IB DP MATHEMATICS

AA Topic 5 · Calculus

MARK SCHEME

5.1 Differentiation Basics · 5.2 Applications of Differentiation
5.3 Integration Basics · 5.4 Applications of Integration

Standard and Higher Level · Syllabus 2025

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Original exam-style questions written for the IB Diploma Programme Mathematics course (first assessment 2025)

Section A — Multiple Choice: Answer Key (15 marks)

1. Answer: **C** — By the power rule, $d \div dx(x^n) = nx^{n-1}$.

2. Answer: **B** — $f'(x) = 3 \times 5x^2 = 15x^2$, using the power rule.

3. Answer: **B** — $dy \div dx = 2x$; at $x = 3$, gradient $= 2(3) = 6$.

4. Answer: **C** — A constant function has a horizontal graph (zero gradient everywhere), so its derivative is 0.

5. Answer: **C** — A stationary point occurs where the gradient of the curve is zero, i.e. $f'(x) = 0$.

6. Answer: **B** — If $f''(x) > 0$ at a stationary point, the curve is concave up there, so the point is a local minimum.

7. Answer: **C** — For two perpendicular lines with gradients m_1 and m_2, $m_1 \times m_2 = -1$.

8. Answer: **C** — The derivative $dy \div dx$ gives the instantaneous rate of change of y with respect to x, i.e. the gradient of the curve at a point.

9. Answer: **B** — The power rule for integration states $\int x^n dx = x^{n+1} \div (n+1) + C$, for $n \neq -1$.

10. Answer: **B** — $\int 6x\, dx = 6 \times x^2 \div 2 + C = 3x^2 + C$.

11. Answer: **B** — A definite integral of a non-negative function represents the area enclosed between the curve and the x-axis over the given interval.

12. Answer: **B** — Integrating a constant, k, with respect to x gives $\int k\, dx = kx + C$; here $\int 5\, dx = 5x + C$.

13. Answer: **B** — The area between two curves is found by integrating the difference between the upper and lower functions: $\int_a^b (f(x) - g(x))\, dx$.

14. Answer: **A** — Velocity is the rate of change of displacement with respect to time, so $v(t) = ds \div dt$ (the derivative of displacement).

15. Answer: **B** — Since $v(t) = ds \div dt$, displacement is the antiderivative (integral) of velocity: $s(t) = \int v(t)\, dt$.

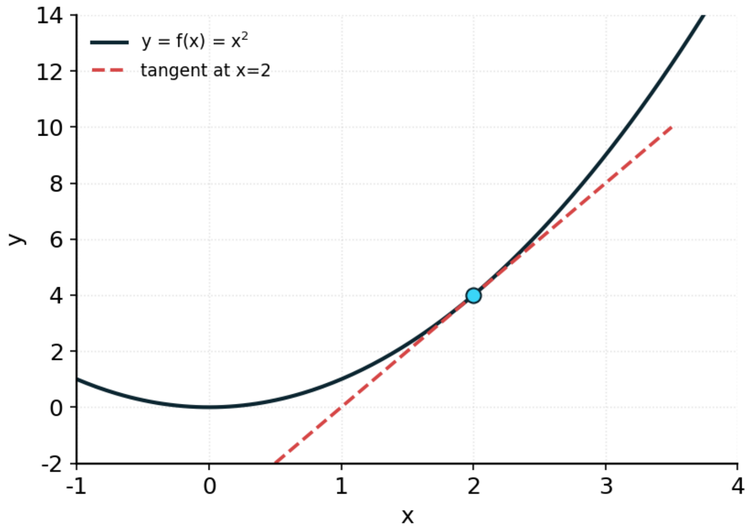
Section B — Structured Questions: Worked Solutions (65 marks)

Mark-scheme notation: M = method mark, A = accuracy mark (dependent on the preceding M mark), R = reasoning mark.

5.1 Differentiation Basics

Question 1 [7 marks]

The graph shows the curve $y = f(x) = x^2$, together with the tangent line at the point where $x = 2$ (computed).



$y = x^2$ with the tangent line at $x = 2$ (computed).

(a) Using the graph, estimate the gradient of the tangent line shown. **[2]**

Reading two points on the tangent line (e.g. (2,4) and (3,8)), the gradient is approximately **4** (the tangent rises 4 units for every 1 unit across). **A1 A1**

(b) Using the power rule, differentiate $f(x) = x^2$, and hence calculate the exact gradient of the tangent at $x = 2$. **[2]**

$f'(x) = \mathbf{2x}$. At $x=2$: $f'(2) = 2(2) = \mathbf{4}$, confirming the graphical estimate in part (a). **M1 A1**

(c) Find the equation of the tangent line to $y = x^2$ at $x = 2$, and confirm it matches the line shown on the graph. **[3]**

At $x=2$, $y = f(2) = \mathbf{4}$, and the gradient is 4 (from part (b)). Using $y-y_1 = m(x-x_1)$: $y-4 = 4(x-2) \Rightarrow \mathbf{y = 4x-4}$, which matches the tangent line shown. **M1 M1 A1**

Question 2 [6 marks]

Polynomial functions can be differentiated term-by-term using the power rule.

(a) State the power rule for differentiating x^n with respect to x. **[2]**

$\mathbf{d \div dx(x^n) = nx^{n-1}}$. **A1 A1**

(b) Differentiate $f(x) = 3x^4-5x^2+7$ with respect to x. **[2]**

$f'(x) = \mathbf{12x^3 - 10x}$ (differentiating term-by-term; the derivative of the constant 7 is 0). **M1 A1**

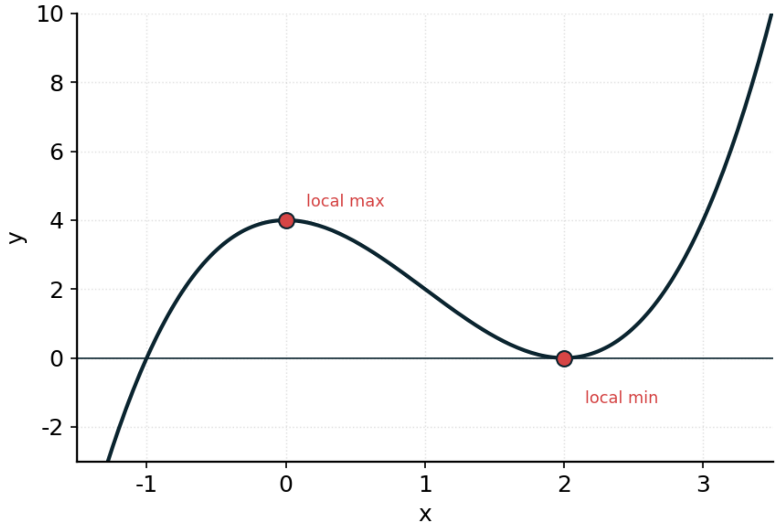
(c) Calculate $f'(1)$. **[2]**

$f'(1) = 12(1)^3 - 10(1) = 12 - 10 = \mathbf{2}$. **M1 A1**

5.2 Applications of Differentiation

Question 3 [7 marks]

The graph shows the curve $y = f(x) = x^3 - 3x^2 + 4$, with its two stationary points marked (computed).



$y = x^3 - 3x^2 + 4$ (computed), with stationary points marked.

(a) Using the graph, write down the x-coordinate of the local maximum and the x-coordinate of the local minimum. **[2]**

Local maximum at $\mathbf{x = 0}$; local minimum at $\mathbf{x = 2}$. **A1 A1**

(b) Find $f'(x)$, and hence find the x-coordinates of the stationary points algebraically. **[2]**

$f'(x) = \mathbf{3x^2 - 6x}$. Setting $f'(x)=0$: $3x(x-2)=0 \Rightarrow$ **$x = 0$ or $x = 2$**, confirming part (a). **M1 A1**

(c) Find $f''(x)$, and use it to classify each stationary point found in part (b). **[3]**

$f''(x) = \mathbf{6x - 6}$. At $x=0$: $f''(0) = -6 < 0$, so this is a **local maximum**. At $x=2$: $f''(2) = 6 > 0$, so this is a **local minimum**, confirming the graph. **M1 A1 A1**

Question 4 [6 marks]

The tangent and normal to a curve at a point are perpendicular to each other.

(a) State the relationship between the gradient of the tangent and the gradient of the normal to a curve at a given point. **[2]**

The gradient of the normal is the **negative reciprocal** of the gradient of the tangent; if the tangent has gradient m, the normal has gradient $\mathbf{-1 \div m}$ (their product is -1). **A1 A1**

(b) For the curve $y = x^3$ at the point (1, 1), find the gradient of the tangent. **[2]**

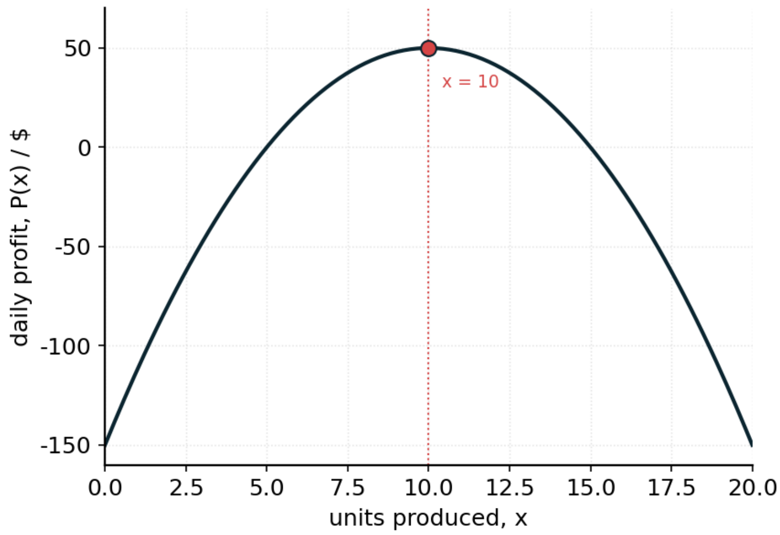
$dy \div dx = 3x^2$. At $x=1$: gradient $= 3(1)^2 =$ **3**. **M1 A1**

(c) Hence find the gradient of the normal to the curve at the point (1, 1). **[2]**

Gradient of normal $= -1 \div 3 =$ **−1/3** (using the result from part (b)). **M1 A1**

Question 5 [7 marks]

The graph shows the daily profit, $P(x)$ (in dollars), of a company producing x units of a product, where $P(x) = -2x^2+40x-150$ (computed).



$P(x) = -2x^2+40x-150$ (computed), showing the maximum profit.

(a) Using the graph, estimate the number of units, x, that maximizes the daily profit. **[2]**

From the graph, the maximum profit occurs at approximately **x = 10** units. **A1 A1**

(b) Find $P'(x)$, and hence find the exact value of x that maximizes the profit. **[2]**

$P'(x) =$ **$-4x+40$**. Setting $P'(x)=0$: $-4x+40=0 \Rightarrow$ **x = 10**, confirming part (a). **M1 A1**

(c) Calculate the maximum daily profit. **[3]**

$P(10) = -2(10)^2+40(10)-150 = -200+400-150 =$ **$50**. **M1 M1 A1**

5.3 Integration Basics

Question 6 [6 marks]

Integration is the reverse process of differentiation, and follows its own power rule.

(a) State the power rule for integration, $\int x^n dx$ (for $n \neq -1$). **[2]**

$\int x^n dx = x^{n+1} \div (n+1) + C$. **A1 A1**

(b) Find $\int(3x^2-4x+5)dx$. **[2]**

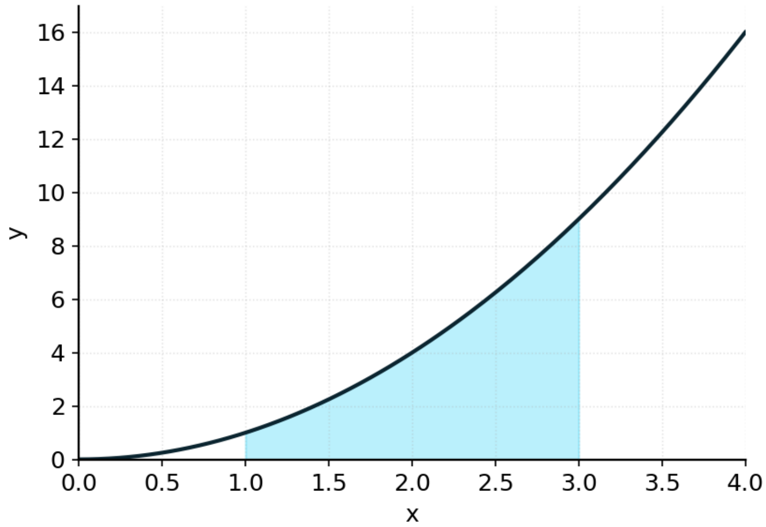
$\int(3x^2-4x+5)dx =$ **$x^3-2x^2+5x + C$**. **M1 A1**

(c) Evaluate the definite integral $\int_0^2(3x^2-4x+5)dx$. **[2]**

$[x^3-2x^2+5x]_0^2 = (8-8+10)-(0) = \mathbf{10}$. **M1 A1**

Question 7 [7 marks]

The graph shows the curve $y = x^2$, with the region between $x = 1$ and $x = 3$ shaded (computed).



$y = x^2$ (computed), with the shaded region between x=1 and x=3.

(a) State what the shaded area on the graph represents, in terms of a definite integral. **[2]**

The shaded area represents the definite integral $\int_1^3 \mathbf{x^2dx}$ (the area between the curve and the x-axis from x=1 to x=3). **A1 A1**

(b) Find the indefinite integral $\int x^2dx$. **[2]**

$\int x^2dx = \mathbf{x^3 \div 3 + C}$. **A1 A1**

(c) Calculate the exact value of the shaded area, giving your answer as an exact fraction. **[3]**

$[x^3\div 3]_1^3 = 27\div 3-1\div 3 = 9-1\div 3 = \mathbf{26/3}$ (square units). **M1 M1 A1**

Question 8 [6 marks]

A curve has gradient function $dy\div dx = 6x-4$, and passes through the point (1, 3).

(a) Explain what the constant of integration, C, represents when finding y from $dy\div dx$ by integration. **[2]**

C represents an **unknown constant**, since integrating $dy\div dx$ only recovers y up to an added constant; C accounts for the **vertical position** of the curve, and is determined using a known point on the curve. **R1 R1**

(b) Find the equation of the curve $y = f(x)$, using the given point to find the value of C. **[2]**

$y = \int(6x-4)dx = 3x^2-4x + C$. Since the curve passes through (1,3): $3 = 3(1)^2-4(1)+C = -1+C \Rightarrow C=\mathbf{4}$. So $\mathbf{y = 3x^2-4x+4}$. **M1 A1**

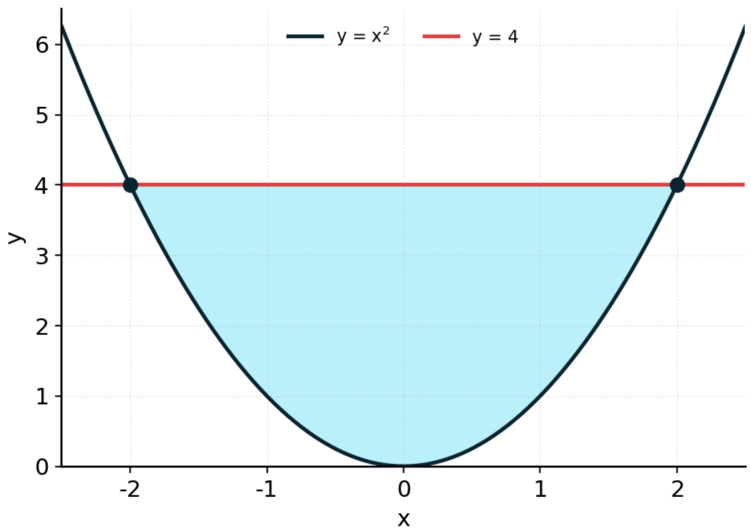
(c) Find the value of y when x = 2. **[2]**

$y = 3(2)^2-4(2)+4 = 12-8+4 = \mathbf{8}$. **M1 A1**

5.4 Applications of Integration

Question 9 [7 marks]

The graph shows the region enclosed between the curve $y = x^2$ and the line $y = 4$ (computed).



Region enclosed between $y = x^2$ and $y = 4$ (computed).

(a) Using the graph, write down the x-coordinates of the points where the curve and the line intersect. **[2]**

The curve and line intersect where $x^2=4$, i.e. at $\mathbf{x = -2}$ and $\mathbf{x = 2}$, matching the graph. **A1 A1**

(b) State the formula used to find the area between two curves $y = f(x)$ (upper) and $y = g(x)$ (lower), for $a \le x \le b$. **[2]**

Area = $\boldsymbol{\int_a^b (f(x)-g(x))dx}$. **A1 A1**

(c) Calculate the exact area of the shaded region, using $\int_{-2}^{2}(4-x^2)dx$. **[3]**

$[4x-x^3\div3]_{-2}^{2} = (8-8/3)-(-8+8/3) = (16/3)-(-16/3) = \mathbf{32/3}$ (square units). **M1 M1 A1**

Question 10 [6 marks]

A particle moves in a straight line with velocity $v(t) = 6t-t^2$ (m/s), and starts at displacement $s(0) = 0$.

(a) State the relationship between displacement $s(t)$, velocity $v(t)$, and acceleration $a(t)$ in terms of derivatives and integrals. **[2]**

$v(t) = \mathbf{ds\div dt}$ (velocity is the derivative of displacement); $a(t) = \mathbf{dv\div dt}$ (acceleration is the derivative of velocity); equivalently, $s(t) = \int v(t)dt$ and $v(t) = \int a(t)dt$. **A1 A1**

(b) Find the displacement function $s(t)$, using the given initial condition $s(0) = 0$. **[2]**

$s(t) = \int(6t-t^2)dt = 3t^2-t^3\div3 + C$. Since $s(0)=0$: $C=0$. So $\mathbf{s(t) = 3t^2-t^3\div3}$. **M1 A1**

(c) Calculate the displacement of the particle at $t = 3$ seconds. **[2]**

$s(3) = 3(3)^2-(3)^3\div3 = 27-9 = \mathbf{18\ m}$. **M1 A1**