IB DP PHYSICS

Theme A: Space, Time and Motion

A.2 Forces and Momentum

Revision Notes · Standard and Higher Level

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Original notes prepared for the IB Diploma Programme Physics course (first assessment 2025)

What the syllabus requires

A.2 is the heart of mechanics: it explains *why* motion changes. The content below is common to SL and HL and is examined heavily in both papers. Use this table as a final pre-exam checklist.

Understanding	You should be able to...
Forces and free-body diagrams	Treat forces as interactions between bodies; represent all forces on one body with a correctly labelled free-body diagram.
Newton's three laws	State and apply all three laws; identify third-law force pairs correctly.
Equilibrium	Recognise translational equilibrium (resultant force zero) and solve problems by resolving forces.
Specific forces	Use weight mg, normal force, tension, Hooke's law $F = -kx$, friction ($F \le \mu_s N$, $F = \mu_d N$), buoyancy $F_b = \rho V g$, and Stokes' law $F = 6\pi\eta r v$.
Momentum and impulse	Use $p = mv$, impulse $J = F\Delta t = \Delta p$, and force as the rate of change of momentum; interpret force-time graphs.
Conservation of momentum	Apply conservation of linear momentum to collisions and explosions; distinguish elastic, inelastic and totally inelastic collisions using kinetic energy.
Circular motion	Use T, f, $\omega = 2\pi/T$, $v = \omega r$, $a = v^2/r = \omega^2 r = 4\pi^2 r/T^2$, and identify the real forces that supply the centripetal force.

> **Exam note:** In data-booklet notation the resultant force is $F = ma$ and also $F = \Delta p/\Delta t$. The second form is more general - it still works when mass changes (rockets, sand falling onto a belt) and it is the form examiners expect in impulse questions.

1. Forces as interactions; free-body diagrams

A force is a push or a pull exerted **by** one body **on** another. No object can exert a force on itself, and every force is one half of an interaction between two bodies. Forces are vectors, measured in newtons (N), and they come in two broad families:

- **Contact forces** - normal force, friction, tension, drag, buoyancy. They act only while the surfaces or media touch.
- **Field forces** - gravitational, electric, magnetic. They act at a distance through a field, with no contact needed.

Drawing a free-body diagram

A free-body diagram (FBD) shows **one** chosen body, reduced to a point or a simple box, with arrows for **every force acting on it** - and nothing else. The rules examiners mark against:

- Each arrow starts on the body and points in the direction the force acts; its length indicates relative magnitude.
- Label forces by name or standard symbol (W or mg, N or R, T, f, F_b), not by vague words like “motion force”.
- Include only forces exerted **on** the body by other objects - never forces the body exerts on something else, and never “the force of motion” or ma itself.

- Do not resolve a force *and* draw the original on the same diagram - that double-counts it.

The cast of forces you will label again and again:

Force	Symbol	Direction	Magnitude
Weight	W, mg	Vertically down, from the centre of mass	mg
Normal force	N, R	Perpendicular to the surface, pushing away from it	Whatever contact demands (from equilibrium or N2)
Tension	T	Along the string, away from the body	Same throughout a light string
Spring force	F_s	Toward the natural length	kx (Hooke's law region)
Friction	f, F_f	Along the surface, opposing relative sliding (or its tendency)	$\le \mu_s N$ static; $\mu_d N$ sliding
Buoyancy	F_b	Vertically up	ρVg (fluid density × volume displaced × g)
Viscous drag	F_d	Opposite to the velocity	$6\pi\eta rv$ for a small slow sphere

> **Common error:** Adding a forward “force of the throw” to a ball in flight. Once contact is lost the hand exerts nothing; a projectile's FBD contains weight (and drag, if not neglected) only. Motion does not require a force - *changing* motion does.

2. Newton's three laws of motion

Newton's first law

A body remains at rest or moves with constant velocity unless acted on by a resultant force. The first law defines what a force does: it changes velocity. It also tells you how to read a situation in reverse - if velocity is constant (including zero), the resultant force must be zero, so the forces on the body balance exactly.

The property of a body that resists changes in its motion is its **inertia**, measured by its mass. Mass (kg) is a scalar and is the same everywhere; weight (N) is the gravitational force on that mass and varies with the local field. A large mass is equally hard to accelerate in deep space, where it weighs nothing.

Newton's second law

The resultant force on a body equals the rate of change of its momentum:

$$\boldsymbol{F = \Delta p / \Delta t} \textbf{ which for constant mass becomes } \boldsymbol{F = ma}$$

Both F and a here are vectors, and F means the **resultant** (net) force - the vector sum of every force on the body. The acceleration is always in the direction of the resultant force, though the velocity need not be.

Newton's third law

If body A exerts a force on body B, then body B exerts a force on body A that is equal in magnitude and opposite in direction. The two forces of a third-law pair always:

- act on **different** bodies (one on A, one on B),
- are of the **same type** (both gravitational, or both normal-contact, etc.),

- are equal in magnitude, opposite in direction, and act simultaneously.

Because they act on different bodies, third-law partners can **never cancel each other** in a free-body diagram.

> **Classic trap:** The weight of a book on a table and the normal force from the table are **not** a third-law pair: both act on the book, and one is gravitational while the other is a contact force. The true partners are (i) Earth pulls book / book pulls Earth, and (ii) table pushes book / book pushes table.

Force on body A	Third-law partner	Type
Earth pulls skydiver down (weight)	Skydiver pulls Earth up	Gravitational (field)
Ground pushes sprinter forward (friction)	Sprinter pushes ground backward	Friction (contact)
Rocket pushes exhaust gas backward	Gas pushes rocket forward	Contact
Magnet A attracts magnet B	Magnet B attracts magnet A	Magnetic (field)

Worked example 1 - apparent weight in a lift

A 70 kg student stands on a balance in a lift. Find the reading of the balance (the normal force) when the lift accelerates (a) upward at 2.0 m s^{-2}, (b) downward at 2.0 m s^{-2}. Take g = 9.8 m s^{-2}.

Forces on the student: normal force N up, weight mg = 686 N down. Take up as positive and apply $F = ma$: $N - mg = ma$.

(a) $N = m(g + a)$ = 70 × 11.8 = **826 N** ≈ **830 N** - the student feels heavier.

(b) $N = m(g - a)$ = 70 × 7.8 = **546 N** ≈ **550 N** - the student feels lighter.

Check: if $a = g$ downward (free fall), N = 0 - apparent weightlessness.

3. Translational equilibrium and resolving forces

A body is in **translational equilibrium** when the resultant force on it is zero. By the first law it is then either at rest (static equilibrium) or moving with constant velocity (dynamic equilibrium) - the two are indistinguishable as far as forces are concerned. A car cruising at a steady 100 km h^{-1} on a straight road is in equilibrium just as much as a parked one.

The standard method

1. Draw the FBD. 2. Choose two perpendicular axes - usually horizontal/vertical, but along/perpendicular to the slope for inclines. 3. Resolve every force into components along the axes (a force F at angle θ to an axis contributes F cos θ along it and F sin θ perpendicular to it). 4. Set the sum of components along each axis to zero (equilibrium) or to ma (acceleration). 5. Solve the simultaneous equations.

The inclined plane

For a body of mass m on a slope at angle θ to the horizontal, take axes along and perpendicular to the slope. The weight mg then resolves into:

mg sin θ down the slope mg cos θ into the slope

Perpendicular to the slope there is no acceleration, so the normal force is $N = mg \cos\theta$ (smaller than the weight). Along the slope, the resultant of $mg \sin\theta$, friction and any applied force determines the acceleration.

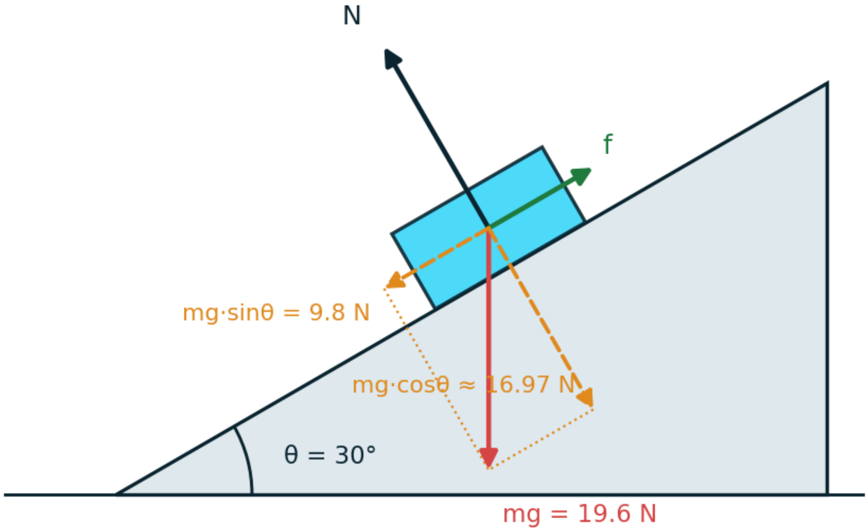

Figure 1. Free-body diagram of a block on a θ = 30° incline (m = 2 kg, g = 9.8 m s^{-2}): weight mg = 19.6 N resolves into mg sinθ = 9.8 N down the slope and mg cosθ ≈ 16.97 N into the slope; the normal force N = mg cosθ and friction f acts up-slope.

Worked example 2 - crate at rest on a slope

An 8.0 kg crate rests on a ramp inclined at 25° to the horizontal. Find (a) the friction force on the crate, (b) the normal force, (c) the minimum coefficient of static friction.

Weight: $mg = 8.0 \times 9.8 = 78.4$ N.

(a) Equilibrium along the slope: $f = mg \sin 25° = 78.4 \times 0.423 =$ **33 N up the slope**.

(b) Perpendicular: $N = mg \cos 25° = 78.4 \times 0.906 =$ **71 N**.

(c) The crate is on the verge of slipping when $f = \mu_s N$, so $\mu_s \geq f / N = 33.1 / 71.1 = \tan 25° =$ **0.47**.

Neat result: for any body on the point of sliding, $\mu_s = \tan\theta$ - independent of mass.

Worked example 3 - sign hanging from two cables

A 12 kg traffic sign hangs from the midpoint of a cable whose two halves each make 20° with the horizontal. Find the tension in the cable.

FBD of the midpoint: weight 12 × 9.8 = 117.6 N down; two tensions T, each at 20° above horizontal.

Horizontal components cancel by symmetry. Vertically: $2T \sin 20° = 117.6$, so $T = 117.6 / (2 \times 0.342)$ = **172 N** ≈ **170 N**.

Note T is larger than the weight: the shallower the cable angle, the larger the tension - which is why a cable can never be pulled perfectly straight by a hanging load.

Connected bodies

When two bodies are joined by a light string, treat them **either** as one system (external forces only - the tension is internal and cancels) **or** as separate bodies each with its own FBD, in which the tension appears explicitly. Use the system view to find the acceleration, then a single-body view to find the tension.

Worked example 4 - Atwood machine

Masses of 3.0 kg and 2.0 kg hang from the ends of a light string over a frictionless pulley. Find the acceleration of the system and the tension in the string.

Take the direction of the 3.0 kg mass falling as positive for the whole system. The resultant external force is the difference of the weights:

$a = (m_1 - m_2)g / (m_1 + m_2) = (3.0 - 2.0) \times 9.8 / 5.0 =$ **$2.0\ \mathrm{m\ s^{-2}}$**.

FBD of the 2.0 kg mass alone (accelerating upward): $T - m_2 g = m_2 a$, so $T = 2.0 \times (9.8 + 1.96)$ = **23.5 N** ≈ **24 N**.

Check with the 3.0 kg mass: $m_1 g - T = 3.0 \times 9.8 - 23.5 = 5.9\ \mathrm{N} = 3.0 \times 2.0\ \mathrm{m\ s^{-2}}$. Consistent. Note T lies between the two weights (19.6 N and 29.4 N), as it must.

4. A tour of the specific forces

Weight, normal force and tension

Weight $W = mg$ is the gravitational pull of the Earth (or other planet) on a body. It acts at the centre of mass, always vertically downward, and depends on the local gravitational field strength g - so mass is the same on the Moon but weight is not.

The **normal force** N is the push of a surface on a body, always perpendicular to the surface. It is a reaction that adjusts itself: it takes whatever value is needed to prevent the body sinking into the surface, up to the point where the surface breaks. It equals mg only in the special case of a horizontal surface with no other vertical forces.

Tension T is the pull transmitted along a stretched string, rope or cable, directed along the string away from the body it acts on. For a light (massless) inextensible string over a frictionless pulley, the tension is the same magnitude throughout.

Elastic restoring force - Hooke's law

An ideal spring stretched or compressed by extension x from its natural length exerts a restoring force

$$F = -kx$$

where k is the spring constant (N m^{-1}), a measure of stiffness. The minus sign says the force always points back toward the natural length - opposite to the displacement. The law holds only up to the limit of proportionality; a graph of F against x is a straight line through the origin with gradient k.

Friction

Friction acts along the surface of contact and opposes relative motion (or the tendency toward it) between the surfaces. IB distinguishes two regimes:

	Static friction	Dynamic (kinetic) friction
When	Surfaces not sliding over each other	Surfaces sliding
Law	$F_f \le \mu_s N$ (an inequality)	$F_f = \mu_d N$ (an equality)
Behaviour	Adjusts from 0 up to a maximum $\mu_s N$, matching the applied force	Roughly constant, independent of speed and contact area
Typical size	$\mu_s > \mu_d$ for the same pair of surfaces	Slightly smaller - once sliding starts it is easier to keep going

The coefficients μ_s and μ_d are dimensionless numbers characterising the *pair* of surfaces, usually between 0 and about 1.2. Static friction is what accelerates a walking person or a driven car wheel forward - friction is not always a hindrance.

> **Interpretation:** The inequality $F \le \mu_s N$ means you may NOT assume friction equals $\mu_s N$ unless the question says the body is on the point of slipping. Below that threshold, find static friction from the equilibrium condition instead.

Worked example 5 - crate sliding with friction

A 6.0 kg crate slides across a horizontal floor, pushed by a horizontal force of 40 N. The coefficient of dynamic friction is 0.35. Find the acceleration.

Vertical equilibrium: $N = mg = 6.0 \times 9.8 = 58.8$ N.

Friction: $F_f = \mu_d N = 0.35 \times 58.8 = 20.6$ N, opposing the motion.

Newton's second law along the floor: $a = (40 - 20.6) / 6.0 =$ **3.2 m s^{-2}**.

Buoyancy

A body wholly or partly immersed in a fluid experiences an upward **buoyant force** equal to the weight of fluid displaced:

$$F_b = \rho V g$$

where ρ is the density of the **fluid** and *V* is the volume of fluid **displaced** (the submerged volume, not necessarily the whole body). A floating body in equilibrium displaces exactly its own weight of fluid; a fully submerged body sinks if its average density exceeds the fluid's.

Viscous drag and Stokes' law

A small smooth sphere of radius *r* moving slowly at speed *v* through a fluid of viscosity η (unit Pa s) experiences a viscous drag force

$$F_d = 6\pi\eta r v$$

directed opposite to the velocity. Key features: drag is proportional to speed (laminar, low-speed flow only), to the radius, and to the viscosity. At high speeds the flow becomes turbulent and drag instead grows roughly with the **square** of the speed - the syllabus requires only this qualitative statement for the high-speed case.

Worked example 6 - terminal speed of a sphere in oil (Stokes' law)

A steel sphere of radius 2.0 mm (density 7800 kg m^{-3}) falls through oil of density 900 kg m^{-3} and viscosity 0.85 Pa s. Find its terminal speed.

Volume: $V = (4/3)\pi r^3 = (4/3)\pi \times (2.0 \times 10^{-3})^3 = 3.35 \times 10^{-8}$ m^3.

At terminal speed: weight = buoyancy + drag, so $6\pi\eta r v = (\rho_s - \rho_f)Vg = 6900 \times 3.35 \times 10^{-8} \times 9.8 = 2.27 \times 10^{-3}$ N.

Drag coefficient: $6\pi\eta r = 6\pi \times 0.85 \times 2.0 \times 10^{-3} = 0.0320$ N s m^{-1}.

$v = 2.27 \times 10^{-3} / 0.0320 =$ **0.071 m s^{-1}** (about 7 cm per second).

5. Linear momentum and impulse

The **linear momentum** of a body is the product of its mass and velocity:

$$p = mv \text{ (unit: kg m s}^{-1}\text{, equivalently N s)}$$

Momentum is a vector in the direction of the velocity. It measures “quantity of motion”: a 10 000 kg lorry at 2 m s^{-1} has the same momentum as a 1000 kg car at 20 m s^{-1}, and the same resultant force takes the same time to stop either.

Impulse

Rearranging Newton's second law $F = \Delta p/\Delta t$ gives the **impulse-momentum theorem**:

$$J = F\Delta t = \Delta p = mv - mu$$

Impulse is the product of the (average) resultant force and the time for which it acts, and it equals the change in momentum it produces. Impulse is a vector: in one-dimensional problems, set a positive direction and give every velocity its sign before subtracting.

Force-time graphs

For a force that varies with time - a bat striking a ball, a collision - the **area under the force-time graph equals the impulse**, and therefore the change of momentum. The peak force can be read directly, and the average force is the constant force that would enclose the same area in the same time.

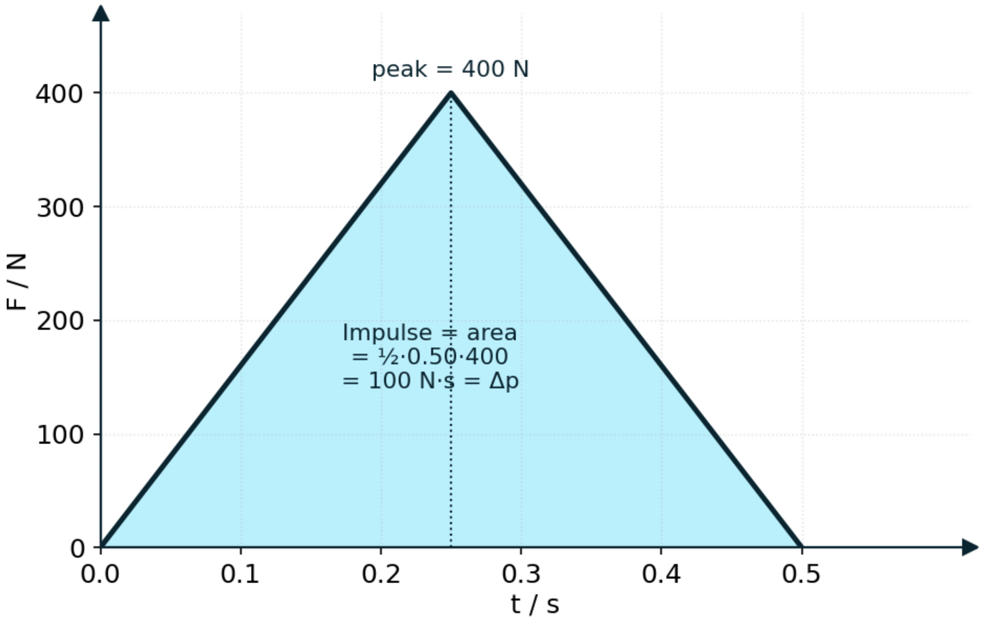

Figure 3. Force–time graph for a triangular pulse (peak 400 N over 0.50 s). The impulse equals the area under the curve, 1/2 · 0.50 s · 400 N = 100 N s, which is the change of momentum Δp.

Safety: stretching out the collision time

In any impact the change of momentum Δp is fixed by the initial and final velocities. Since $F = \Delta p/\Delta t$, the only way to reduce the force is to **increase the time** over which the momentum changes:

- **Crumple zones** deform progressively, extending the stopping time of the car so the deceleration of the occupants is smaller.
- **Airbags** and seat belts stop the passenger over a longer time (and spread the force over a larger area) compared with hitting the dashboard.
- The same physics explains bending your knees on landing, cushioned running shoes, and catching a hard ball by drawing your hands back.

Worked example 7 - ball rebounding from a wall

A 0.16 kg hockey ball strikes a wall horizontally at 25 m s^{-1} and rebounds along the same line at 20 m s^{-1}. Contact lasts 12 ms. Find (a) the impulse on the ball, (b) the average force exerted by the wall.

Take the rebound direction as positive: u = -25 m s^{-1}, v = +20 m s^{-1}.

(a) $J = m(v - u)$ = 0.16 × (20 - (-25)) = 0.16 × 45 = **7.2 N s** away from the wall.

(b) $F = J / \Delta t$ = 7.2 / 0.012 = **600 N** - about 380 times the ball's weight, which is why the wall, not gravity, dominates during contact.

Sign trap: forgetting the minus on u gives 0.8 N s - a factor-of-nine error examiners look for.

Force on a steady stream of matter

When mass flows steadily - water striking a wall, sand landing on a moving belt - Newton's second law in the form $F = \Delta p/\Delta t$ gives $F = (\Delta m/\Delta t)v$: the mass flow rate times the velocity change of each kilogram. For example, a hose delivering 3.0 kg of water per second horizontally at 12 m s^{-1} against a wall (water not rebounding) exerts a force of 3.0 × 12 = 36 N on the wall - and by Newton's third law the wall exerts 36 N back on the water.

Worked example 8 - why airbags work

In a crash, a 65 kg driver moving at 14 m s^{-1} is brought to rest. Compare the average force if the driver is stopped (a) by an airbag over 0.25 s, (b) by a rigid windscreen over 0.015 s.

The change of momentum is the same in both cases: Δp = 65 × 14 = 910 kg m s^{-1}.

(a) F = 910 / 0.25 = 3640 N ≈ **3.6 kN** - about 5.7 times body weight, survivable.

(b) F = 910 / 0.015 = 60 700 N ≈ **61 kN** - roughly 17 times larger, and likely fatal.

Identical impulse, very different force: only Δt changed. This one-line argument, $F = \Delta p/\Delta t$ with Δp fixed, is the model answer for every safety-device question.

6. Conservation of linear momentum

Where the law comes from

Consider two bodies A and B that collide. During contact, A exerts force F on B, and by Newton's third law B exerts $-F$ on A, for the same contact time Δt. The impulses are therefore equal and opposite: $\Delta p_B = F\Delta t$ and $\Delta p_A = -F\Delta t$. Whatever momentum one body gains, the other loses, so the total is unchanged. Hence:

The total momentum of a system is constant provided no resultant external force acts on it.

The condition matters. Internal forces (the collision forces themselves) can never change the total; external forces (friction from the ground, gravity during a long interaction) can. During a brief, violent collision the external impulse is usually negligible, so momentum is conserved *across the instant of collision* even on rough ground.

Classifying collisions with kinetic energy

Momentum is conserved in **every** collision and explosion. Kinetic energy is the quantity that distinguishes them:

Type	Momentum	Kinetic energy	Signature
Elastic	conserved	conserved	Bodies separate; total KE after = before. Ideal case - approached by colliding gas molecules, magnetic bumpers.
Inelastic	conserved	decreases	Some KE becomes internal (thermal) energy, sound, deformation. Most real collisions.
Totally inelastic	conserved	maximum possible loss	Bodies stick and move with one common velocity.

Type	Momentum	Kinetic energy	Signature
Explosion	conserved (often zero)	increases	Stored chemical or elastic energy becomes KE; fragments move apart.

To test elasticity: compute total KE = ½mv^2 summed over the bodies, before and after, and compare. Never assume a collision is elastic unless told (or shown by the numbers).

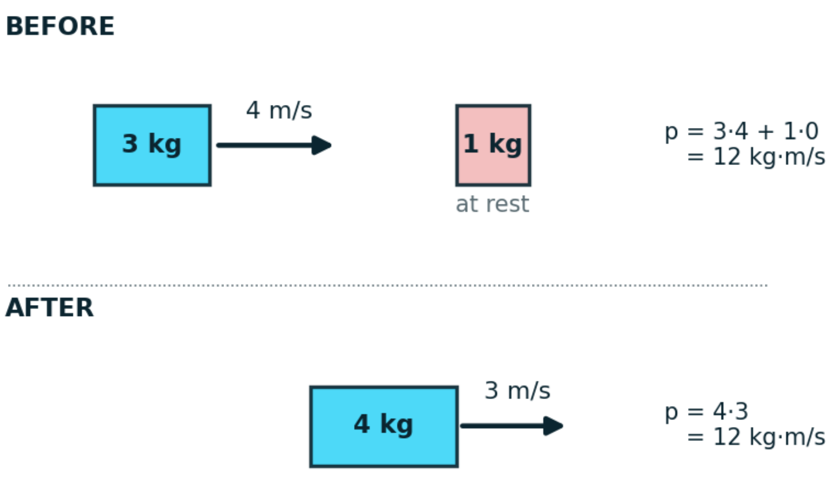

Figure 2. Perfectly inelastic collision: a 3 kg body at 4 m s^{-1} strikes a stationary 1 kg body and they move off together. Momentum before = 12 kg m s^{-1} equals momentum after, so the common velocity is v = 3 m s^{-1}.

Momentum or kinetic energy? Keeping the two bookkeepings straight:

	Momentum $p = mv$	Kinetic energy $E_k = ½mv^2$
Nature	Vector - signs and directions matter	Scalar - always positive, no direction
Unit	kg m s^{-1} (= N s)	J
Conserved in a collision?	Always (no external resultant force)	Only if the collision is elastic
Can it cancel?	Yes - two opposite momenta sum to zero	Never - energies of moving bodies always add
Useful link	$E_k = p^2 / 2m$	for the same p, the smaller mass carries more KE

Worked example 9 - totally inelastic car crash

A 1200 kg car travelling at 15 m s^{-1} runs into the back of a stationary 800 kg car. The vehicles lock together. Find (a) their common velocity just after impact, (b) the kinetic energy converted to other forms.

(a) Momentum: 1200 × 15 + 800 × 0 = (1200 + 800)v, so v = 18 000 / 2000 = **9.0 m s^{-1}** in the original direction.

(b) KE before = ½ × 1200 × 15^2 = 135 kJ. KE after = ½ × 2000 × 9.0^2 = 81 kJ.

Loss = 135 - 81 = **54 kJ** (40% of the original KE) - transferred to deformation, sound and heating. Momentum is conserved; kinetic energy is not.

Worked example 10 - recoil (an explosion in miniature)

A 4.0 kg rifle, initially at rest, fires a 20 g bullet at 300 m s^{-1}. Find the recoil speed of the rifle, and compare the kinetic energies of bullet and rifle.

Total momentum before = 0, so after: 0.020 × 300 + 4.0 × v = 0, giving v = -1.5 m s^{-1}: the rifle recoils at **1.5 m s^{-1}** backwards.

KE of bullet = ½ × 0.020 × 300^2 = 900 J; KE of rifle = ½ × 4.0 × 1.5^2 = 4.5 J.

Although the momenta are equal and opposite, the lighter body carries almost all the kinetic energy (KE = $p^2/2m$, so for equal p, the smaller mass has the larger KE).

Collisions in two dimensions (brief)

Momentum is a vector, so in an oblique collision it is conserved **independently along each axis**. Resolve every momentum into x and y components, write one conservation equation per axis, and solve. A useful check: if two bodies move apart after the collision, the vector sum of their momenta must still equal the original total - drawing a momentum vector triangle often settles the geometry quickly.

Special results worth knowing for elastic collisions in 1D: two **equal masses** exchange velocities (the moving one stops, the target moves off at the original speed); a light body striking a very heavy one rebounds with almost unchanged speed; a very heavy body striking a light one continues almost unaffected while the light one is flicked forward at up to twice the incoming speed.

Worked example 11 - collision in two dimensions

A 0.17 kg billiard ball moving at 4.0 m s^{-1} strikes an identical stationary ball. After the (elastic) collision the first ball moves at 30° to its original direction. Find the speeds of both balls and the direction of the second.

For an elastic collision between equal masses, one initially at rest, the two final velocities are perpendicular, so the second ball moves at 60° on the other side of the line.

Resolve along the original (x) and transverse (y) directions with speeds v_1, v_2:

y: $v_1 \sin 30° = v_2 \sin 60°$; x: $v_1 \cos 30° + v_2 \cos 60° = 4.0$

Solving: $v_1 = 4.0 \cos 30°$ = **3.5 m s^{-1}** and $v_2 = 4.0 \sin 30°$ = **2.0 m s^{-1}** at **60°** to the original line.

Check KE: ½m($3.46^2 + 2.0^2$) = ½m(12 + 4) = ½m × 16 = ½m × 4.0^2 - elastic, as stated.

Exam technique: Momentum questions are sign questions. Write the positive direction at the top of your answer, attach a sign to every velocity before substituting, and interpret a negative answer as motion in the negative direction - never discard the sign.

7. Circular motion

Describing rotation

For a body moving in a circle of radius *r*: the **period** *T* is the time for one revolution, the **frequency** $f = 1/T$ is the number of revolutions per second (Hz), and the **angular velocity** ω is the angle swept per unit time (rad s^{-1}):

$$\omega = 2\pi / T = 2\pi f \quad v = \omega r = 2\pi r / T$$

Angles here are in **radians** (2π rad per revolution). Rotation rates quoted in revolutions per minute must be converted: ω = rpm × 2π/60. A hard disk at 7200 rpm, for instance, has ω = 7200 × 2π/60 = 754 rad s^{-1}.

Centripetal acceleration

Even at constant speed, a body on a circular path is accelerating, because the **direction** of its velocity changes continuously. The acceleration points toward the centre of the circle (centripetal = centre-seeking) and has magnitude:

$$a = v^2 / r = \omega^2 r = 4\pi^2 r / T^2$$

Centripetal force is a resultant, not a new force

By Newton's second law, this acceleration requires a resultant force of magnitude $F = mv^2/r = m\omega^2 r$ directed toward the centre. **Never draw a separate arrow labelled centripetal force on an FBD** - it is a job description filled by real forces:

Situation	Force(s) supplying the centripetal force
Car cornering on a flat road	Sideways friction from the road on the tyres

Situation	Force(s) supplying the centripetal force
Planet or satellite in orbit	Gravitational attraction
Mass whirled on a string (horizontal circle)	Horizontal component of the tension
Car on a banked curve (qualitative)	Horizontal component of the normal force, plus friction if needed; banking lets a car corner faster, and at the design speed no friction is required at all
Bottom of a vertical circle	Tension (or normal force) minus weight: $T - mg = mv^2/r$, so tension is greatest at the bottom
Top of a vertical circle	Tension (or normal force) plus weight: $T + mg = mv^2/r$; minimum speed for a taut string is $v = (gr)^{1/2}$, when $T = 0$ and weight alone turns the body

Because the centripetal force is always perpendicular to the velocity, it does **no work**: the speed and kinetic energy of a body in uniform circular motion are constant even though a resultant force acts throughout.

Worked example 12 - conical pendulum

A 0.30 kg ball on a 0.80 m string swings in a horizontal circle, the string making 30° with the vertical. Find (a) the tension, (b) the speed of the ball, (c) the period.

Radius: $r = 0.80 \sin 30° = 0.40$ m. Forces on the ball: tension T along the string, weight $mg = 2.94$ N.

(a) Vertical equilibrium: $T \cos 30° = mg$, so $T = 2.94 / 0.866 =$ **3.4 N**.

(b) Horizontally, the resultant is $T \sin 30° = mv^2/r$. Dividing the two equations: $\tan 30° = v^2/(rg)$, so $v^2 = 0.40 \times 9.8 \times 0.577 = 2.26$, giving $v =$ **1.5 m s^{-1}**.

(c) Period = $2\pi r / v = 2\pi \times 0.40 / 1.50 =$ **1.7 s**.

Language check: Centrifugal force earns no marks. In the lab frame there is no outward force on the body; passengers thrown outward in a turning car are simply continuing in a straight line (Newton's first law) while the car turns beneath them.

8. Common pitfalls

- Pairing weight with the normal force as action-reaction. Third-law partners act on different bodies and are of the same type.
- Drawing ma, centripetal force, or a force of motion as arrows on a free-body diagram. Only real pushes and pulls from identifiable objects belong there.
- Assuming $N = mg$ everywhere. On slopes $N = mg \cos \theta$; in accelerating lifts $N = m(g \pm a)$; on curved tracks it is set by the circular motion.
- Using $F = \mu_s N$ when the body is not on the point of slipping - static friction takes whatever value equilibrium demands, up to that maximum.
- Dropping signs in impulse and momentum calculations, especially rebounds, where $\Delta p = m(v - u)$ with u negative.

- Claiming kinetic energy is conserved in every collision. Only momentum always survives; KE must be checked numerically.
- Forgetting the condition for momentum conservation - no resultant **external** force - or applying it along a direction in which an external force acts (e.g. vertically during an impact with the ground).
- Saying a body in uniform circular motion is in equilibrium. It accelerates toward the centre; the resultant force is non-zero.
- Mixing degrees and radians: ω must be in rad s^{-1} in $v = \omega r$ and $a = \omega^2 r$.

9. Quick reference

Result	Statement
Newton's second law	$F = ma = \Delta p / \Delta t$ (F is the resultant force)
Equilibrium	Resultant force zero: components along each axis sum to zero
Inclined plane	Along slope: $mg \sin\theta$; normal force: $N = mg \cos\theta$
Hooke's law	$F = -kx$ (restoring, gradient of F-x graph gives k)
Friction	Static: $F \leq \mu_s N$; dynamic: $F = \mu_d N$
Buoyancy	$F_b = \rho V g$ (ρ = fluid density, V = volume displaced)
Stokes' law (small sphere, low speed)	$F_d = 6\pi\eta r v$; at high speed drag grows roughly as v^2
Momentum and impulse	$p = mv$; $J = F\Delta t = \Delta p$; area under F-t graph = impulse
Conservation of momentum	Total p constant if no resultant external force; holds for all collisions and explosions
Elastic vs inelastic	Elastic: KE conserved. Inelastic: KE decreases. Totally inelastic: bodies coalesce
Circular motion	$\omega = 2\pi/T$; $v = \omega r$; $a = v^2/r = \omega^2 r = 4\pi^2 r/T^2$; $F = mv^2/r$ toward the centre

10. Test yourself

Attempt these without notes; full answers below. Take g = 9.8 m s^{-2}.

1. A 1500 kg car experiences a driving force of 4500 N and total resistive forces of 1200 N. Find its acceleration.

2. A 20 N horizontal force pulls a 3.0 kg block along a frictionless floor; a light string connects a 2.0 kg block behind it. Find the acceleration of the system and the tension in the string.

3. A 5.0 kg block slides down a 35° incline with $\mu_d = 0.20$. Find its acceleration.

4. A spring of constant 250 N m^{-1} is stretched by 12 cm. Find the magnitude of the restoring force and state its direction.

5. During a kick, the force on a stationary 2.0 kg object rises linearly from zero to 400 N and falls back to zero over a total of 0.50 s (a triangular F-t graph). Find the impulse and the final speed.

6. A 2.0 kg trolley moving at 6.0 m s^{-1} strikes a stationary 4.0 kg trolley. Afterwards the 2.0 kg trolley rebounds at 2.0 m s^{-1} and the 4.0 kg trolley moves forward at 4.0 m s^{-1}. Show that momentum is conserved

and determine whether the collision is elastic.

7. Two ice skaters, 55 kg and 75 kg, stand at rest and push apart. The 55 kg skater moves off at 1.8 m s^{-1}. Find the speed of the other skater.

8. A car crosses the top of a humpbacked bridge whose radius of curvature is 25 m. Find the maximum speed at which the wheels stay in contact with the road.

9. A 0.50 kg mass on a string moves in a horizontal circle of radius 0.75 m at 90 revolutions per minute. Find the resultant (centripetal) force on it.

10. A wooden block of density 600 kg m^{-3} floats in water (density 1000 kg m^{-3}). What fraction of its volume is submerged, and why?

Answers

1. Resultant = 4500 - 1200 = 3300 N; a = 3300 / 1500 = **2.2 m s^{-2}**.

2. Whole system: a = 20 / (3.0 + 2.0) = **4.0 m s^{-2}**. FBD of the 2.0 kg block alone: the only horizontal force is the tension, so T = 2.0 × 4.0 = **8.0 N**.

3. a = g(sin 35° - μ_d cos 35°) = 9.8 × (0.574 - 0.20 × 0.819) = 9.8 × 0.410 = **4.0 m s^{-2}** down the slope.

4. F = kx = 250 × 0.12 = **30 N**, directed back toward the natural length (opposite to the extension) - the meaning of the minus sign in F = -kx.

5. Impulse = area of triangle = ½ × 0.50 × 400 = **100 N s**. v = J / m = 100 / 2.0 = **50 m s^{-1}**.

6. Before: p = 2.0 × 6.0 = 12 kg m s^{-1}. After: 2.0 × (-2.0) + 4.0 × 4.0 = -4.0 + 16 = 12 kg m s^{-1} - conserved. KE before = ½ × 2.0 × 6.0^2 = 36 J; KE after = ½ × 2.0 × 2.0^2 + ½ × 4.0 × 4.0^2 = 4.0 + 32 = 36 J. KE is also conserved, so the collision is **elastic**.

7. Momentum: 55 × 1.8 = 75 × v, so v = 99 / 75 = **1.3 m s^{-1}** in the opposite direction.

8. Contact is lost when N = 0, i.e. weight alone supplies the centripetal force: mg = mv^2/r, so v = $(gr)^{1/2}$ = $(9.8 \times 25)^{1/2}$ = **16 m s^{-1}** (about 56 km h^{-1}).

9. ω = 90 × 2π / 60 = 9.42 rad s^{-1}. F = $m\omega^2 r$ = 0.50 × 9.42^2 × 0.75 = **33 N** toward the centre.

10. Floating equilibrium: $\rho_w V_{sub} g = \rho_{block} V g$, so V_{sub}/V = 600 / 1000 = **0.60**. The block sinks until the weight of the displaced water equals its own weight.