IB DP MATHEMATICS

AA Topic 3 · Geometry and Trigonometry

MARK SCHEME

3.1 Right-Angled Trigonometry · 3.2 Sine & Cosine Rules
3.3 Radian Measure & Circular Functions · 3.4 Trig Graphs & Equations

Standard and Higher Level · Syllabus 2025

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Original exam-style questions written for the IB Diploma Programme Mathematics course (first assessment 2025)

Section A — Multiple Choice: Answer Key (15 marks)

1. Answer: **C** — By definition (SOH-CAH-TOA), $\tan\theta = \text{opposite} \div \text{adjacent}$.

2. Answer: **A** — By Pythagoras' theorem, hypotenuse = $\sqrt{(6^2+8^2)} = \sqrt{(36+64)} = \sqrt{100} = 10$.

3. Answer: **C** — $\sin\theta = \text{opposite} \div \text{hypotenuse} = 3 \div 5 = 0.6$.

4. Answer: **B** — Since the angles of a triangle sum to 180° and one angle is 90°, the remaining two (acute) angles must sum to $180° - 90° = 90°$.

5. Answer: **A** — The cosine rule is $a^2 = b^2+c^2 - 2bc\cos A$, a generalisation of Pythagoras' theorem for any triangle.

6. Answer: **B** — The sine rule states $a \div \sin A = b \div \sin B = c \div \sin C$, relating each side to the sine of its opposite angle.

7. Answer: **C** — The area of a triangle can be calculated as Area = $\frac{1}{2}ab\sin C$, using two sides and the included angle.

8. Answer: **B** — When two angles and one side are known (AAS), all three angles can be found (angles sum to 180°), and the sine rule directly relates the known side to the unknown side via their opposite angles.

9. Answer: **B** — A full circle corresponds to an angle of 2π radians, equivalent to 360°.

10. Answer: **A** — The arc length formula is $s = r\theta$, where θ is measured in radians.

11. Answer: **C** — The sector area formula is $A = \frac{1}{2}r^2\theta$, where θ is measured in radians.

12. Answer: **D** — Since $180° = \pi$ radians, $90° = \pi \div 2$ radians.

13. Answer: **C** — For $y = a\sin(bx) + d$, the amplitude is $|a|$; here $a = 3$, so the amplitude is 3.

14. Answer: **B** — For $y = a\sin(bx) + d$, the period is $2\pi \div b$; here $b = 2$, so the period is $2\pi \div 2 = \pi$.

15. Answer: **B** — For $y = a\sin(bx) + d$, the principal axis is $y = d$; here $d = 1$, so the principal axis is $y = 1$.

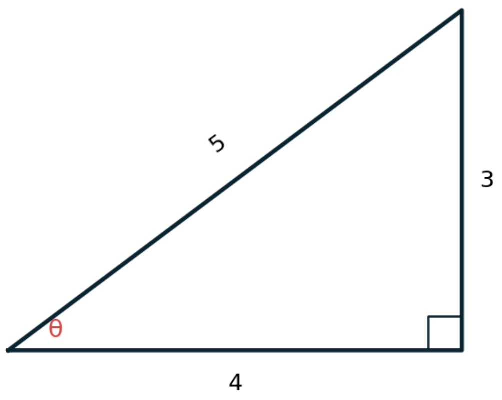
Section B — Structured Questions: Worked Solutions (65 marks)

Mark-scheme notation: M = method mark, A = accuracy mark (dependent on the preceding M mark), R = reasoning mark.

3.1 Right-Angled Trigonometry

Question 1 [7 marks]

The diagram shows a right-angled triangle with base 4, height 3, and hypotenuse 5 (computed, a 3-4-5 triangle).



A right-angled triangle with sides 3, 4, and 5 (computed).

(a) Using the diagram, identify the hypotenuse of the triangle, and state which side is opposite angle θ. **[2]**

The hypotenuse (the longest side, opposite the right angle) has length **5**. The side **opposite θ has length 3** (the vertical side). **A1 A1**

(b) Using tanθ = opposite÷adjacent, calculate θ, giving your answer to 1 decimal place. **[2]**

$\tan\theta = 3 \div 4 = 0.75 \Rightarrow \theta = \tan^{-1}(0.75) =$ **36.9°**. **M1 A1**

(c) Verify your answer to part (b) using sinθ = opposite÷hypotenuse instead. **[3]**

$\sin\theta = 3 \div 5 = 0.6 \Rightarrow \theta = \sin^{-1}(0.6) =$ **36.9°**, which **matches the answer to part (b)**, as expected. **M1 M1 A1**

Question 2 [6 marks]

Pythagoras' theorem and the basic trigonometric ratios apply to any right-angled triangle.

(a) State Pythagoras' theorem for a right-angled triangle with legs a and b and hypotenuse c. **[2]**

$\mathbf{c^2 = a^2 + b^2}$. **A1 A1**

(b) A right-angled triangle has legs of length 6 and 8. Calculate the length of the hypotenuse. **[2]**

$c = \sqrt{(6^2+8^2)} = \sqrt{(36+64)} = \sqrt{100} =$ **10**. **M1 A1**

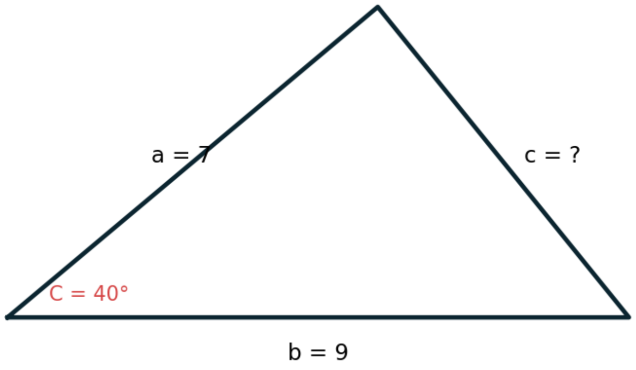
(c) For this triangle, calculate the angle opposite the side of length 8. **[2]**

$\sin\theta = 8 \div 10 = 0.8 \Rightarrow \theta = \sin^{-1}(0.8) =$ **53.1°**. **M1 A1**

3.2 Sine & Cosine Rules

Question 3 [7 marks]

The diagram shows a triangle with sides a = 7, b = 9, and included angle C = 40° (computed, SAS case).



Triangle with a = 7, b = 9, C = 40° (computed).

(a) Using the diagram, state which rule (sine or cosine) is appropriate for finding side c, and explain why. **[2]**

The **cosine rule** is appropriate, since two sides (a and b) and the **included angle (C)** are known (an SAS case). **A1 R1**

(b) State the cosine rule used to find side c in terms of a, b, and C. **[2]**

$\mathbf{c^2 = a^2+b^2-2ab\cos C}$. **A1 A1**

(c) Calculate the length of side c, giving your answer to 3 significant figures. **[3]**

$c^2 = 7^2+9^2-2(7)(9)\cos 40° = 49+81-126(0.766) = 130-96.52 = 33.48$. $c = \sqrt{33.48} =$ **5.79**. **M1 M1 A1**

Question 4 [6 marks]

A triangle has a = 10, A = 30°, and B = 70°.

(a) State the sine rule relating sides a and b to angles A and B. **[2]**

$\mathbf{a \div \sin A = b \div \sin B}$. **A1 A1**

(b) Using the sine rule, calculate the length of side b, giving your answer to 3 significant figures. **[2]**

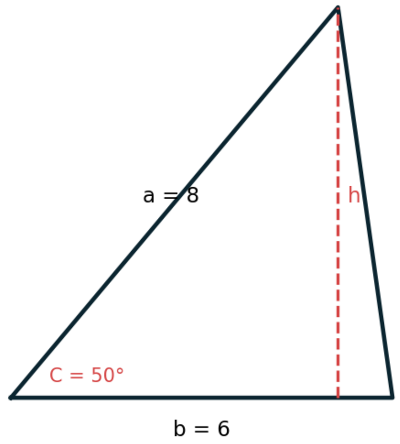
$b = a\sin B \div \sin A = 10 \times \sin 70° \div \sin 30° = 10 \times 0.9397 \div 0.5 =$ **18.8**. **M1 A1**

(c) Calculate the size of angle C. **[2]**

Since the angles of a triangle sum to 180°: $C = 180° - 30° - 70° =$ **80°**. **M1 A1**

Question 5 [7 marks]

The diagram shows a triangle with sides $a = 8$, $b = 6$, and included angle $C = 50°$ (computed), with the perpendicular height h from the vertex opposite b shown.



Triangle with a = 8, b = 6, C = 50°, showing the height h (computed).

(a) Using the diagram, state the formula for the area of this triangle in terms of a, b, and C. **[2]**

Area = ½**absinC**. **A1 A1**

(b) Calculate the area of the triangle, giving your answer to 3 significant figures. **[2]**

Area = ½(8)(6)sin50° = 24×0.766 = **18.4** (square units). **M1 A1**

(c) Calculate the height h shown on the diagram, and hence verify your answer to part (b) using Area = ½ × base × height. **[3]**

h = asinC = 8sin50° = **6.13**. Area = ½(6)(6.13) = **18.4**, which **matches part (b)**. **M1 M1 A1**

3.3 Radian Measure & Circular Functions

Question 6 [6 marks]

Angles can be measured in degrees or in radians; the arc length formula uses radian measure.

(a) Convert 60° to radians, giving your answer as an exact multiple of π. **[2]**

60° = 60÷180 × π = π÷**3** radians. **M1 A1**

(b) State the formula for the arc length s subtended by an angle θ (in radians) in a circle of radius r. **[2]**

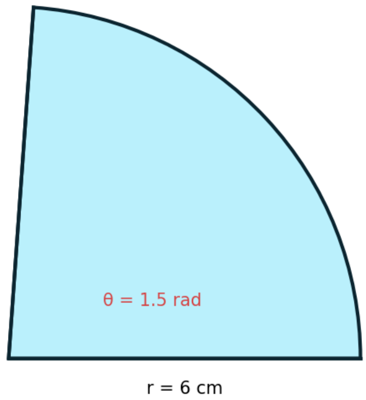
s = **r**θ. **A1 A1**

(c) Calculate the arc length in a circle of radius 5 cm, subtended by an angle of 1.2 radians. **[2]**

$s = r\theta = 5 \times 1.2 =$ **6 cm**. **M1 A1**

Question 7 [7 marks]

The diagram shows a sector of a circle with radius r = 6 cm and angle θ = 1.5 radians (computed).



A circular sector, r = 6 cm, θ = 1.5 rad (computed).

(a) Using $s = r\theta$, calculate the arc length of this sector. **[2]**

$s = r\theta = 6 \times 1.5 =$ **9 cm**. **M1 A1**

(b) State the formula for the area of a sector, in terms of r and θ. **[2]**

$A = \frac{1}{2}\mathbf{r^2}\theta$. **A1 A1**

(c) Calculate the area of the sector shown in the diagram. **[3]**

$A = \frac{1}{2}(6)^2(1.5) = \frac{1}{2}(36)(1.5) =$ **27 cm^2**. **M1 M1 A1**

Question 8 [6 marks]

Radians and degrees are related by 180° = π radians.

(a) State the relationship between degrees and radians for a full circle (360°). **[2]**

360° = **2π radians**. **A1 A1**

(b) Convert 5π÷6 radians to degrees. **[2]**

5π÷6 radians = 5÷6 × 180° = **150°**. **M1 A1**

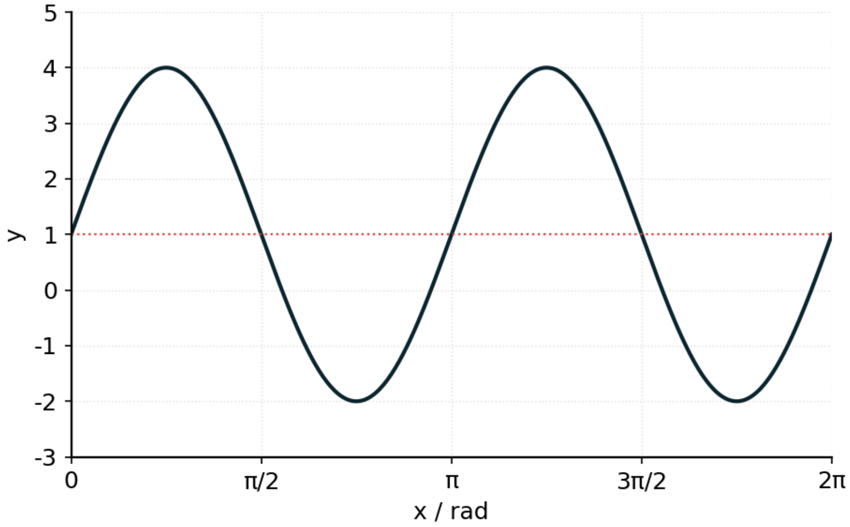
(c) A sector has area 40 cm^2 and radius 8 cm. Find the angle θ, in radians, that this sector subtends. **[2]**

$A = \frac{1}{2}r^2\theta \Rightarrow \theta = 2A \div r^2 = 2(40) \div 8^2 = 80 \div 64 =$ **1.25 rad**. **M1 A1**

3.4 Trig Graphs & Equations

Question 9 [7 marks]

The graph shows the function $y = 3\sin(2x) + 1$, for $0 \le x \le 2\pi$ (computed).



y = 3sin(2x) + 1 for 0 ≤ x ≤ 2π (computed).

(a) Using the graph, state the amplitude and the period of this function. **[2]**

The amplitude is **3** and the period is π (the graph completes 2 full cycles over the interval $[0, 2\pi]$). **A1 A1**

(b) State the maximum and minimum values of y, and the equation of the principal axis. **[2]**

The maximum value is $\mathbf{y = 4}$ and the minimum value is $\mathbf{y = -2}$; the principal axis is $\mathbf{y = 1}$ (the midline, halfway between the maximum and minimum). **A1 A1**

(c) Solve the equation $3\sin(2x) + 1 = 1$ for $0 \le x \le 2\pi$. **[3]**

$3\sin(2x)+1 = 1 \Rightarrow \sin(2x) = 0 \Rightarrow 2x = 0, \pi, 2\pi, 3\pi, 4\pi \Rightarrow \mathbf{x = 0, \pi\div 2, \pi, 3\pi\div 2, 2\pi}$. **M1 M1 A1**

Question 10 [6 marks]

The Pythagorean identity relates $\sin\theta$ and $\cos\theta$ for any angle θ.

(a) State the Pythagorean identity relating $\sin\theta$ and $\cos\theta$. **[2]**

$\mathbf{\sin^2\theta + \cos^2\theta = 1}$. **A1 A1**

(b) Given that $\sin\theta = 0.6$, and θ is acute, calculate the exact value of $\cos\theta$. **[2]**

$\cos^2\theta = 1-0.6^2 = 1-0.36 = 0.64 \Rightarrow \cos\theta = \sqrt{0.64} = \mathbf{0.8}$ (positive, since θ is acute). **M1 A1**

(c) Solve the equation $2\sin\theta = 1$ for $0° \le \theta \le 360°$. **[2]**

$2\sin\theta = 1 \Rightarrow \sin\theta = 0.5 \Rightarrow \mathbf{\theta = 30°}$ **or** $\mathbf{\theta = 150°}$ (since sine is positive in the first and second quadrants). **M1 A1**