IB DP MATHEMATICS

AI Topic 1 · Number and Algebra

MARK SCHEME

1.1 Approximation & Estimation · 1.2 Sequences & Series
1.3 Exponential Growth & Decay · 1.4 Financial Mathematics

Standard and Higher Level · Syllabus 2025

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Original exam-style questions written for the IB Diploma Programme Mathematics course (first assessment 2025)

Section A — Multiple Choice: Answer Key (15 marks)

1. Answer: **A** — Standard form expresses a number as $a\times10^n$, where $1 \le a < 10$ and n is an integer.

2. Answer: **C** — The third decimal digit (7) rounds the second decimal digit up, giving $4.567 \approx 4.57$ (2 d.p.).

3. Answer: **A** — Percentage error = $|\text{estimate}-\text{actual}|\div\text{actual} \times 100\%$, expressing the size of the error relative to the true (actual) value.

4. Answer: **C** — $45\,000 = 4.5 \times 10\,000 = 4.5 \times 10^4$, with $1 \le 4.5 < 10$ as required for standard form.

5. Answer: **B** — This is an arithmetic sequence with $u_1=20$, $d=2$. $u_{10} = 20+(10-1)\times2 = 20+18 = 38$.

6. Answer: **C** — This is a geometric sequence with $u_1=50$, $r=2$. Population after 4 hours = $50\times2^4 = 50\times16 = 800$.

7. Answer: **C** — An infinite geometric series converges to a finite sum only when $|r| < 1$.

8. Answer: **A** — For a convergent geometric series, $S_\infty = u_1\div(1-r)$.

9. Answer: **B** — In $A(t) = A_0b^t$, A_0 is the value of A when $t=0$, i.e. the initial amount.

10. Answer: **B** — Value = $20\,000\times(0.85)^2 = 20\,000\times0.7225$ = \$14 450.00.

11. Answer: **B** — Half-life is the time required for the mass (or amount) of a radioactive substance to decrease to half its original value.

12. Answer: **B** — When multiplying powers with the same base, the exponents are added: $a^m\times a^n = a^{m+n}$.

13. Answer: **A** — Compound interest is calculated on the accumulated balance each period, giving the exponential formula $FV = PV(1+r)^n$.

14. Answer: **A** — Amount in EUR = 200×0.9 = €180.00.

15. Answer: **B** — Amortization is the process of paying off a loan over time through a series of regular payments, each of which covers part of the interest owed and part of the principal.

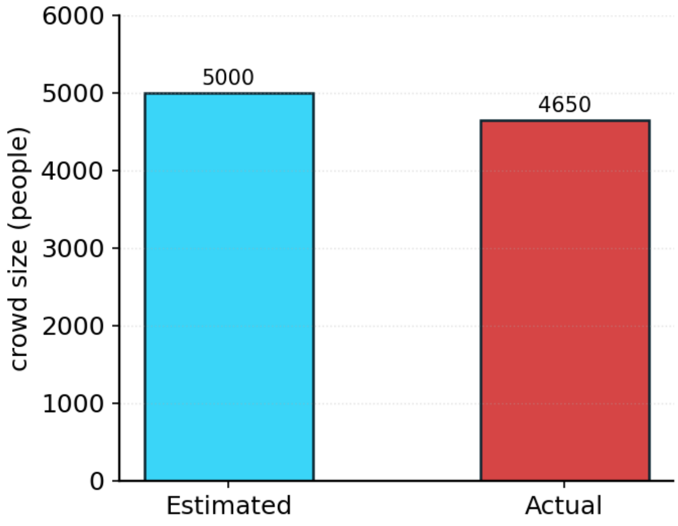
Section B — Structured Questions: Worked Solutions (65 marks)

Mark-scheme notation: M = method mark, A = accuracy mark (dependent on the preceding M mark), R = reasoning mark.

1.1 Approximation & Estimation

Question 1 [7 marks]

A news report estimates the crowd size at a public event; a more careful count afterwards gives the actual number (given data).



Estimated vs. actual crowd size (given data).

(a) Using the bar chart, write down the estimated crowd size and the actual crowd size. **[2]**

Estimated crowd size = **5000**; actual crowd size = **4650**. **A1 A1**

(b) Calculate the percentage error of the estimate, giving your answer to 3 significant figures. **[2]**

Percentage error = $|5000-4650| \div 4650 \times 100\%$ = **7.53%**. **M1 A1**

(c) Round the actual crowd size to 2 significant figures. **[3]**

The first 2 significant figures of 4650 are 4 and 6; the next digit is 5, which rounds the 6 up to 7. So 4650 rounded to 2 significant figures is **4700**. **M1 A1 A1**

Question 2 [6 marks]

Rounding and standard form are used to represent numbers to an appropriate degree of accuracy.

(a) Round 3456.789 to 3 significant figures. **[2]**

3456.789 rounded to 3 significant figures is **3460** (the 4th significant digit, 6, rounds the 3rd digit up from 5 to 6). **M1 A1**

(b) Write 0.000456 in standard form. **[2]**

0.000456 = $\mathbf{4.56 \times 10^{-4}}$. **M1 A1**

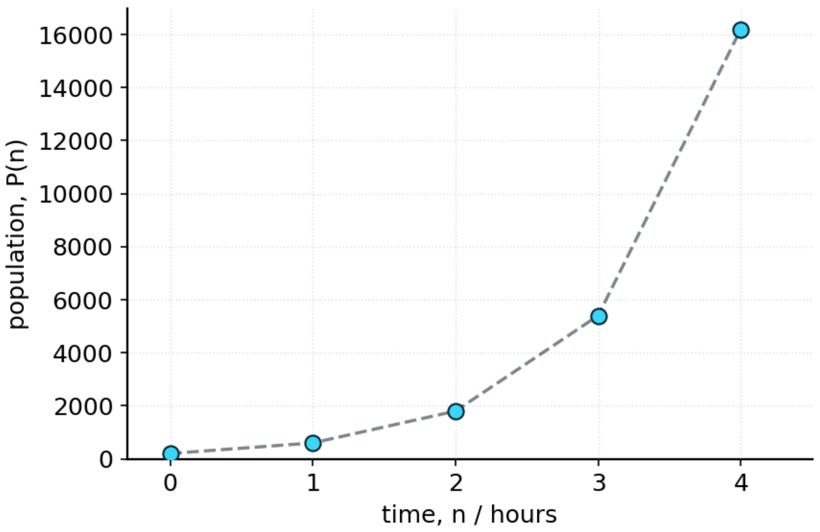
(c) A length is measured as 25 cm, when its true value is 24 cm. Calculate the percentage error of this measurement. **[2]**

Percentage error = $|25-24| \div 24 \times 100\% = 1 \div 24 \times 100\%$ = **4.17%** (3 s.f.). **M1 A1**

1.2 Sequences & Series

Question 3 [7 marks]

A bacteria culture triples in size every hour. The graph shows the population, P(n), after n hours, starting with 200 bacteria (computed).



Bacteria population $P(n) = 200\times3^n$ (computed).

(a) Using the graph, write down the initial population, P(0), and the common ratio of this geometric sequence. **[2]**

P(0) = **200**; common ratio = **3** (the population triples every hour). **A1 A1**

(b) Using $P(n) = 200\times3^n$, calculate the population after 5 hours. **[2]**

$P(5) = 200\times3^5 = 200\times243 =$ **48600** bacteria. **M1 A1**

(c) Find the smallest whole number of hours, n, after which the population first exceeds 100 000. **[3]**

Testing values: P(5) = 48600 (≤100 000), $P(6) = 200\times3^6$ = **145800** (>100 000). So the population first exceeds 100 000 after **n = 6** hours. **M1 M1 A1**

Question 4 [6 marks]

A ball is dropped from a height of 10 m. Each time it bounces, it rebounds to 60% of the height of the previous bounce, forming a geometric sequence of bounce heights (first bounce height $u_1 = 6$ m).

(a) State the formula for the sum to infinity, S∞, of a convergent geometric series, and explain why this series (r = 0.6) converges. **[2]**

S∞ = $\mathbf{u_1 \div (1-r)}$. This series converges because r = 0.6, and |0.6| < 1. **A1 R1**

(b) Calculate the height of the 4th bounce. **[2]**

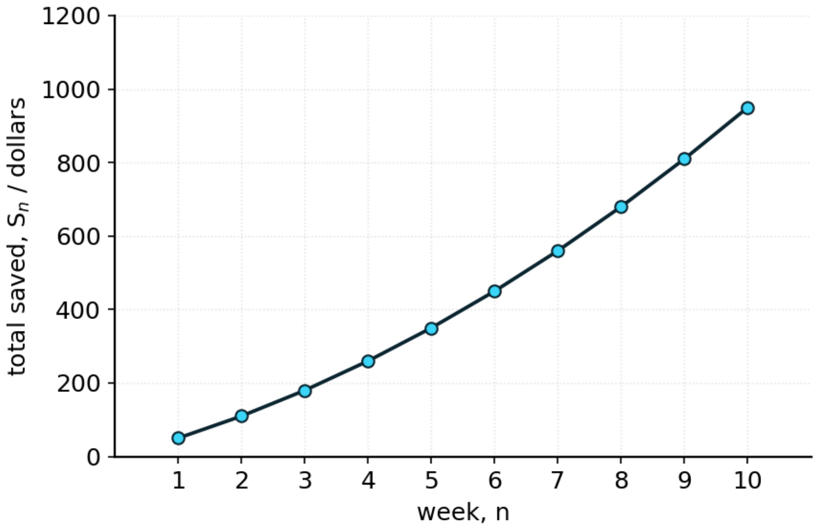
$u_4 = u_1 r^3 = 6\times(0.6)^3 = 6\times0.216 =$ **1.3 m**. **M1 A1**

(c) Calculate S∞ for the sequence of bounce heights, and hence find the total vertical distance travelled by the ball (including the initial 10 m drop). **[2]**

$S_\infty = 6\div(1-0.6) = 6\div0.4 =$ **15.0 m**. Total distance = initial drop + $2\times S_\infty$ (each bounce height is travelled up and down) = $10 + 2\times15 =$ **40.0 m**. **M1 M1**

Question 5 [7 marks]

The graph shows a savings plan: $50 is saved in week 1, increasing by $10 each subsequent week, with the cumulative total, S_n, plotted against week number, n (computed).



Cumulative savings, S_n, u_1=$50, d=$10 (computed).

(a) Using the graph, estimate the total amount saved after 8 weeks. **[2]**

From the graph, the total saved after 8 weeks is approximately **$680**. **A1 A1**

(b) Using $S_n = n\div2(2u_1+(n-1)d)$, calculate the exact total saved after 8 weeks. **[2]**

$S_8 = 8\div2(2\times50+(8-1)\times10) = 4(100+70) = 4\times170 =$ **$680**, confirming part (a). **M1 A1**

(c) Find the smallest whole number of weeks, n, after which the total savings first exceed $1000. **[3]**

Testing values: S_{10} = $950 (≤1000), $S_{11} = 11\div2(100+100) =$ **$1100** (>1000). So savings first exceed $1000 after **n = 11** weeks. **M1 M1 A1**

1.3 Exponential Growth & Decay

Question 6 [6 marks]

The population of a town is modelled by $P(t) = 500\times(1.08)^t$, where t is the time in years.

(a) State the initial population of the town (at t=0). **[2]**

Initial population = $P(0) = 500\times(1.08)^0 =$ **500**. **A1 A1**

(b) State the annual percentage growth rate of the population, as modelled by this function. **[2]**

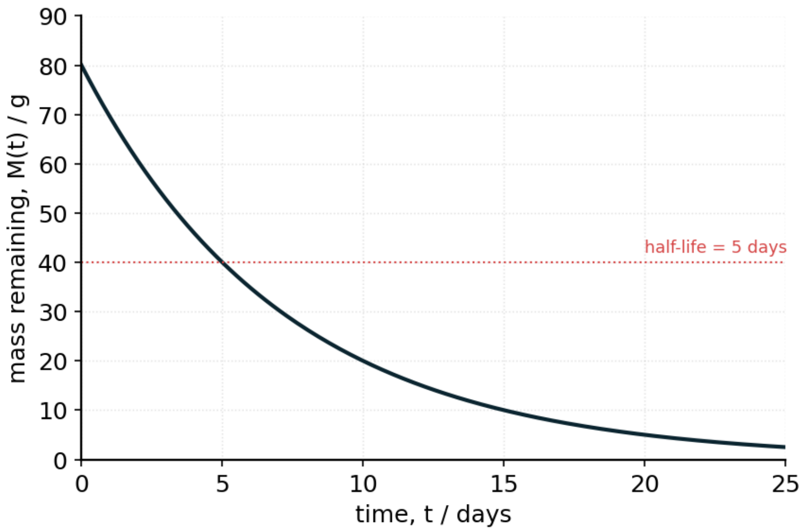
Since the base is 1.08 = 1+0.08, the annual growth rate is **8%** per year. **A1 A1**

(c) Calculate the population predicted by this model after 10 years, giving your answer to the nearest whole number. **[2]**

$P(10) = 500\times(1.08)^{10}$ = **1079** (to the nearest whole number). **M1 A1**

Question 7 [7 marks]

The graph shows the mass, M(t), of a radioactive sample remaining after t days, where $M(t) = 80\times(0.5)^{t/5}$ grams (computed).



Radioactive decay, $M(t) = 80\times(0.5)^{t/5}$ (computed).

(a) Using the graph, state the initial mass of the sample, and estimate the half-life of the substance. **[2]**

Initial mass = **80 g** (at t=0). The half-life (time for the mass to halve, e.g. from 80g to 40g) is approximately **5 days**. **A1 A1**

(b) Calculate the mass remaining after 10 days. **[2]**

$M(10) = 80\times(0.5)^{10/5} = 80\times(0.5)^2 = 80\times0.25$ = **20 g**. **M1 A1**

(c) Find the time taken for the mass to decay to 5 grams. **[3]**

$5 = 80\times(0.5)^{t/5} \Rightarrow (0.5)^{t/5} = 5\div80 = 1\div16 = (0.5)^4 \Rightarrow t\div5 = 4 \Rightarrow$ **t = 20 days**. **M1 M1 A1**

Question 8 [6 marks]

A radioactive isotope has a half-life of 20 years, and an initial mass of 100 g.

(a) State the general half-life decay formula $M(t) = M_0(0.5)^{t/h}$, and identify what h represents. **[2]**

$\mathbf{M(t) = M_0(0.5)^{t/h}}$, where h is the **half-life** (the time for the mass to reduce to half its value). **A1 A1**

(b) Calculate the mass remaining after 60 years. **[2]**

$M(60) = 100\times(0.5)^{60/20} = 100\times(0.5)^3 = 100\times0.125$ = **12.5 g**. **M1 A1**

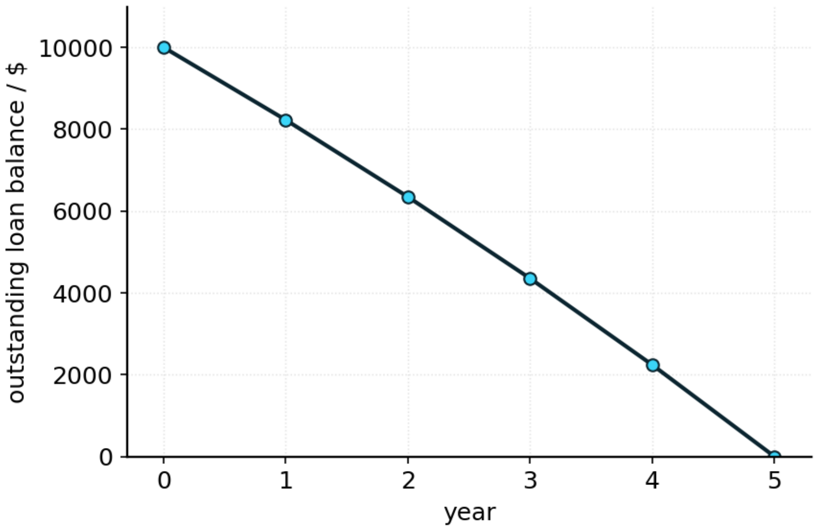
(c) Find the number of half-lives that have passed when the mass has decayed to 6.25 g, and hence find the corresponding time. **[2]**

$100\times(0.5)^n = 6.25 \Rightarrow (0.5)^n = 0.0625 = (0.5)^4 \Rightarrow$ **n = 4 half-lives**, corresponding to a time of 4×20 = **80 years**. **M1 A1**

1.4 Financial Mathematics

Question 9 [7 marks]

A $10 000 loan is taken out at an annual interest rate of 6%, to be repaid with equal annual payments over 5 years. The graph shows the outstanding loan balance at the end of each year (computed).



Loan amortization schedule, P=$10 000, r=6%, n=5 years (computed).

(a) Using the graph, state the initial loan balance, and state after how many years the loan is fully repaid. **[2]**

Initial loan balance = **$10 000**; the loan is fully repaid (balance reaches $0) after **5 years**. **A1 A1**

(b) Explain how this 'amortization' graph differs from a simple compound-interest growth graph. **[2]**

In an amortization schedule, the balance **decreases** over time (towards zero) because regular payments are made that exceed the interest charged each period; in simple compound-interest growth, the balance **increases** over time because interest is added and no withdrawals or payments are made. **R1 R1**

(c) Using the amortization formula $A = Pr(1+r)^n\div((1+r)^n-1)$, calculate the annual payment, A, giving your answer to the nearest dollar. **[3]**

$A = 10000\times0.06\times(1.06)^5\div((1.06)^5-1)$ = **$2374** per year (to the nearest dollar). **M1 M1 A1**

Question 10 [6 marks]

A traveler converts USD to EUR at an exchange rate of 1 USD = 0.92 EUR.

(a) Calculate the amount in EUR received for $500 USD, before any commission. **[2]**

Amount = 500×0.92 = **€460.00**. **M1 A1**

(b) The bank charges a 2% commission on the converted amount. Calculate the actual amount of EUR received after commission. **[2]**

Amount after commission = $460.00 \times (1 - 0.02) = 460.00 \times 0.98$ = **€450.80**. **M1 A1**

(c) Using the same exchange rate, calculate the equivalent USD value of a €200 purchase, giving your answer to 2 decimal places. **[2]**

USD equivalent = $200 \div 0.92$ = **$217.39**. **M1 A1**