecules in 0.044 kg of carbon dioxide (molar mass 44 g mol^{-1}).

3. A gas occupies 250 cm^3 at 100 kPa. It is compressed slowly at constant temperature to 100 cm^3. Find the new pressure.

4. At constant pressure, 1.20 L of gas at 27 °C is heated to 127 °C. Find the new volume.

5. A sealed rigid aerosol can holds gas at 105 kPa at 290 K. It is thrown into a fire and reaches 580 K. Find the pressure, and explain microscopically why it rises.

6. A fixed mass of gas occupies 3.0 L at 100 kPa and 300 K. It is compressed to 2.0 L while cooling to 250 K. Find the final pressure.

7. A balloon of volume 0.030 m^3 contains gas at 100 kPa and 300 K. Find the number of molecules and the number of moles.

8. Calculate the average translational kinetic energy of a gas molecule at 400 K. By what factor does the rms speed of a gas change when its kelvin temperature rises from 100 K to 400 K?

9. Calculate the rms speed of nitrogen molecules (molar mass 28 g mol^{-1}) at 293 K.

10. Explain (a) why the internal energy of an ideal gas contains no potential energy term, and (b) under what two conditions a real gas deviates most from ideal behaviour.

Answers

1. $T = 25 + 273 = 298$ K. Gas-law relations are proportionalities through the origin of the absolute scale (e.g. $P \propto T$ requires $P = 0$ at $T = 0$); the Celsius zero is arbitrary, so ratios of Celsius temperatures have no physical meaning.

2. $n = 44\text{ g} / 44\text{ g mol}^{-1} = 1.0$ mol; $N = 1.0 \times 6.02 \times 10^{23} = 6.0 \times 10^{23}$ molecules.

3. Boyle: $P_2 = 100 \times 250/100 = 250$ kPa.

4. Charles: $T_1 = 300$ K, $T_2 = 400$ K; $V_2 = 1.20 \times 400/300 = 1.60$ L.

5. $P_2 = 105 \times 580/290 = 210$ kPa. At higher T the molecules move faster: each wall collision transfers more momentum, and collisions are more frequent, so the average force per unit area — the pressure — increases (volume unchanged).

6. $P_2 = P_1V_1T_2 / (T_1V_2) = 100 \times 3.0 \times 250 / (300 \times 2.0) = 125$ kPa.

7. $N = PV/k_BT = (1.0 \times 10^5 \times 0.030) / (1.38 \times 10^{-23} \times 300) = 7.2 \times 10^{23}$ molecules; $n = N/N_A = 1.2$ mol.

8. $E_k = (3/2) \times 1.38 \times 10^{-23} \times 400 = 8.3 \times 10^{-21}$ J. Since $c_{rms} \propto \sqrt{T}$, the factor is $\sqrt{(400/100)} = 2$.

9. $c_{rms} = \sqrt{(3 \times 8.31 \times 293 / 0.028)} = \sqrt{(2.61 \times 10^5)} \approx 510$ m s^{-1}.

10. (a) The ideal model assumes no intermolecular forces (except instantaneous collisions); with no forces there is no way to store intermolecular potential energy, so U is entirely molecular kinetic energy. (b) At high pressure/density (molecular volume no longer negligible) and at low temperature near the condensation point (intermolecular attractions become significant).