MEGA LECTURE

IB DP MATHEMATICS

AI Topic 1 · Number and Algebra

TOPICAL WORKSHEET

1.1 Approximation & Estimation · 1.2 Sequences & Series
1.3 Exponential Growth & Decay · 1.4 Financial Mathematics

Standard and Higher Level · Syllabus 2025

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Original exam-style questions written for the IB Diploma Programme Mathematics course (first assessment 2025)

Section A — Multiple Choice (15 marks)

Circle the letter of the best answer to each question. Each question is worth 1 mark.

1. A number written in standard form (scientific notation) has the form **[1]**

A $a \times 10^n$, where $1 \leq a < 10$

B $a \times 10^n$, where a is any integer

C $10 \times a^n$

D $n \times 10^a$

2. 4.567 rounded to 2 decimal places is **[1]**

A 4.5

B 4.56

C 4.57

D 4.6

3. The percentage error of an estimate is calculated as **[1]**

A $|\text{estimate} - \text{actual}| \div \text{actual} \times 100\%$

B $(\text{estimate} - \text{actual}) \times 100\%$

C $\text{actual} \div \text{estimate} \times 100\%$

D $|\text{estimate}| \div |\text{actual}|$

4. 45 000 written in standard form is **[1]**

A 4.5×10^3

B 45×10^3

C 4.5×10^4

D 0.45×10^5

5. A theatre has 20 seats in the first row, increasing by 2 seats in each subsequent row. The number of seats in row 10 is **[1]**

A 36

B 38

C 40

D 22

6. A bacteria culture doubles in size every hour, starting with 50 bacteria. The population after 4 hours is **[1]**

A 200

B 400

C 800

D 1600

7. An infinite geometric series has a finite sum (converges) only when **[1]**

A $r > 1$

B $r = 1$

C $|r| < 1$

D $r < 0$

8. The sum to infinity of a convergent geometric series with first term u_1 and common ratio r is given by **[1]**

A $S\infty = u_1 \div (1-r)$

B $S\infty = u_1(1-r)$

C $S\infty = u_1 \div r$

D $S\infty = nu_1$

9. An exponential growth model has the general form $A(t) = A_0 b^t$. In this model, A_0 represents **[1]**

A the growth rate

B the initial (starting) amount, at t=0

C the amount after 1 time period

D the time taken to double

10. A car valued at $20 000 depreciates by 15% per year. Its value after 2 years is **[1]**

A $14 000.00

B $14 450.00

C $17 000.00

D $20 300.00

11. The 'half-life' of a radioactive substance is best described as **[1]**

A the time for half of the sample to decay to zero mass

B the time taken for the mass of the substance to reduce to half its original value

C half of the total decay time of the substance

D the mass remaining after 1 year

12. Using the laws of exponents, $a^m \times a^n$ simplifies to **[1]**

A $a^{m \times n}$

B a^{m+n}

C a^{mn}

D a^{m-n}

13. The future value FV of an investment of PV, earning compound interest at rate r per period for n periods, is given by **[1]**

A $FV = PV(1+r)^n$

B $FV = PV(1+rn)$

C $FV = PV \times rn$

D $FV = PV + r^n$

14. A traveler converts $200 USD to EUR at an exchange rate of 1 USD = 0.9 EUR. The amount received, before any commission, is **[1]**

A €180.00

B €200.90

C €222.22

D €0.90

15. 'Amortization' of a loan refers to **[1]**

A a single lump-sum repayment made at the end of the loan term

B the process of paying off a loan gradually through regular payments, covering both interest and principal

C the interest rate charged on a loan

D converting a loan from one currency to another

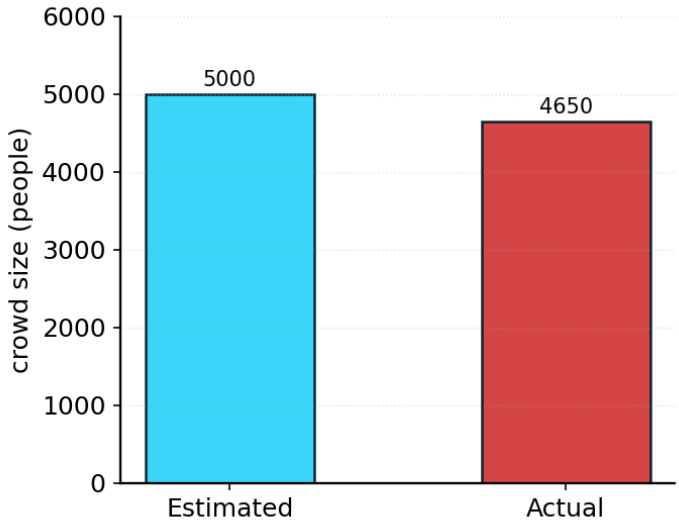
Section B — Structured Questions (65 marks)

Answer all questions in the space provided. Working must be shown for full marks.

1.1 Approximation & Estimation

Question 1 [7 marks]

A news report estimates the crowd size at a public event; a more careful count afterwards gives the actual number (given data).



Estimated vs. actual crowd size (given data).

(a) Using the bar chart, write down the estimated crowd size and the actual crowd size. **[2]**

(b) Calculate the percentage error of the estimate, giving your answer to 3 significant figures. **[2]**

(c) Round the actual crowd size to 2 significant figures. **[3]**

Question 2 [6 marks]

Rounding and standard form are used to represent numbers to an appropriate degree of accuracy.

(a) Round 3456.789 to 3 significant figures. **[2]**

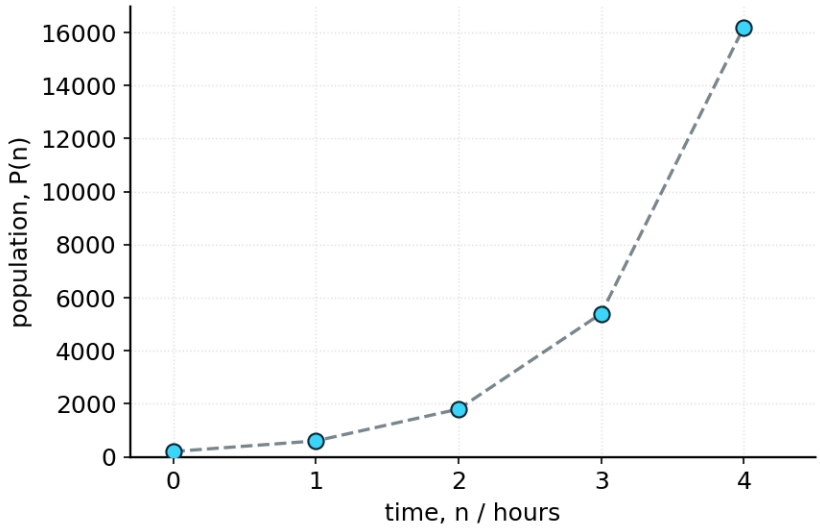
(b) Write 0.000456 in standard form. **[2]**

(c) A length is measured as 25 cm, when its true value is 24 cm. Calculate the percentage error of this measurement. **[2]**

1.2 Sequences & Series

Question 3 [7 marks]

A bacteria culture triples in size every hour. The graph shows the population, P(n), after n hours, starting with 200 bacteria (computed).



Bacteria population $P(n) = 200\times3^n$ (computed).

(a) Using the graph, write down the initial population, P(0), and the common ratio of this geometric sequence. **[2]**

(b) Using $P(n) = 200\times3^n$, calculate the population after 5 hours. **[2]**

(c) Find the smallest whole number of hours, n, after which the population first exceeds 100 000. **[3]**

Question 4 [6 marks]

A ball is dropped from a height of 10 m. Each time it bounces, it rebounds to 60% of the height of the previous bounce, forming a geometric sequence of bounce heights (first bounce height $u_1 = 6$ m).

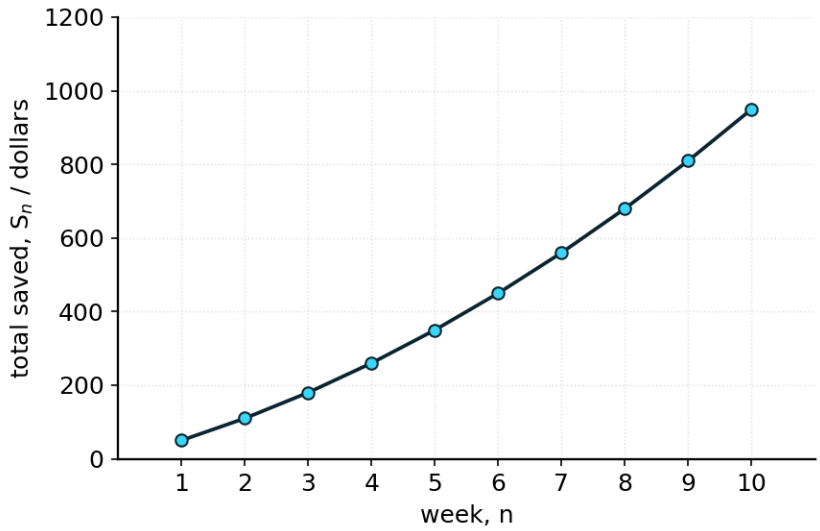
(a) State the formula for the sum to infinity, S_∞, of a convergent geometric series, and explain why this series ($r = 0.6$) converges. **[2]**

(b) Calculate the height of the 4th bounce. **[2]**

(c) Calculate S_∞ for the sequence of bounce heights, and hence find the total vertical distance travelled by the ball (including the initial 10 m drop). **[2]**

Question 5 [7 marks]

The graph shows a savings plan: \$50 is saved in week 1, increasing by \$10 each subsequent week, with the cumulative total, S_n, plotted against week number, n (computed).



Cumulative savings, S_n, u_1=\$50, d=\$10 (computed).

(a) Using the graph, estimate the total amount saved after 8 weeks. **[2]**

(b) Using $S_n = n \div 2(2u_1+(n-1)d)$, calculate the exact total saved after 8 weeks. **[2]**

(c) Find the smallest whole number of weeks, n, after which the total savings first exceed \$1000. **[3]**

1.3 Exponential Growth & Decay

Question 6 [6 marks]

The population of a town is modelled by $P(t) = 500 \times (1.08)^t$, where t is the time in years.

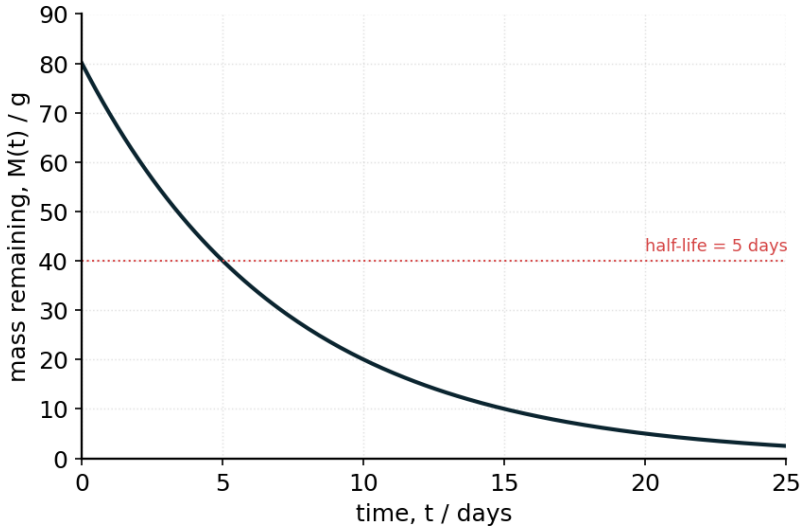
(a) State the initial population of the town (at t=0). **[2]**

(b) State the annual percentage growth rate of the population, as modelled by this function. **[2]**

(c) Calculate the population predicted by this model after 10 years, giving your answer to the nearest whole number. **[2]**

Question 7 [7 marks]

The graph shows the mass, M(t), of a radioactive sample remaining after t days, where $M(t) = 80\times(0.5)^{t/5}$ grams (computed).



Radioactive decay, $M(t) = 80\times(0.5)^{t/5}$ (computed).

(a) Using the graph, state the initial mass of the sample, and estimate the half-life of the substance. **[2]**

(b) Calculate the mass remaining after 10 days. **[2]**

(c) Find the time taken for the mass to decay to 5 grams. **[3]**

Question 8 [6 marks]

A radioactive isotope has a half-life of 20 years, and an initial mass of 100 g.

(a) State the general half-life decay formula $M(t) = M_0(0.5)^{t/h}$, and identify what h represents. **[2]**

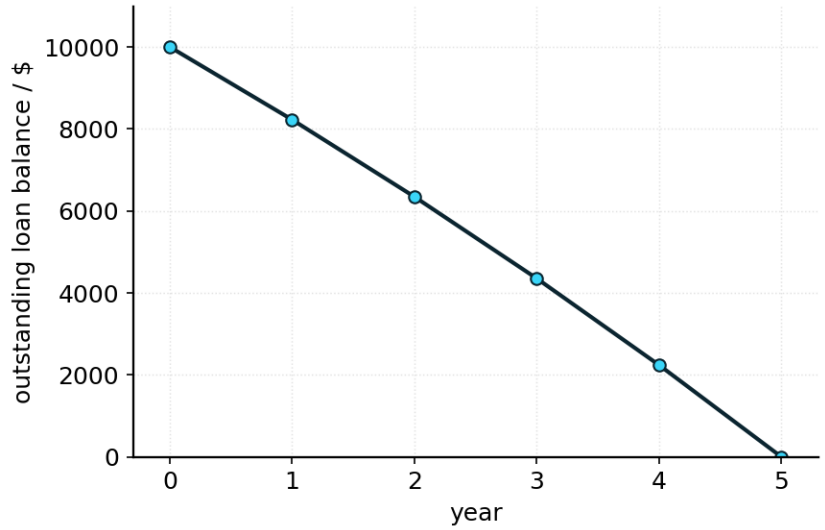
(b) Calculate the mass remaining after 60 years. **[2]**

(c) Find the number of half-lives that have passed when the mass has decayed to 6.25 g, and hence find the corresponding time. **[2]**

1.4 Financial Mathematics

Question 9 [7 marks]

A $10 000 loan is taken out at an annual interest rate of 6%, to be repaid with equal annual payments over 5 years. The graph shows the outstanding loan balance at the end of each year (computed).



Loan amortization schedule, P=$10 000, r=6%, n=5 years (computed).

(a) Using the graph, state the initial loan balance, and state after how many years the loan is fully repaid. **[2]**

(b) Explain how this 'amortization' graph differs from a simple compound-interest growth graph. **[2]**

(c) Using the amortization formula $A = Pr(1+r)^n \div ((1+r)^n - 1)$, calculate the annual payment, A, giving your answer to the nearest dollar. **[3]**

Question 10 [6 marks]

A traveler converts USD to EUR at an exchange rate of 1 USD = 0.92 EUR.

(a) Calculate the amount in EUR received for $500 USD, before any commission. **[2]**

(b) The bank charges a 2% commission on the converted amount. Calculate the actual amount of EUR received after commission. **[2]**

(c) Using the same exchange rate, calculate the equivalent USD value of a €200 purchase, giving your answer to 2 decimal places. **[2]**