IB DP MATHEMATICS

AI Topic 2 · Functions

MARK SCHEME

2.1 Linear Models · 2.2 Quadratic Models
2.3 Other Function Models · 2.4 Optimization

Standard and Higher Level · Syllabus 2025

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Original exam-style questions written for the IB Diploma Programme Mathematics course (first assessment 2025)

Section A — Multiple Choice: Answer Key (15 marks)

1. Answer: **C** — In $C(n) = 30+5n$, the constant term 30 is the fixed cost (the value of C when $n=0$).

2. Answer: **B** — Gradient = $(100-40)\div(30-10) = 60\div20 = 3$.

3. Answer: **D** — A linear model only passes through the origin if $c=0$; in general the y-intercept c can be any value.

4. Answer: **C** — $V(3) = 25\,000-2000\times3 = 25\,000-6000 = \$19\,000$.

5. Answer: **C** — Maximum height occurs at $t = -b\div2a = -30\div(2\times-5) = 3$ s.

6. Answer: **C** — Axis of symmetry: $x = -b\div2a = 8\div(2\times2) = 2$.

7. Answer: **C** — The vertex (4, 80) of a downward-opening parabola (since it represents a maximum) gives the maximum value directly: 80 m.

8. Answer: **C** — A negative discriminant means the quadratic equation has no real roots, so the graph never crosses (or touches) the x-axis.

9. Answer: **D** — $k = xy = 5\times20 = 100$.

10. Answer: **B** — x must be positive, and $10-2x$ must also be positive (so the base has positive side length), giving $x < 5$. So the valid domain is $0 < x < 5$.

11. Answer: **A** — For $y = A\sin(x)+D$, the amplitude is A, the coefficient of the sine term: here $A = 4$.

12. Answer: **D** — Period = $2\pi\div(\pi\div6) = 2\pi\times6\div\pi = 12$ hours.

13. Answer: **B** — The maximum or minimum value of a function corresponds to its turning point (vertex), which can be located directly using a GDC's graphing tools.

14. Answer: **C** — Break-even occurs when total revenue equals total cost, i.e. when profit $P(x) = R(x)-C(x) = 0$.

15. Answer: **B** — For a downward-opening quadratic, the vertex is the highest point on the graph, representing the maximum value of the profit model.

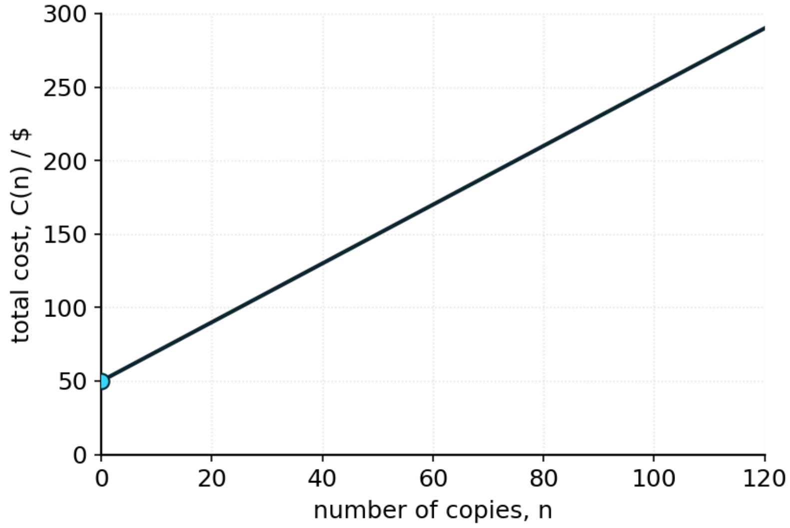
Section B — Structured Questions: Worked Solutions (65 marks)

Mark-scheme notation: M = method mark, A = accuracy mark (dependent on the preceding M mark), R = reasoning mark.

2.1 Linear Models

Question 1 [7 marks]

A printing company charges a fixed set-up fee plus a cost per copy. The graph shows the total cost, $C(n)$, for printing n copies, where $C(n) = 50+2n$ (computed).



Linear cost model, C(n) = 50+2n (computed).

(a) Using the graph, state the fixed cost (the cost when $n=0$), and the gradient of the line (the cost per copy). **[2]**

Fixed cost = **$50**; gradient (cost per copy) = **$2** per copy. **A1 A1**

(b) Using $C(n) = 50+2n$, calculate the total cost of printing 80 copies. **[2]**

$C(80) = 50+2\times80 = 50+160 =$ **$210**. **M1 A1**

(c) Find the number of copies, n, for which the total cost is $250. **[3]**

$250 = 50+2n \Rightarrow 2n = 200 \Rightarrow$ **$n = 100$** copies. **M1 M1 A1**

Question 2 [6 marks]

A car is purchased for $20 000, and its value depreciates linearly by $1500 each year, so that $V(t) = 20\,000-1500t$.

(a) State the initial value of the car (at $t=0$). **[2]**

Initial value = $V(0)$ = **$20 000**. **A1 A1**

(b) Calculate the value of the car after 4 years. **[2]**

$V(4) = 20\,000-1500\times4 = 20\,000-6000 =$ **$14 000**. **M1 A1**

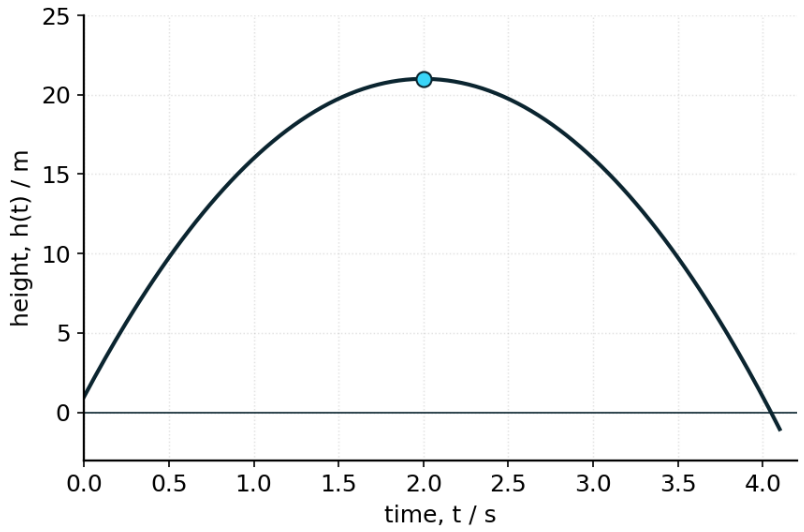
(c) The car is sold for scrap once its value reaches \$5000. Find the value of t at which this occurs. **[2]**

$5000 = 20\,000 - 1500t \Rightarrow 1500t = 15\,000 \Rightarrow$ **$t = 10$ years**. **M1 A1**

2.2 Quadratic Models

Question 3 [7 marks]

A ball is thrown into the air. The graph shows its height, h(t), after t seconds, where $h(t) = -5t^2+20t+1$ (computed).



Projectile height, $h(t) = -5t^2+20t+1$ (computed).

(a) Using the graph, write down the maximum height reached by the ball, and the time at which this occurs. **[2]**

Maximum height = **21 m**, reached at **t = 2 s**. **A1 A1**

(b) Calculate the height of the ball after 1 second. **[2]**

$h(1) = -5(1)^2+20(1)+1 = -5+20+1 =$ **16 m**. **M1 A1**

(c) Find the time at which the ball hits the ground (h(t)=0), giving your answer to 3 significant figures. **[3]**

$-5t^2+20t+1 = 0 \Rightarrow 5t^2-20t-1 = 0 \Rightarrow t = (20+\sqrt{420})\div 10 \Rightarrow$ **$t \approx 4.05$ s** (rejecting the negative root). **M1 M1 A1**

Question 4 [6 marks]

A farmer has 40 m of fencing to enclose a rectangular field, using an existing wall as one side (so fencing is only needed for the other three sides). If x is the length of each of the two sides perpendicular to the wall, the enclosed area is $A(x) = 40x-2x^2$.

(a) Calculate the enclosed area when x=5. **[2]**

$A(5) = 40(5)-2(5)^2 = 200-50 =$ **150 m^2**. **M1 A1**

(b) Using the vertex of this quadratic model, find the value of x that maximizes the enclosed area. **[2]**

$x = -b \div 2a = -40 \div (2\times -2) =$ **$x = 10$ m**. **M1 A1**

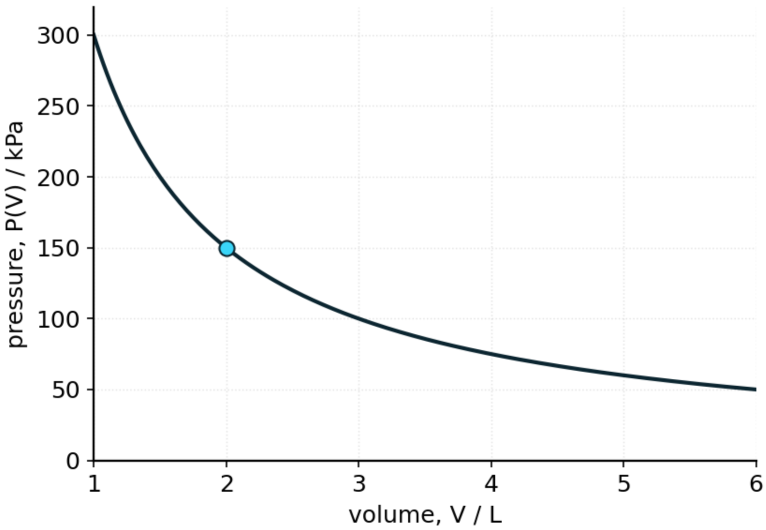
(c) Calculate the maximum enclosed area. **[2]**

$A(10) = 40(10) - 2(10)^2 = 400 - 200 =$ **200 m²**. **M1 A1**

2.3 Other Function Models

Question 5 [7 marks]

Boyle's Law states that, at constant temperature, the pressure P of a fixed mass of gas is inversely proportional to its volume V, so that $P(V) = k \div V$. The graph shows this relationship (computed).



Boyle's Law, P(V) = 300÷V (computed).

(a) Using the point marked on the graph (V=2, P=150), find the value of the constant k. **[2]**

$k = PV = 150\times 2 =$ **$k = 300$**. **M1 A1**

(b) Using $P(V) = 300 \div V$, calculate the pressure when V=5 L. **[2]**

$P(5) = 300 \div 5 =$ **60 kPa**. **M1 A1**

(c) Calculate the volume when the pressure is 100 kPa. **[3]**

$100 = 300 \div V \Rightarrow V = 300 \div 100 =$ **3 L**. **M1 M1 A1**

Question 6 [6 marks]

An open-topped box is formed from a square sheet of card of side 20 cm, by cutting squares of side x cm from each corner and folding up the sides, giving volume $V(x) = x(20-2x)^2$.

(a) Calculate the volume of the box when x=2. **[2]**

$V(2) = 2(20-4)^2 = 2(16)^2 = 2\times 256 =$ **512 cm³**. **M1 A1**

(b) State the valid domain of x for this model, giving a reason. **[2]**

Since x must be positive, and $(20-2x)$ must also be positive (for the base to have positive side length), the valid domain is **$0 < x < 10$**. **R1 R1**

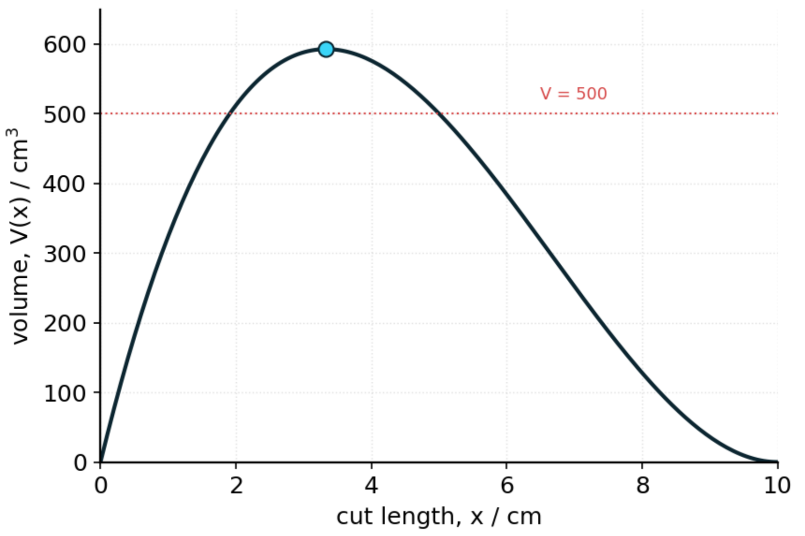
(c) Calculate the volume of the box when $x=4$. **[2]**

$V(4) = 4(20-8)^2 = 4(12)^2 = 4\times144 =$ **576 cm^3**. **M1 A1**

2.4 Optimization

Question 7 [7 marks]

The graph shows the volume, $V(x)$, of the open-topped box from Question 6, for $0 < x < 10$ (computed).



Open-box volume, $V(x) = x(20-2x)^2$ (computed).

(a) Using the graph, write down the value of x that gives the maximum volume, and state this maximum volume, both to 3 significant figures. **[2]**

From the graph, the maximum volume occurs at **$x \approx 3.33$ cm**, giving a maximum volume of **$V \approx 593$ cm^3**. **A1 A1**

(b) State the maximum volume to the nearest whole number. **[2]**

Maximum volume = **593 cm^3** (to the nearest whole number). **A1 A1**

(c) Using the graph, state the range of values of x for which $V(x) > 500$ cm^3. **[3]**

From the graph, $V(x) > 500$ for **$1.91 < x < 5$** (3 s.f.). **M1 A1 A1**

Question 8 [6 marks]

A company's profit, in dollars, from selling x units is modelled by $P(x) = -0.1x^2+30x-500$.

(a) Find the break-even points (the values of x for which the profit is zero), giving your answers to the nearest whole unit. **[2]**

Solving $-0.1x^2+30x-500 = 0$ gives **$x \approx 18$** and **$x \approx 282$** (to the nearest whole unit). **M1 A1**

(b) Find the number of units that maximizes the profit, and state this maximum profit. **[2]**

Maximum profit occurs at x = −b÷2a = −30÷(2×−0.1) = **x = 150** units, giving maximum profit P(150) = **$1750**. **M1 A1**

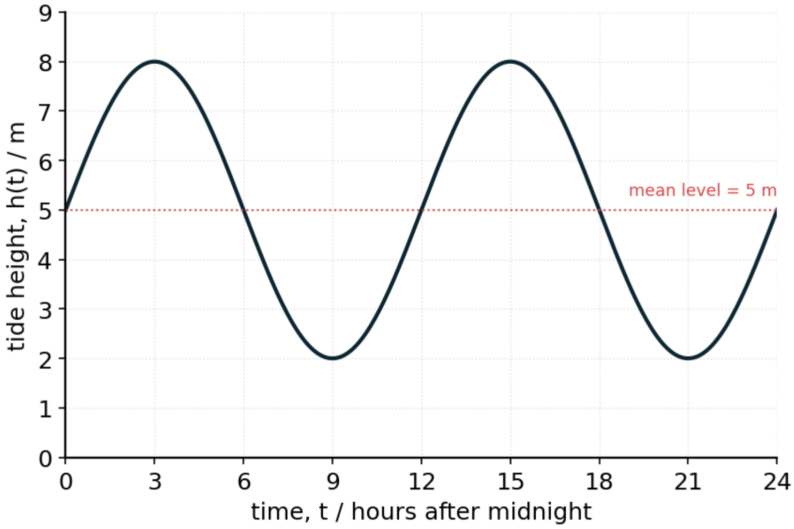
(c) State the range of values of x for which the company makes a profit (P(x) > 0). **[2]**

Since P(x) is a downward-opening parabola with roots at x ≈ 18 and x ≈ 282, the company makes a profit for **18 < x < 282**. **R1 A1**

2.3 Other Function Models

Question 9 [7 marks]

The height of the tide, h(t) metres, t hours after midnight, is modelled by the sinusoidal function h(t) = 3sin(πt÷6)+5. The graph shows this model over 24 hours (computed).



Tide height model, h(t) = 3sin(πt÷6)+5 (computed).

(a) Using the graph (or the equation), state the amplitude and the mean (sea) level of this model. **[2]**

Amplitude = **3 m**; mean level = **5 m**. **A1 A1**

(b) Calculate the tide height at t=3 (3 a.m.). **[2]**

h(3) = 3sin(π÷6×3)+5 = 3sin(π÷2)+5 = 3(1)+5 = **8 m**. **M1 A1**

(c) Find the first time after midnight, t > 0, at which the tide height is 6.5 m. **[3]**

3sin(πt÷6)+5 = 6.5 ⇒ sin(πt÷6) = 0.5 ⇒πt÷6 = π÷6 ⇒ **t = 1 hour** (i.e. 1 a.m.). **M1 M1 A1**

2.1 Linear Models

Question 10 [6 marks]

A mobile phone plan charges a flat fee of $20 for the first 100 minutes of calls each month, then $0.25 per minute for any additional minutes, so that C(m) = 20 for m≤100, and C(m) = 20+0.25(m−100) for m > 100.

(a) Calculate the cost of 60 minutes of calls. **[2]**

Since $60 \leq 100$, $C(60) =$ **$20**. **A1 A1**

(b) Calculate the cost of 150 minutes of calls. **[2]**

Since $150 > 100$, $C(150) = 20+0.25(150-100) = 20+0.25\times 50 = 20+12.5 =$ **$32.50**. **M1 A1**

(c) Find the number of minutes, m, for which the cost is $45. **[2]**

$45 = 20+0.25(m-100) \Rightarrow 25 = 0.25(m-100) \Rightarrow m-100 = 100 \Rightarrow$ **m = 200 minutes**. **M1 A1**