IB DP MATHEMATICS

AI Topic 5 · Calculus

TOPICAL WORKSHEET

5.1 Differentiation & Rates of Change · 5.2 Kinematics & Optimization
5.3 Integration & Area · 5.4 Kinematics using Integration

Standard and Higher Level · Syllabus 2025

Fahad H. Ahmad

+92 323 509 4443 | Megalecture.com

Original exam-style questions written for the IB Diploma Programme Mathematics course (first assessment 2025)

Section A — Multiple Choice (15 marks)

Circle the letter of the best answer to each question. Each question is worth 1 mark.

1. If s(t) is the displacement of a particle at time t, the velocity function v(t) is given by **[1]**

A $v(t) = s(t)\times t$

B $v(t) = ds\div dt$

C $v(t) = \int s(t)dt$

D $v(t) = s(t)\div t$

2. Given $y = 7x^2-3x+5$, the value of $dy\div dx$ at $x=2$ is **[1]**

A 11

B 22

C 25

D 31

3. The second derivative of a displacement function, $s''(t)$, represents **[1]**

A the displacement

B the velocity

C the acceleration

D the total distance travelled

4. For $f(x) = 4x^3-2x$, the derivative $f'(x)$ is **[1]**

A $12x^2-2$

B $4x^2-2$

C $12x^2$

D $4x^3-2$

5. At a local maximum or minimum point of a function, the value of the first derivative is **[1]**

A always positive

B always negative

C equal to zero

D undefined

6. After finding a critical point using $f'(x)=0$, the nature of the stationary point (maximum, minimum, or neither) can be confirmed using **[1]**

A the value of f(x) at that point

B the second derivative test (or a sign change of $f'(x)$)

C the y-intercept of f(x)

D the domain of f(x)

7. If, at a certain time, a particle's velocity $v(t)=0$ but its acceleration $a(t)\neq 0$, the particle is **[1]**

A permanently at rest

B moving at constant speed

C instantaneously at rest, and about to change direction

D accelerating forever

8. To minimize $f(x) = x^2+16\div x$ for $x > 0$, the value of x at the minimum is **[1]**

A 1

B 2

C 4

D 8

9. $\int 3x^2 dx$ is equal to **[1]**

A $6x+C$

B x^3+C

C $3x^3+C$

D x^2+C

10. An indefinite integral always includes **[1]**

A a definite numerical answer

B a constant of integration, C

C the limits of integration

D a negative sign

11. $\int(4x-1)dx$ is equal to **[1]**

A $4x^2-x+C$

B $2x^2-x+C$

C $4x-x^2+C$

D $2x^2+C$

12. The definite integral $\int_a^b f(x)dx$ represents **[1]**

A the gradient of $f(x)$ at $x=a$

B the (net signed) area between the curve $y=f(x)$ and the x-axis, from $x=a$ to $x=b$

C the maximum value of $f(x)$

D the value of $f(x)$ at $x=b$

13. If $v(t)$ is the velocity of a particle, $\int v(t)dt$ represents **[1]**

A the acceleration

B the displacement

C the speed

D the jerk (rate of change of acceleration)

14. The value of $\int_0^2 3x^2 dx$ is **[1]**

A 4

B 6

C 8

D 12

15. Unlike displacement, calculating the total distance travelled by a particle over an interval where its velocity changes sign requires **[1]**

A integrating v(t) directly over the whole interval

B integrating |v(t)| (splitting the interval at each sign change and taking absolute values)

C differentiating v(t) instead of integrating

D using only the initial and final velocities

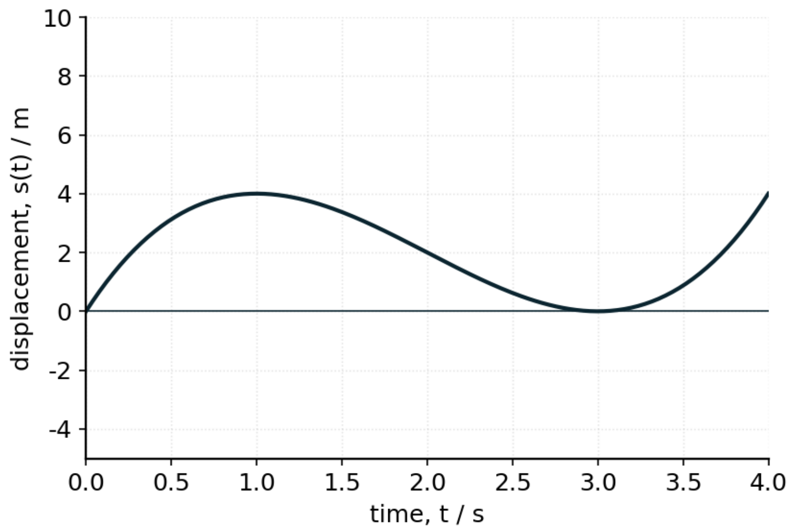
Section B — Structured Questions (65 marks)

Answer all questions in the space provided. Working must be shown for full marks.

5.1 Differentiation: Rates of Change

Question 1 [7 marks]

A particle moves in a straight line so that its displacement, s(t) metres, after t seconds, is given by $s(t) = t^3 - 6t^2 + 9t$. The graph shows s(t) (computed).



Displacement, $s(t) = t^3 - 6t^2 + 9t$ (computed).

(a) Find an expression for the velocity, v(t), of the particle. **[2]**

(b) Calculate the velocity of the particle at $t=2$. **[2]**

(c) Find the times at which the particle is momentarily at rest ($v(t)=0$). **[3]**

Question 2 [6 marks]

For the particle in Question 1, the velocity is $v(t) = 3t^2 - 12t + 9$.

(a) Find an expression for the acceleration, a(t), of the particle. **[2]**

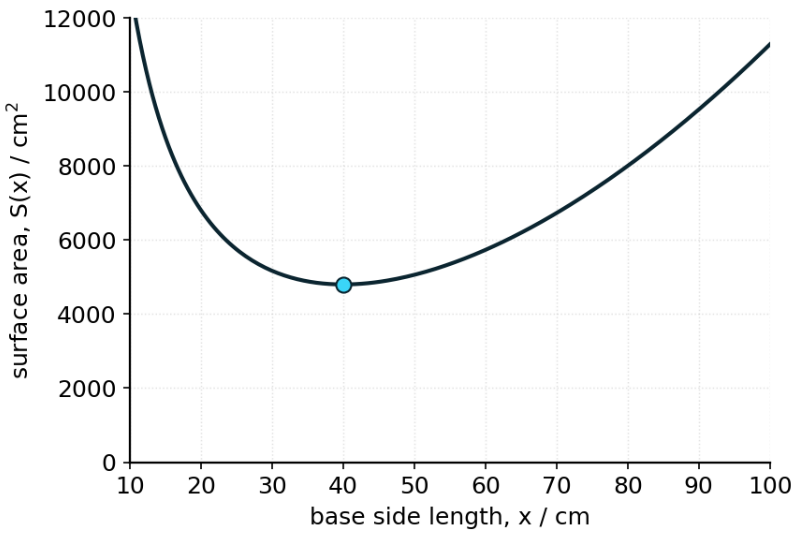
(b) Calculate the acceleration of the particle at t=0. **[2]**

(c) Find the time at which the acceleration is zero. **[2]**

5.2 Applications: Kinematics & Optimization

Question 3 [7 marks]

An open-topped box with a square base of side x cm and height h cm is to be built with a fixed volume of 32 000 cm^3. The surface area of the box is given by $S(x) = x^2 + 128000 \div x$. The graph shows S(x) (computed).



Box surface area, $S(x) = x^2+128000\div x$ (computed).

(a) Using the graph, estimate the value of x that minimizes the surface area. **[2]**

(b) Using calculus, find $dS \div dx$, and hence find the exact value of x that minimizes the surface area. **[3]**

(c) Calculate the minimum surface area. **[2]**

Question 4 [6 marks]

A ball is thrown vertically upwards. Its height, $h(t)$ metres, after t seconds, is given by $h(t) = 30t-5t^2$.

(a) Find an expression for the velocity, $v(t) = dh \div dt$, of the ball. **[2]**

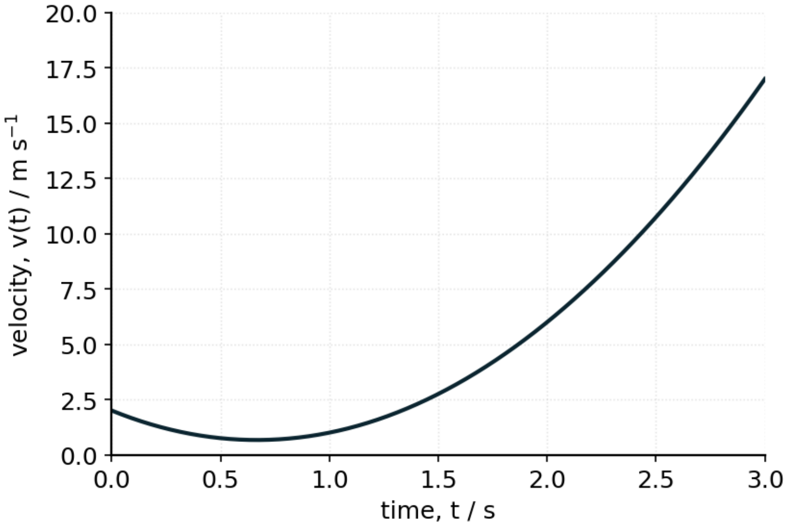
(b) Find the time at which the ball reaches its maximum height ($v(t)=0$). **[2]**

(c) Calculate the maximum height reached by the ball. **[2]**

5.3 Integration: Anti-differentiation & Area

Question 5 [7 marks]

A particle has acceleration $a(t) = 6t-4$ (in m s^{-2}), and an initial velocity $v(0)=2$ m s^{-1}. The graph shows the velocity function $v(t)$ (computed).



Velocity, $v(t) = 3t^2 - 4t + 2$ (computed).

(a) By integrating a(t), and using the given initial condition, find an expression for the velocity, v(t). **[3]**

(b) Calculate the velocity at $t=2$. **[2]**

(c) By considering the discriminant of $3t^2-4t+2$, explain why the particle's velocity is never zero. **[2]**

Question 6 [6 marks]

Consider the function $f(x) = 4x^3-6x+5$.

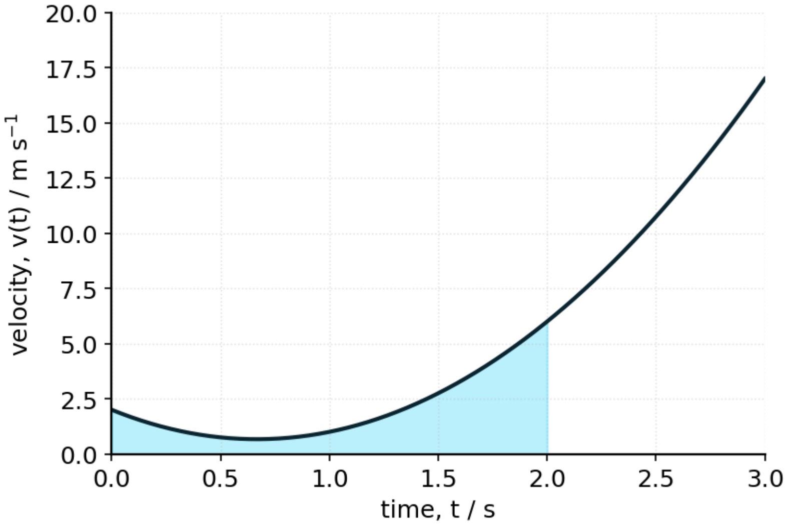
(a) Find $\int(4x^3-6x+5)dx$. **[2]**

(b) Evaluate the definite integral $\int_1^3(4x^3-6x+5)dx$. **[4]**

5.4 Applications: Kinematics using Integration

Question 7 [7 marks]

For the particle in Question 5, with velocity $v(t) = 3t^2 - 4t + 2$ and initial displacement $s(0)=5$ m, the graph shows $v(t)$ with the region representing the displacement between $t=0$ and $t=2$ shaded (computed).



Velocity, $v(t) = 3t^2 - 4t + 2$, shaded area [0,2] (computed).

(a) By integrating $v(t)$, and using the given initial condition, find an expression for the displacement, $s(t)$. **[3]**

(b) Calculate the displacement of the particle at $t=2$. **[2]**

(c) Using the shaded area on the graph (i.e. the definite integral $\int_0^2 v(t)dt$), find the change in displacement (the distance moved) between $t=0$ and $t=2$. **[2]**

Question 8 [6 marks]

The curve $y = 6x - x^2 - 5$ intersects the x-axis at two points.

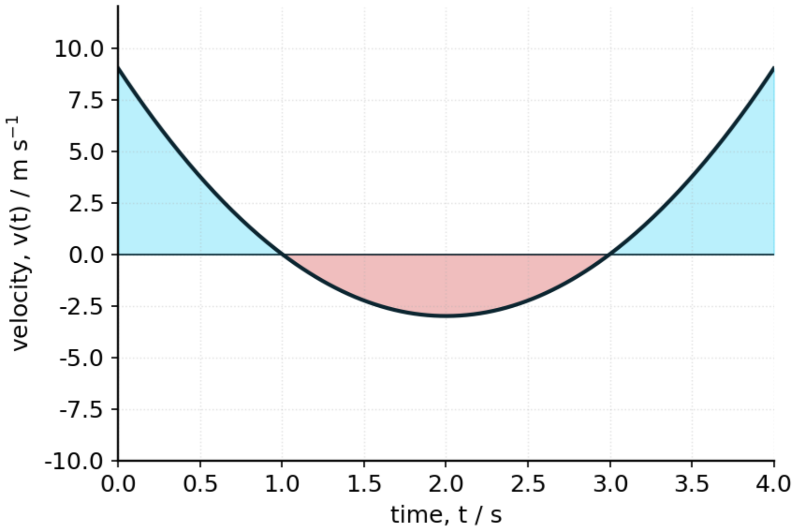
(a) Find the x-coordinates of the points where the curve intersects the x-axis. **[2]**

(b) Calculate the area enclosed between the curve and the x-axis, between these two points. **[4]**

5.1 Differentiation: Rates of Change

Question 9 [7 marks]

For the particle in Question 1 (with velocity $v(t) = 3t^2 - 12t + 9$), the graph shows v(t) over $0 \le t \le 4$, with the regions where the particle moves forward and backward shaded differently (computed).



Velocity, $v(t) = 3t^2 - 12t + 9$, direction shading (computed).

(a) Using the graph, write down the time intervals during which the particle is moving forward ($v(t) > 0$) and backward ($v(t) < 0$). **[2]**

(b) Using $s(t) = t^3 - 6t^2 + 9t$, calculate the total displacement of the particle between t=0 and t=4. **[2]**

(c) Using s(0)=0, s(1)=4, s(3)=0 and s(4)=4, calculate the total distance travelled by the particle between t=0 and t=4. **[3]**

Question 10 [6 marks]

Consider the function $f(x) = 5x^4 - 3x^2 + 7x - 2$.

(a) Find $f'(x)$. **[2]**

(b) Calculate $f'(1)$. **[2]**

(c) State, with a reason, whether $f(x)$ is increasing or decreasing at $x=1$. **[2]**