IB DP MATHEMATICS

AI Topic 2 · Functions

TOPICAL WORKSHEET

2.1 Linear Models · 2.2 Quadratic Models

2.3 Other Function Models · 2.4 Optimization

Standard and Higher Level · Syllabus 2025

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Original exam-style questions written for the IB Diploma Programme Mathematics course (first assessment 2025)

Section A — Multiple Choice (15 marks)

Circle the letter of the best answer to each question. Each question is worth 1 mark.

1. A linear model has equation $C(n) = 30 + 5n$. The fixed (constant) cost, independent of n, is **[1]**

A 5

B 25

C 30

D 35

2. A linear model passes through the points (10, 40) and (30, 100). The gradient of this line is **[1]**

A 2

B 3

C 4

D 60

3. Which of the following is NOT a property of a linear model $y = mx+c$? **[1]**

A The rate of change (gradient) is constant

B The graph is a straight line

C First differences of equally-spaced y-values are constant

D The graph must pass through the origin

4. A car's value decreases linearly from \$25 000 by \$2000 per year. Its value after 3 years is **[1]**

A \$6 000

B \$17 000

C \$19 000

D \$23 000

5. A ball's height is modelled by $h(t) = -5t^2+30t$. The time at which the ball reaches its maximum height is **[1]**

A 1 s

B 2 s

C 3 s

D 6 s

6. The axis of symmetry of the quadratic model $y = 2x^2-8x+3$ is the line **[1]**

A $x = -2$

B $x = 1$

C $x = 2$

D $x = 4$

7. A quadratic model for a ball's height has vertex (4, 80), where x is time in seconds and y is height in metres. The maximum height reached by the ball is **[1]**

A 4 m

B 40 m

C 80 m

D 84 m

8. A quadratic model $y = ax^2+bx+c$ has a negative discriminant ($b^2-4ac < 0$). This means the graph of the model **[1]**

A has two distinct x-intercepts

B has exactly one x-intercept (touches the x-axis)

C has no x-intercepts (does not cross the x-axis)

D is a straight line

9. For an inverse variation model $y = k \div x$, if $y = 20$ when $x = 5$, the constant k is **[1]**

A 4

B 15

C 25

D 100

10. An open box is formed by cutting squares of side x from a 10 cm square of card and folding up the sides, giving volume $V(x) = x(10-2x)^2$. The valid domain for this model is **[1]**

A $x > 0$ only

B $0 < x < 5$

C $0 < x < 10$

D $x < 10$

11. For the sinusoidal model $y = 4\sin(x)+7$, the amplitude of the model is **[1]**

A 4

B 7

C 8

D 11

12. The period, in hours, of the sinusoidal model $h(t) = 3\sin(\pi t \div 6)+5$ is **[1]**

A 3

B 6

C $\pi \div 6$

D 12

13. To find the maximum or minimum value of a function model using a graphing calculator (GDC), you would **[1]**

A locate the y-intercept of the graph

B locate the turning point (vertex) of the graph

C find where the graph crosses the x-axis

D find the gradient of the graph at $x=0$

14. For a business model, the 'break-even point(s)' occur where **[1]**

A Revenue = 0

B Cost = 0

C Revenue = Cost (Profit = 0)

D Profit is at its maximum

15. For a profit model $P(x)$ that is a downward-opening (negative leading coefficient) quadratic, the vertex of the graph represents **[1]**

A the break-even point

B the maximum profit

C the minimum cost

D the fixed cost

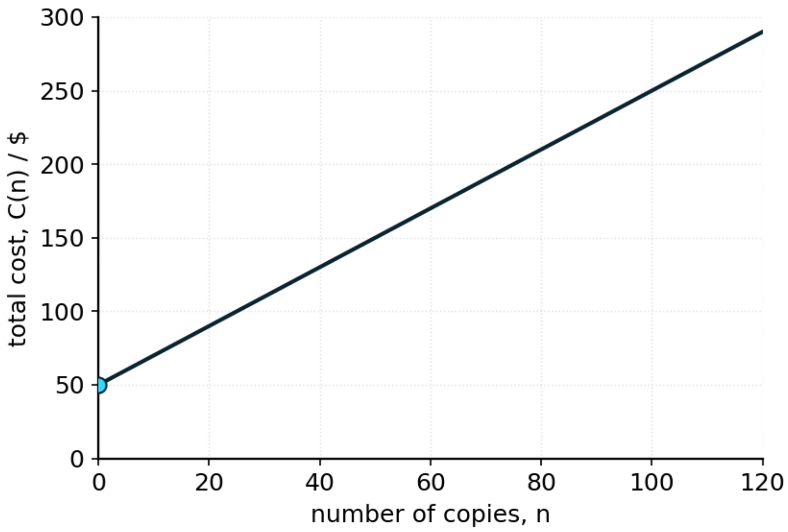
Section B — Structured Questions (65 marks)

Answer all questions in the space provided. Working must be shown for full marks.

2.1 Linear Models

Question 1 [7 marks]

A printing company charges a fixed set-up fee plus a cost per copy. The graph shows the total cost, C(n), for printing n copies, where C(n) = 50+2n (computed).



Linear cost model, C(n) = 50+2n (computed).

(a) Using the graph, state the fixed cost (the cost when n=0), and the gradient of the line (the cost per copy). **[2]**

(b) Using C(n) = 50+2n, calculate the total cost of printing 80 copies. **[2]**

(c) Find the number of copies, n, for which the total cost is $250. **[3]**

Question 2 [6 marks]

A car is purchased for $20 000, and its value depreciates linearly by $1500 each year, so that V(t) = 20 000−1500t.

(a) State the initial value of the car (at t=0). **[2]**

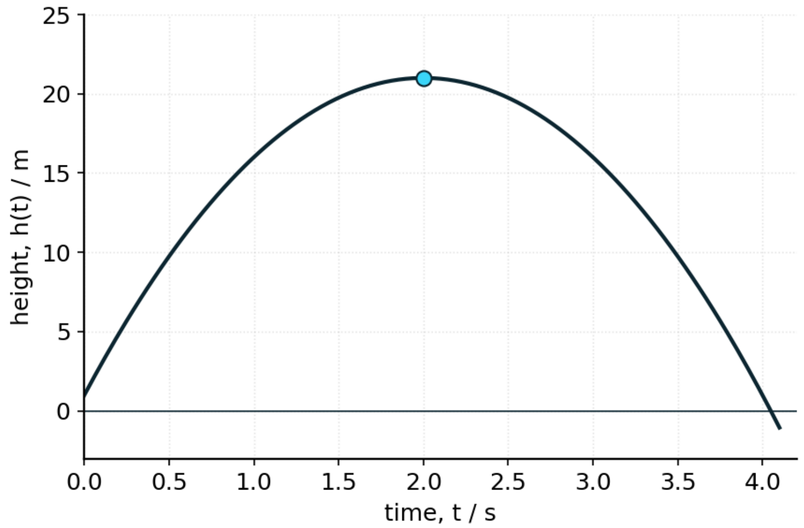
(b) Calculate the value of the car after 4 years. **[2]**

(c) The car is sold for scrap once its value reaches \$5000. Find the value of t at which this occurs. **[2]**

2.2 Quadratic Models

Question 3 [7 marks]

A ball is thrown into the air. The graph shows its height, h(t), after t seconds, where $h(t) = -5t^2+20t+1$ (computed).



Projectile height, $h(t) = -5t^2+20t+1$ (computed).

(a) Using the graph, write down the maximum height reached by the ball, and the time at which this occurs. **[2]**

(b) Calculate the height of the ball after 1 second. **[2]**

(c) Find the time at which the ball hits the ground ($h(t)=0$), giving your answer to 3 significant figures. **[3]**

Question 4 [6 marks]

A farmer has 40 m of fencing to enclose a rectangular field, using an existing wall as one side (so fencing is only needed for the other three sides). If x is the length of each of the two sides perpendicular to the wall, the enclosed area is $A(x) = 40x - 2x^2$.

(a) Calculate the enclosed area when $x=5$. **[2]**

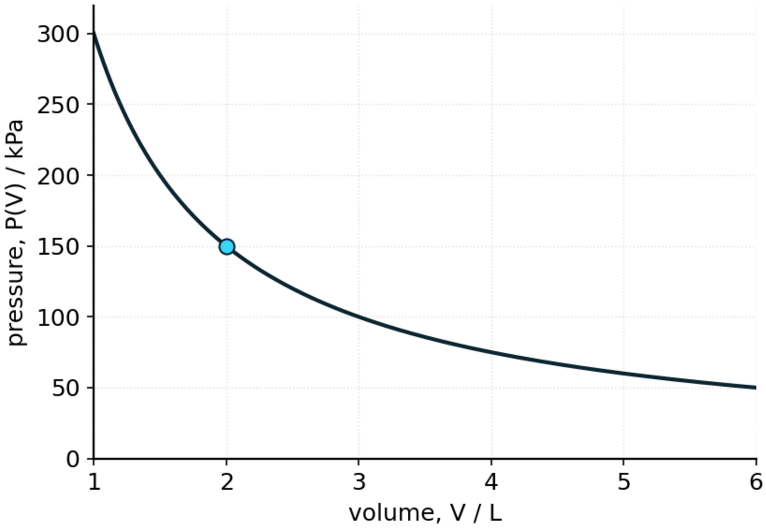
(b) Using the vertex of this quadratic model, find the value of x that maximizes the enclosed area. **[2]**

(c) Calculate the maximum enclosed area. **[2]**

2.3 Other Function Models

Question 5 [7 marks]

Boyle's Law states that, at constant temperature, the pressure P of a fixed mass of gas is inversely proportional to its volume V, so that $P(V) = k \div V$. The graph shows this relationship (computed).



Boyle's Law, P(V) = 300÷V (computed).

(a) Using the point marked on the graph (V=2, P=150), find the value of the constant k. **[2]**

(b) Using P(V) = 300÷V, calculate the pressure when V=5 L. **[2]**

(c) Calculate the volume when the pressure is 100 kPa. **[3]**

Question 6 [6 marks]

An open-topped box is formed from a square sheet of card of side 20 cm, by cutting squares of side x cm from each corner and folding up the sides, giving volume $V(x) = x(20-2x)^2$.

(a) Calculate the volume of the box when x=2. **[2]**

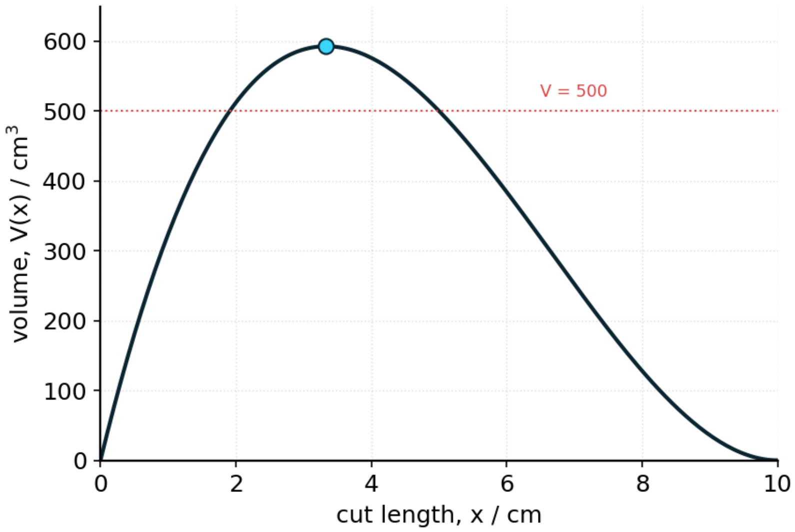
(b) State the valid domain of x for this model, giving a reason. **[2]**

(c) Calculate the volume of the box when x=4. **[2]**

2.4 Optimization

Question 7 [7 marks]

The graph shows the volume, $V(x)$, of the open-topped box from Question 6, for $0 < x < 10$ (computed).



Open-box volume, $V(x) = x(20-2x)^2$ (computed).

(a) Using the graph, write down the value of x that gives the maximum volume, and state this maximum volume, both to 3 significant figures. **[2]**

(b) State the maximum volume to the nearest whole number. **[2]**

(c) Using the graph, state the range of values of x for which $V(x) > 500\ \text{cm}^3$. **[3]**

Question 8 [6 marks]

A company's profit, in dollars, from selling x units is modelled by $P(x) = -0.1x^2+30x-500$.

(a) Find the break-even points (the values of x for which the profit is zero), giving your answers to the nearest whole unit. **[2]**

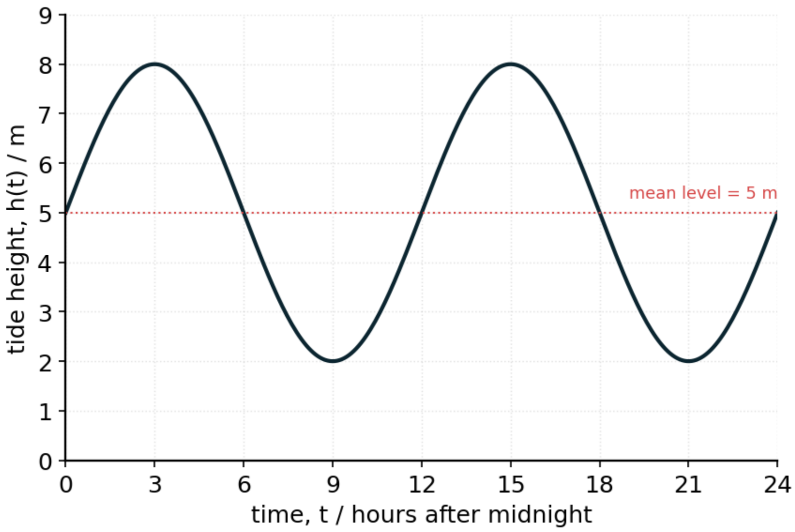
(b) Find the number of units that maximizes the profit, and state this maximum profit. **[2]**

(c) State the range of values of x for which the company makes a profit ($P(x) > 0$). **[2]**

2.3 Other Function Models

Question 9 [7 marks]

The height of the tide, h(t) metres, t hours after midnight, is modelled by the sinusoidal function $h(t) = 3\sin(\pi t \div 6)+5$. The graph shows this model over 24 hours (computed).



Tide height model, $h(t) = 3\sin(\pi t \div 6)+5$ (computed).

(a) Using the graph (or the equation), state the amplitude and the mean (sea) level of this model. **[2]**

(b) Calculate the tide height at t=3 (3 a.m.). **[2]**

(c) Find the first time after midnight, $t > 0$, at which the tide height is 6.5 m. **[3]**

2.1 Linear Models

Question 10 [6 marks]

A mobile phone plan charges a flat fee of \$20 for the first 100 minutes of calls each month, then \$0.25 per minute for any additional minutes, so that $C(m) = 20$ for $m \leq 100$, and $C(m) = 20 + 0.25(m - 100)$ for $m > 100$.

(a) Calculate the cost of 60 minutes of calls. **[2]**

(b) Calculate the cost of 150 minutes of calls. **[2]**

(c) Find the number of minutes, m, for which the cost is \$45. **[2]**