

IB DP MATHEMATICS

AI Topic 3 · Geometry and Trigonometry

MARK SCHEME

3.1 Volume & Surface Area · 3.2 Trigonometry in Context
3.3 Sine & Cosine Rules · 3.4 Navigation & Construction

Standard and Higher Level · Syllabus 2025

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Section A — Multiple Choice: Answer Key (15 marks)

1. Answer: **B** — $V = \pi r^2 h = \pi \times 2^2 \times 6 = 24\pi \text{ m}^3$.
2. Answer: **B** — The volume of a cone is one third of the volume of the cylinder with the same base and height: $V = \frac{1}{3} \pi r^2 h$.
3. Answer: **D** — Surface area of a sphere $= 4\pi r^2 = 4\pi \times 3^2 = 36\pi \text{ cm}^2$.
4. Answer: **A** — Side length = cube root of 64 = **4 cm** (since $4^3 = 64$).
5. Answer: **B** — The angle of elevation is the angle measured upwards from the horizontal to a point above the observer's eye level.
6. Answer: **C** — Height $= 20 \times \tan(40^\circ) = \mathbf{16.8 \text{ m}}$ (3 s.f.).
7. Answer: **B** — A bearing is measured clockwise from north, and is always given as a three-figure angle from 000° to 360° .
8. Answer: **C** — Since $130^\circ < 180^\circ$, the back bearing $= 130^\circ + 180^\circ = 310^\circ$.
9. Answer: **A** — The sine rule states $a \div \sin A = b \div \sin B = c \div \sin C$ for any triangle.
10. Answer: **B** — The cosine rule states $a^2 = b^2 + c^2 - 2bc \cdot \cos A$.
11. Answer: **B** — Angles in a triangle sum to 180° : third angle $= 180^\circ - 50^\circ - 70^\circ = 60^\circ$.
12. Answer: **A** — The area of a triangle is $\frac{1}{2} ab \cdot \sin C$, where C is the angle between sides a and b.
13. Answer: **B** — The angle between the two bearings is $130^\circ - 040^\circ = 90^\circ$, so the ship turns through a right angle.
14. Answer: **C** — Using Pythagoras' theorem (the two legs are perpendicular): distance $= \sqrt{(30^2 + 40^2)} = \sqrt{2500} = 50 \text{ km}$.
15. Answer: **C** — Since the cliff-top horizontal and the sea-level horizontal are parallel, the angle of depression and the angle of elevation are alternate angles, and so are equal.

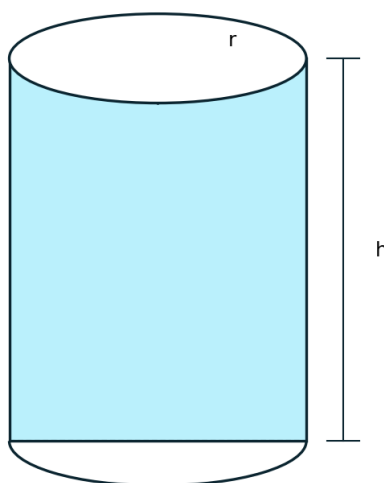
Section B — Structured Questions: Worked Solutions (65 marks)

Mark-scheme notation: M = method mark, A = accuracy mark (dependent on the preceding M mark), R = reasoning mark.

3.1 Volume & Surface Area of 3D Solids

Question 1 [7 marks]

A cylindrical water tank has radius 3 m and height 5 m (given data).



Cylindrical water tank, $r=3$ m, $h=5$ m (given data).

- (a) Using the formula $V = \pi r^2 h$, calculate the volume of the tank, giving your answer to 3 significant figures. [2]

$$V = \pi \times 3^2 \times 5 = 45\pi = \mathbf{141 \text{ m}^3}. \quad \text{M1 A1}$$

- (b) Calculate the curved (lateral) surface area of the tank, giving your answer to 3 significant figures. [2]

$$\text{Curved SA} = 2\pi r h = 2\pi \times 3 \times 5 = 30\pi = \mathbf{94.2 \text{ m}^2}. \quad \text{M1 A1}$$

- (c) Calculate the total surface area of the tank (including the top and bottom circular ends), giving your answer to 3 significant figures. [3]

$$\text{Total SA} = 2\pi r h + 2\pi r^2 = 30\pi + 2\pi \times 3^2 = 30\pi + 18\pi = 48\pi = \mathbf{151 \text{ m}^2}. \quad \text{M1 M1 A1}$$

Question 2 [6 marks]

A solid cone has base radius 4 cm and height 9 cm.

- (a) Using $V = \frac{1}{3} \pi r^2 h$, calculate the volume of the cone, giving your answer to 3 significant figures. [2]

$$V = \frac{1}{3} \pi \times 4^2 \times 9 = 48\pi = \mathbf{151 \text{ cm}^3}. \quad \text{M1 A1}$$

- (b) Using Pythagoras' theorem, calculate the slant height, l , of the cone, giving your answer to 3 significant figures. [2]

$$l = \sqrt{(r^2 + h^2)} = \sqrt{(4^2 + 9^2)} = \sqrt{97} = \mathbf{9.85 \text{ cm.}}$$

M1 A1

(c) Using the curved surface area formula, $CSA = \pi rl$, calculate the curved surface area of the cone, giving your answer to 3 significant figures. [2]

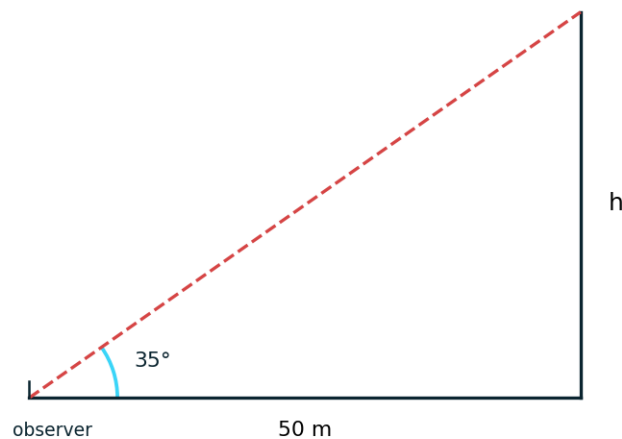
$$CSA = \pi \times 4 \times 9.849 = \mathbf{124 \text{ cm}^2}.$$

M1 A1

3.2 Right-Angled Trigonometry in Context

Question 3 [7 marks]

An observer stands 50 m from the base of a tower. The angle of elevation from the observer to the top of the tower is 35° (given data).



Angle of elevation diagram, base=50 m, angle= 35° (given data).

(a) Calculate the height, h , of the tower, giving your answer to 3 significant figures. [2]

$$h = 50 \cdot \tan(35^\circ) = \mathbf{35 \text{ m.}}$$

M1 A1

(b) Calculate the distance from the observer to the top of the tower (the line of sight distance), giving your answer to 3 significant figures. [2]

$$\text{Distance} = 50 \div \cos(35^\circ) = \mathbf{61 \text{ m.}}$$

M1 A1

(c) The observer walks 20 m closer to the tower (now 30 m from the base). Calculate the new angle of elevation, giving your answer to 3 significant figures. [3]

$$\text{New angle} = \tan^{-1}(35.010 \div 30) = \mathbf{49.4^\circ}.$$

M1 M1 A1

Question 4 [6 marks]

A hiker walks 8 km on a bearing of 065° from point A to point B.

(a) State the bearing required to walk directly back from B to A. [2]

$$\text{Back bearing} = 065^\circ + 180^\circ = \mathbf{245^\circ}.$$

A1 A1

(b) Calculate how far east of A point B is, giving your answer to 3 significant figures. [2]

East displacement = $8 \cdot \sin(65^\circ) = 7.25 \text{ km}$.

M1 A1

(c) Calculate how far north of A point B is, giving your answer to 3 significant figures.

[2]

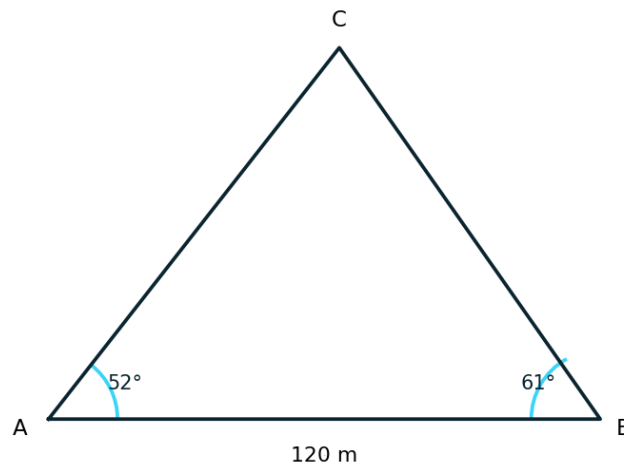
North displacement = $8 \cdot \cos(65^\circ) = 3.38 \text{ km}$.

M1 A1

3.3 Sine & Cosine Rules in Context

Question 5 [7 marks]

A triangular plot of land has vertices A, B and C, with $AB=120 \text{ m}$, angle $A=52^\circ$ and angle $B=61^\circ$ (given data).



Survey triangle diagram, $AB=120 \text{ m}$, $A=52^\circ$, $B=61^\circ$ (given data).

(a) Calculate the size of angle C.

[2]

Angle $C = 180^\circ - 52^\circ - 61^\circ = 67^\circ$.

A1 A1

(b) Using the sine rule, calculate the length of BC, giving your answer to 3 significant figures.

[3]

$BC \div \sin A = AB \div \sin C \Rightarrow BC = 120 \cdot \sin(52^\circ) \div \sin(67^\circ) = 103 \text{ m}$.

M1 M1 A1

(c) Using the sine rule, calculate the length of AC, giving your answer to 3 significant figures.

[2]

$AC = 120 \cdot \sin(61^\circ) \div \sin(67^\circ) = 114 \text{ m}$.

M1 A1

Question 6 [6 marks]

A triangular garden has two sides of length 15 m and 18 m, with an included angle of 75° between them.

(a) Using the cosine rule, calculate the length of the third side, c, giving your answer to 3 significant figures.

[3]

$c^2 = 15^2 + 18^2 - 2 \times 15 \times 18 \cdot \cos(75^\circ) \Rightarrow c = 20.2 \text{ m}$.

M1 M1 A1

(b) Using the formula $\text{Area} = \frac{1}{2} ab \cdot \sin C$, calculate the area of the garden, giving your answer to 3 significant figures.

[3]

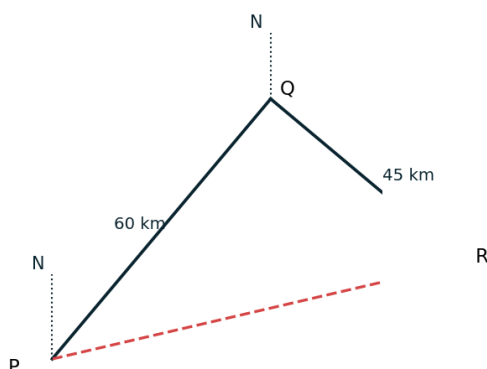
$$\text{Area} = \frac{1}{2} \times 15 \times 18 \cdot \sin(75^\circ) = \mathbf{130 \text{ m}^2}.$$

M1 A1 A1

3.4 Applications: Navigation & Construction

Question 7 [7 marks]

A ship sails from port P on a bearing of 040° for 60 km to point Q, then changes course to a bearing of 130° and sails 45 km to point R (given data).



Navigation diagram, $PQ=60 \text{ km}$ (bearing 040°), $QR=45 \text{ km}$ (bearing 130°) (given data).

(a) Show that angle $PQR = 90^\circ$, explaining your reasoning using the given bearings. [2]

The direction from Q back to P is $040^\circ + 180^\circ = 220^\circ$. The angle between this direction (220°) and the direction QR (130°) is $220^\circ - 130^\circ = 90^\circ$, so angle $PQR = 90^\circ$. R1 R1

(b) Using Pythagoras' theorem, calculate the distance PR. [2]

$$PR = \sqrt{60^2 + 45^2} = \sqrt{5625} = \mathbf{75 \text{ km}}.$$

M1 A1

(c) Calculate the bearing of R from P, giving your answer to 3 significant figures. [3]

$$\text{Angle } QPR = \tan^{-1}(45 \div 60) = 36.9^\circ. \text{ Bearing of R from P} = 040^\circ + 36.9^\circ = \mathbf{76.9^\circ}.$$

M1 M1 A1

3.1 Volume & Surface Area of 3D Solids

Question 8 [6 marks]

A spherical water storage tank has radius 2.5 m.

(a) Using $V = \frac{4}{3} \pi r^3$, calculate the volume of the tank, giving your answer to 3 significant figures. [2]

$$V = \frac{4}{3} \pi \times 2.5^3 = \mathbf{65.4 \text{ m}^3}.$$

M1 A1

(b) Using $SA = 4\pi r^2$, calculate the surface area of the tank, giving your answer to 3 significant figures. [2]

$$SA = 4\pi \times 2.5^2 = 25\pi = \mathbf{78.5 \text{ m}^2}.$$

M1 A1

(c) The tank is filled with water at a rate of 2 m^3 per minute. Calculate the time taken to fill the tank, giving your answer to the nearest minute. [2]

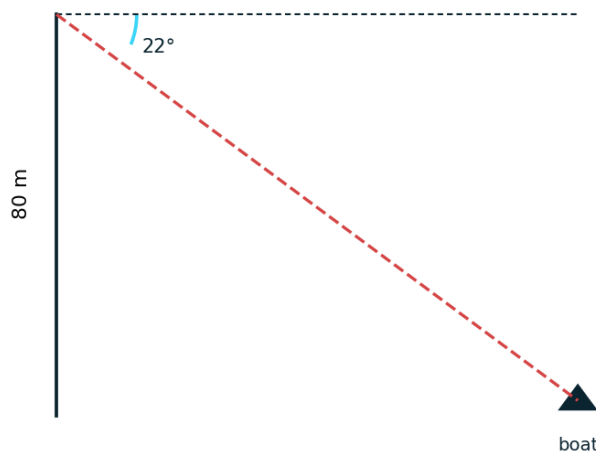
$$\text{Time} = 65.450 \div 2 = \mathbf{33 \text{ minutes}} \text{ (to the nearest minute).}$$

M1 A1

3.2 Right-Angled Trigonometry in Context

Question 9 [7 marks]

From the top of a cliff 80 m high, the angle of depression to a boat at sea is 22° (given data).



Angle of depression diagram, cliff height=80 m, angle= 22° (given data).

(a) Calculate the horizontal distance from the base of the cliff to the boat, giving your answer to 3 significant figures. [2]

$$\text{Distance} = 80 \div \tan(22^\circ) = \mathbf{198 \text{ m.}}$$

M1 A1

(b) Calculate the distance from the top of the cliff to the boat (the line of sight distance), giving your answer to 3 significant figures. [2]

$$\text{Distance} = 80 \div \sin(22^\circ) = \mathbf{214 \text{ m.}}$$

M1 A1

(c) The boat sails directly away from the base of the cliff until the angle of depression becomes 15° . Calculate how far the boat has sailed, giving your answer to 3 significant figures. [3]

$$\text{New horizontal distance} = 80 \div \tan(15^\circ) = 299 \text{ m. Distance sailed} = 298.564 - 198.007 = \mathbf{101 \text{ m.}}$$

M1 M1 A1

3.1 Volume & Surface Area of 3D Solids

Question 10 [6 marks]

A grain silo is made from a cylinder of radius 3 m and height 8 m, topped with a cone of the same radius and height 2 m.

(a) Calculate the volume of the cylindrical part of the silo, giving your answer to 3 significant figures. [2]

$$V_{\text{cyl}} = \pi \times 3^2 \times 8 = 72\pi = \mathbf{226 \text{ m}^3}. \quad \text{M1 A1}$$

(b) Calculate the volume of the conical part of the silo, giving your answer to 3 significant figures. [2]

$$V_{\text{cone}} = \frac{1}{3} \pi \times 3^2 \times 2 = 6\pi = \mathbf{18.8 \text{ m}^3}. \quad \text{M1 A1}$$

(c) Calculate the total volume of the silo, giving your answer to 3 significant figures. [2]

$$\text{Total volume} = 226.195 + 18.850 = 78\pi = \mathbf{245 \text{ m}^3}. \quad \text{M1 A1}$$