IB DP MATHEMATICS

AI Topic 5 · Calculus

MARK SCHEME

5.1 Differentiation & Rates of Change · 5.2 Kinematics & Optimization
5.3 Integration & Area · 5.4 Kinematics using Integration

Standard and Higher Level · Syllabus 2025

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Original exam-style questions written for the IB Diploma Programme Mathematics course (first assessment 2025)

Section A — Multiple Choice: Answer Key (15 marks)

1. Answer: **B** — Velocity is the rate of change of displacement with respect to time: $v(t) = ds \div dt$.

2. Answer: **C** — $dy \div dx = 14x - 3$. At $x=2$: $dy \div dx = 14(2) - 3 = 28 - 3 = 25$.

3. Answer: **C** — The second derivative of displacement with respect to time is the rate of change of velocity, i.e. the acceleration.

4. Answer: **A** — Using the power rule: $f'(x) = 3 \times 4x^2 - 2 = 12x^2 - 2$.

5. Answer: **C** — At a local maximum or minimum (a stationary point), the gradient of the tangent is zero, so $f'(x) = 0$.

6. Answer: **B** — The second derivative test (checking the sign of $f''(x)$, or checking for a sign change in $f'(x)$) confirms whether a critical point is a maximum, minimum, or neither.

7. Answer: **C** — If $v(t)=0$ but $a(t) \neq 0$, the particle is momentarily at rest but its velocity is changing, so it is about to move off (typically reversing direction).

8. Answer: **B** — $f'(x) = 2x - 16 \div x^2 = 0 \Rightarrow 2x^3 = 16 \Rightarrow x^3 = 8 \Rightarrow x = 2$.

9. Answer: **B** — $\int 3x^2 dx = x^3 + C$ (reversing the power rule).

10. Answer: **B** — Since differentiation removes constant terms, an indefinite integral must include an arbitrary constant of integration, C, to represent the full family of antiderivatives.

11. Answer: **B** — $\int (4x - 1)dx = 4x^2 \div 2 - x + C = 2x^2 - x + C$.

12. Answer: **B** — A definite integral gives the net signed area enclosed between the curve and the x-axis over the interval [a,b].

13. Answer: **B** — Since $v(t) = ds \div dt$, integrating $v(t)$ reverses the differentiation and gives the displacement, $s(t)$.

14. Answer: **C** — $\int_0^2 3x^2 dx = [x^3]_0^2 = 8 - 0 = 8$.

15. Answer: **B** — Total distance accounts for direction changes, so the interval must be split at each point where $v(t)=0$ (changes sign), and the absolute value of each part's integral summed.

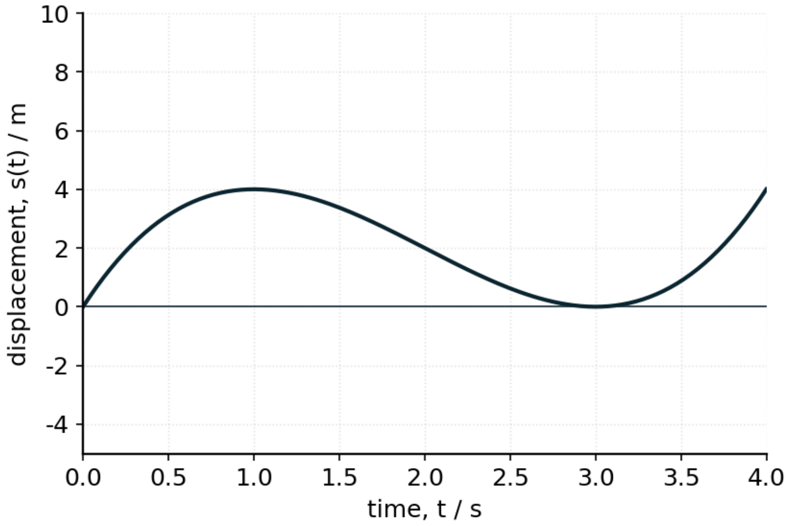
Section B — Structured Questions: Worked Solutions (65 marks)

Mark-scheme notation: M = method mark, A = accuracy mark (dependent on the preceding M mark), R = reasoning mark.

5.1 Differentiation: Rates of Change

Question 1 [7 marks]

A particle moves in a straight line so that its displacement, $s(t)$ metres, after t seconds, is given by $s(t) = t^3 - 6t^2 + 9t$. The graph shows $s(t)$ (computed).



Displacement, $s(t) = t^3 - 6t^2 + 9t$ (computed).

(a) Find an expression for the velocity, $v(t)$, of the particle. **[2]**

$v(t) = ds \div dt = \mathbf{3t^2 - 12t + 9}$. **M1 A1**

(b) Calculate the velocity of the particle at $t=2$. **[2]**

$v(2) = 3(2)^2 - 12(2) + 9 = 12 - 24 + 9 = \mathbf{-3\ m\ s^{-1}}$. **M1 A1**

(c) Find the times at which the particle is momentarily at rest ($v(t)=0$). **[3]**

$3t^2 - 12t + 9 = 0 \Rightarrow 3(t-1)(t-3) = 0 \Rightarrow$ **$t = 1$** or **$t = 3$** (seconds). **M1 A1 A1**

Question 2 [6 marks]

For the particle in Question 1, the velocity is $v(t) = 3t^2 - 12t + 9$.

(a) Find an expression for the acceleration, $a(t)$, of the particle. **[2]**

$a(t) = dv \div dt = \mathbf{6t - 12}$. **M1 A1**

(b) Calculate the acceleration of the particle at $t=0$. **[2]**

$a(0) = 6(0) - 12 = \mathbf{-12\ m\ s^{-2}}$. **M1 A1**

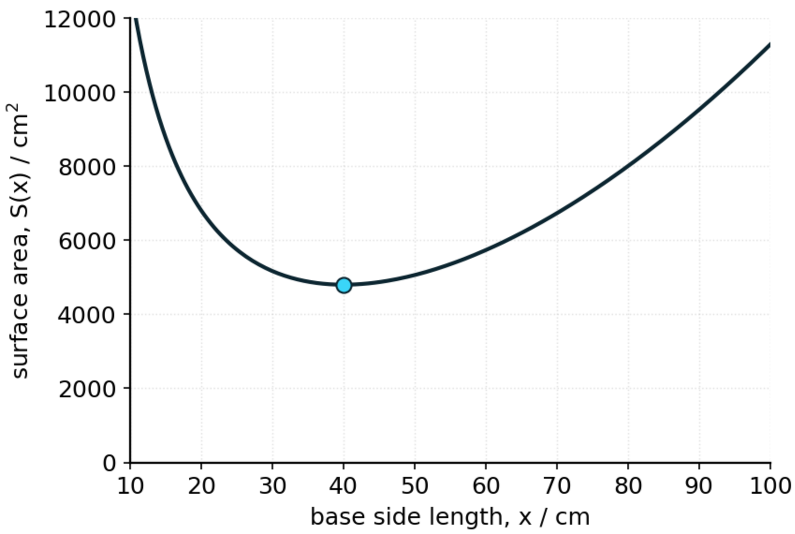
(c) Find the time at which the acceleration is zero. **[2]**

$6t-12 = 0 \Rightarrow$ **$t = 2$ s**. **M1 A1**

5.2 Applications: Kinematics & Optimization

Question 3 [7 marks]

An open-topped box with a square base of side x cm and height h cm is to be built with a fixed volume of 32 000 cm^3. The surface area of the box is given by $S(x) = x^2+128000÷x$. The graph shows S(x) (computed).



Box surface area, $S(x) = x^2+128000÷x$ (computed).

(a) Using the graph, estimate the value of x that minimizes the surface area. **[2]**

From the graph, the minimum occurs at approximately **$x = 40$ cm**. **A1 A1**

(b) Using calculus, find $dS÷dx$, and hence find the exact value of x that minimizes the surface area. **[3]**

$dS÷dx = 2x-128000÷x^2$. Setting $dS÷dx = 0$: $2x^3 = 128000 \Rightarrow x^3 = 64000 \Rightarrow$ **$x = 40$ cm**. **M1 M1 A1**

(c) Calculate the minimum surface area. **[2]**

$S(40) = 40^2+128000÷40 = 1600+3200 =$ **4800 cm^2**. **M1 A1**

Question 4 [6 marks]

A ball is thrown vertically upwards. Its height, h(t) metres, after t seconds, is given by $h(t) = 30t-5t^2$.

(a) Find an expression for the velocity, $v(t) = dh÷dt$, of the ball. **[2]**

$v(t) =$ **$30-10t$**. **M1 A1**

(b) Find the time at which the ball reaches its maximum height ($v(t)=0$). **[2]**

$30-10t = 0 \Rightarrow$ **$t = 3$ s**. **M1 A1**

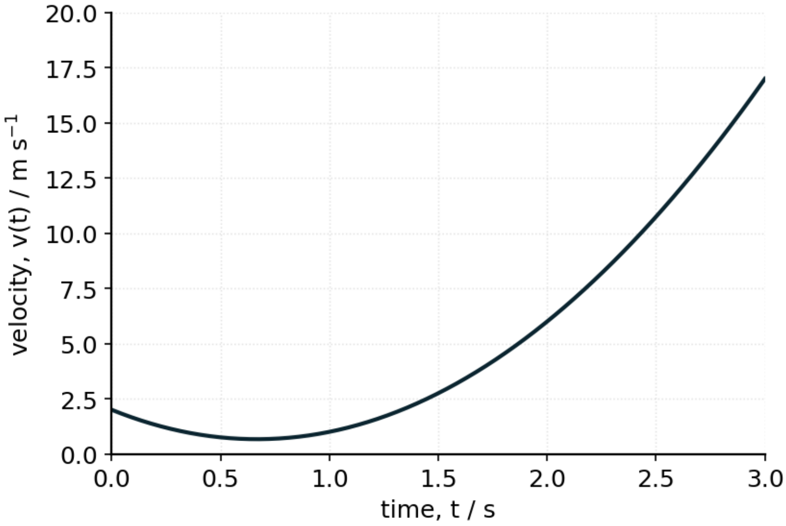
(c) Calculate the maximum height reached by the ball. **[2]**

$h(3) = 30(3)-5(3)^2 = 90-45 =$ **45 m**. **M1 A1**

5.3 Integration: Anti-differentiation & Area

Question 5 [7 marks]

A particle has acceleration $a(t) = 6t-4$ (in m s^{-2}), and an initial velocity $v(0)=2$ m s^{-1}. The graph shows the velocity function $v(t)$ (computed).



Velocity, $v(t) = 3t^2-4t+2$ (computed).

(a) By integrating $a(t)$, and using the given initial condition, find an expression for the velocity, $v(t)$. **[3]**

$v(t) = \int(6t-4)dt = 3t^2-4t+C$. Using $v(0)=2$: $C =$ **2**, so $\mathbf{v(t) = 3t^2-4t+2}$. **M1 M1 A1**

(b) Calculate the velocity at $t=2$. **[2]**

$v(2) = 3(2)^2-4(2)+2 = 12-8+2 =$ **6 m s^{-1}**. **M1 A1**

(c) By considering the discriminant of $3t^2-4t+2$, explain why the particle's velocity is never zero. **[2]**

Discriminant $= (-4)^2-4(3)(2) = 16-24 = \mathbf{-8}$, which is negative. Since the discriminant is negative, $v(t)=0$ has no real solutions, so **the velocity is never zero**. **M1 R1**

Question 6 [6 marks]

Consider the function $f(x) = 4x^3-6x+5$.

(a) Find $\int(4x^3-6x+5)dx$. **[2]**

$\int(4x^3-6x+5)dx = \mathbf{x^4-3x^2+5x+C}$. **M1 A1**

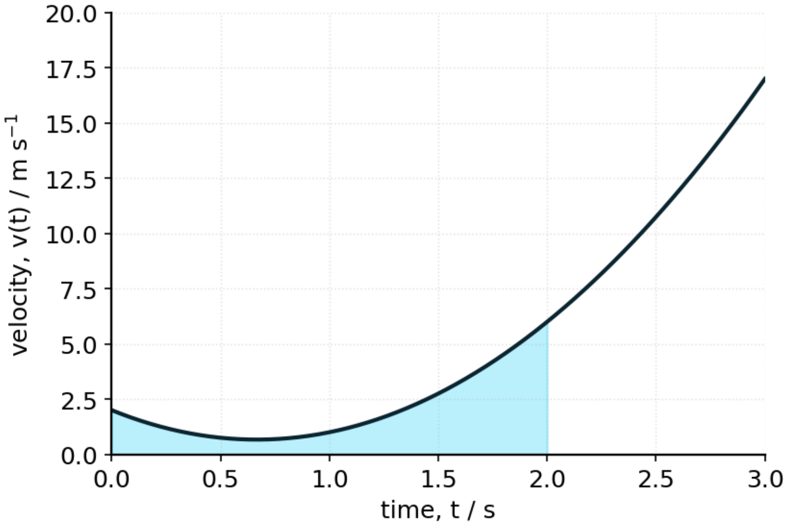
(b) Evaluate the definite integral $\int_1^3(4x^3-6x+5)dx$. **[4]**

$[x^4-3x^2+5x]_1^3 = (81-27+15)-(1-3+5) = 69-3 =$ **66**. **M1 M1 M1 A1**

5.4 Applications: Kinematics using Integration

Question 7 [7 marks]

For the particle in Question 5, with velocity $v(t) = 3t^2-4t+2$ and initial displacement $s(0)=5$ m, the graph shows v(t) with the region representing the displacement between t=0 and t=2 shaded (computed).



Velocity, $v(t) = 3t^2-4t+2$, shaded area [0,2] (computed).

(a) By integrating v(t), and using the given initial condition, find an expression for the displacement, s(t). **[3]**

$s(t) = \int(3t^2-4t+2)dt = t^3-2t^2+2t+C$. Using $s(0)=5$: C = **5**, so $\mathbf{s(t) = t^3-2t^2+2t+5}$. **M1 M1 A1**

(b) Calculate the displacement of the particle at t=2. **[2]**

$s(2) = 2^3-2(2)^2+2(2)+5 = 8-8+4+5 =$ **9 m**. **M1 A1**

(c) Using the shaded area on the graph (i.e. the definite integral $\int_0^2 v(t)dt$), find the change in displacement (the distance moved) between t=0 and t=2. **[2]**

Change in displacement = $s(2)-s(0) = 9-5 =$ **4 m**. **M1 A1**

Question 8 [6 marks]

The curve $y = 6x-x^2-5$ intersects the x-axis at two points.

(a) Find the x-coordinates of the points where the curve intersects the x-axis. **[2]**

$6x-x^2-5 = 0 \Rightarrow x^2-6x+5 = 0 \Rightarrow (x-1)(x-5) = 0 \Rightarrow$ **x = 1** or **x = 5**. **M1 A1**

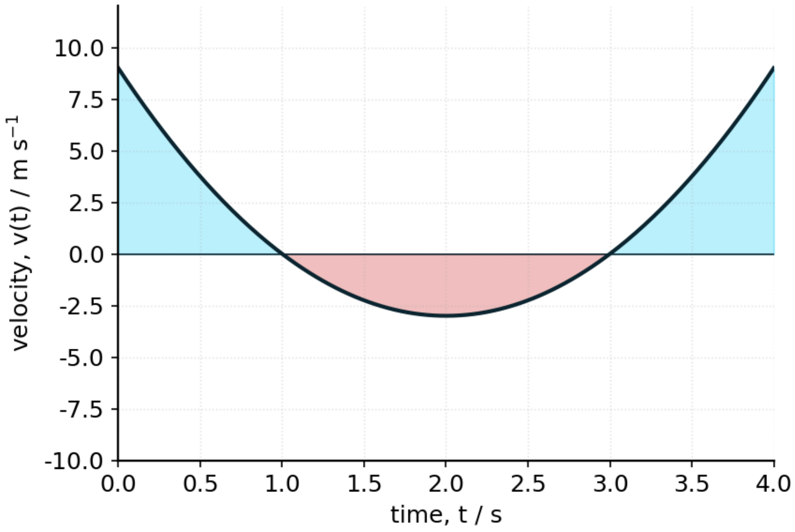
(b) Calculate the area enclosed between the curve and the x-axis, between these two points. **[4]**

Area = $\int_1^5(6x-x^2-5)dx = [3x^2-x^3\div3-5x]_1^5$ = **32÷3** = **10.67 units^2** (3 s.f.). **M1 M1 M1 A1**

5.1 Differentiation: Rates of Change

Question 9 [7 marks]

For the particle in Question 1 (with velocity $v(t) = 3t^2-12t+9$), the graph shows $v(t)$ over $0 \le t \le 4$, with the regions where the particle moves forward and backward shaded differently (computed).



Velocity, $v(t) = 3t^2-12t+9$, direction shading (computed).

(a) Using the graph, write down the time intervals during which the particle is moving forward ($v(t) > 0$) and backward ($v(t) < 0$). **[2]**

The particle moves **forward for $0 < t < 1$ and $t > 3$**; it moves **backward for $1 < t < 3$**. **A1 A1**

(b) Using $s(t) = t^3-6t^2+9t$, calculate the total displacement of the particle between $t=0$ and $t=4$. **[2]**

Displacement = $s(4)-s(0) = (64-96+36)-0 =$ **4 m**. **M1 A1**

(c) Using $s(0)=0$, $s(1)=4$, $s(3)=0$ and $s(4)=4$, calculate the total distance travelled by the particle between $t=0$ and $t=4$. **[3]**

Total distance = $|s(1)-s(0)|+|s(3)-s(1)|+|s(4)-s(3)| = |4|+|-4|+|4| = 4+4+4 =$ **12 m**. **M1 M1 A1**

Question 10 [6 marks]

Consider the function $f(x) = 5x^4-3x^2+7x-2$.

(a) Find $f'(x)$. **[2]**

$f'(x) = \mathbf{20x^3-6x+7}$. **M1 A1**

(b) Calculate $f'(1)$. **[2]**

$f'(1) = 20(1)^3-6(1)+7 = 20-6+7 =$ **21**. **M1 A1**

(c) State, with a reason, whether $f(x)$ is increasing or decreasing at $x=1$. **[2]**

Since $f'(1) = 21 > 0$, **f(x) is increasing** at $x=1$. **R1 R1**