IB DP MATHEMATICS

AA Topic 2 · Functions

MARK SCHEME

2.1 Function Basics · 2.2 Quadratic Functions
2.3 Transformations of Graphs · 2.4 Rational Functions

Standard and Higher Level · Syllabus 2025

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Original exam-style questions written for the IB Diploma Programme Mathematics course (first assessment 2025)

Section A — Multiple Choice: Answer Key (15 marks)

1. Answer: **B** — A function is a relation in which every element of the domain (input) is mapped to exactly one element of the range (output); no input value can produce more than one output.

2. Answer: **B** — The expression under a square root must be non-negative, so $x-1 \geq 0$, giving the domain $x \geq 1$.

3. Answer: **B** — $(f \circ g)(2) = f(g(2)) = f(2^2) = f(4) = 2(4)+3 = 11$.

4. Answer: **C** — The inverse function undoes the mapping of the original function; graphically, $y = f^{-1}(x)$ is the reflection of $y = f(x)$ in the line $y = x$.

5. Answer: **B** — Vertex form, $y = a(x-h)^2 + k$, directly shows the vertex of the parabola at the point (h, k).

6. Answer: **A** — The discriminant of a quadratic equation $ax^2+bx+c=0$ is $\Delta = b^2 - 4ac$, and its sign determines the number of real roots.

7. Answer: **C** — A positive discriminant means the quadratic curve crosses the x-axis at two distinct points, giving two distinct real roots.

8. Answer: **C** — Factorising, $x^2-5x+6 = (x-2)(x-3) = 0$, so $x = 2$ or $x = 3$.

9. Answer: **B** — Adding a constant k to the output of a function shifts every point on the graph vertically upward by k units, i.e. a translation by the vector (0, k).

10. Answer: **B** — Replacing x with $(x-k)$ inside the function shifts the graph k units to the right (a horizontal translation).

11. Answer: **B** — Multiplying the whole function by -1 negates every y-value, which reflects the graph in the x-axis.

12. Answer: **A** — Multiplying the output of the function by $a > 1$ stretches the graph vertically (away from the x-axis) by a scale factor of a.

13. Answer: **C** — The function is undefined when the denominator is zero, i.e. when $x-3 = 0$, so $x = 3$, giving a vertical asymptote at $x = 3$.

14. Answer: **B** — As $x \to \pm\infty$, the term $1 \div (x-3) \to 0$, so $y \to 2$; the horizontal asymptote is $y = 2$.

15. Answer: **C** — The function is undefined when $x+5 = 0$, i.e. when $x = -5$, so the domain is all real x except $x = -5$.

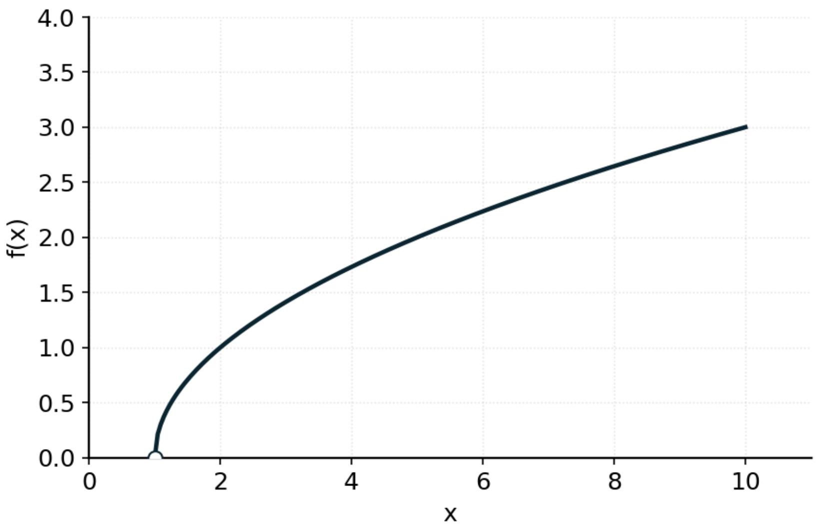
Section B — Structured Questions: Worked Solutions (65 marks)

Mark-scheme notation: M = method mark, A = accuracy mark (dependent on the preceding M mark), R = reasoning mark.

2.1 Function Basics

Question 1 [7 marks]

The graph shows the function $f(x) = \sqrt{(x-1)}$ (computed).



The function $f(x) = \sqrt{(x-1)}$ (computed).

(a) Using the graph, state the domain and range of f(x). **[2]**

From the graph, the domain is $\mathbf{x \geq 1}$ and the range is $\mathbf{f(x) \geq 0}$. **A1 A1**

(b) Calculate f(5), and confirm it agrees with the graph. **[2]**

$f(5) = \sqrt{(5-1)} = \sqrt{4} = \mathbf{2}$, which matches the value shown on the graph at x = 5. **M1 A1**

(c) Find the value of x for which f(x) = 3. **[3]**

$f(x) = 3 \Rightarrow \sqrt{(x-1)} = 3 \Rightarrow x-1 = 3^2 = 9 \Rightarrow x = \mathbf{10}$. **M1 M1 A1**

Question 2 [6 marks]

Composite and inverse functions combine or reverse the effect of individual functions.

(a) Given f(x) = 2x + 3, state the value of f(0) and explain what this represents on the graph of y = f(x). **[2]**

f(0) = 2(0)+3 = **3**, which is the **y-intercept** of the graph of y = f(x). **A1 A1**

(b) Given f(x) = 2x + 3 and $g(x) = x^2$, find (f∘g)(2). **[2]**

$(f \circ g)(2) = f(g(2)) = f(2^2) = f(4) = 2(4)+3 = \mathbf{11}$. **M1 A1**

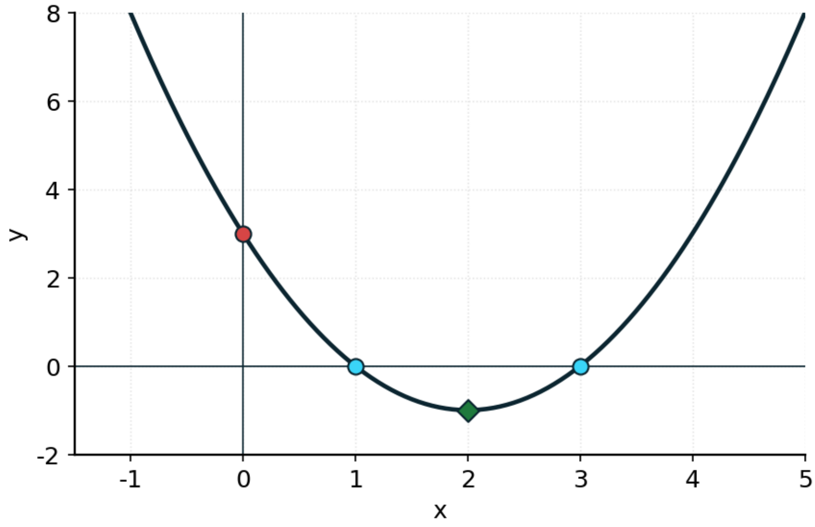
(c) Find the inverse function, $f^{-1}(x)$, of f(x) = 2x + 3. **[2]**

Let $y = 2x+3$. Swap x and y: $x = 2y+3 \Rightarrow y = (x-3)\div 2$. So $\mathbf{f^{-1}(x) = (x-3)\div 2}$. **M1 A1**

2.2 Quadratic Functions

Question 3 [7 marks]

The graph shows the quadratic function $y = x^2 - 4x + 3$ (computed).



$y = x^2 - 4x + 3$ (computed), with roots, y-intercept, and vertex marked.

(a) Using the graph, state the roots (x-intercepts) and the y-intercept of the function. **[2]**

The roots are $\mathbf{x = 1}$ and $\mathbf{x = 3}$; the y-intercept is $\mathbf{y = 3}$. **A1 A1**

(b) By completing the square, write $x^2 - 4x + 3$ in vertex form, and state the coordinates of the vertex. **[2]**

$x^2-4x+3 = (x-2)^2-4+3 = \mathbf{(x-2)^2-1}$, so the vertex is at **(2, −1)**, matching the graph. **M1 A1**

(c) State the equation of the axis of symmetry of this quadratic. **[3]**

The axis of symmetry of a quadratic in vertex form $(x-h)^2+k$ passes through the vertex, so the axis of symmetry is $\mathbf{x = 2}$; this can also be confirmed as the midpoint of the roots x=1 and x=3, since $(1+3)\div 2 = 2$. **M1 R1 A1**

Question 4 [6 marks]

The quadratic formula and the discriminant allow the roots of any quadratic equation to be found or classified.

(a) State the quadratic formula for solving $ax^2+bx+c = 0$. **[2]**

$\mathbf{x = (-b \pm \sqrt{(b^2-4ac)})\div 2a}$. **A1 A1**

(b) State the discriminant formula, and state what a negative discriminant tells us about the roots of the equation. **[2]**

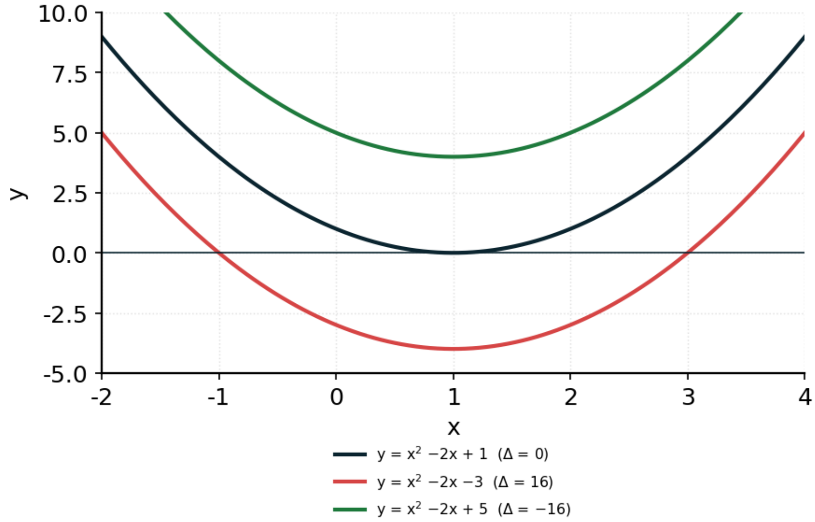
The discriminant is $\Delta = \mathbf{b^2-4ac}$. If $\Delta < 0$, the equation has **no real roots** (the graph does not cross the x-axis). **A1 A1**

(c) Calculate the discriminant of $2x^2+3x-5 = 0$, and state how many real roots this equation has. **[2]**

$\Delta = 3^2-4(2)(-5) = 9+40 = \mathbf{49}$. Since $\Delta > 0$, this equation has **two distinct real roots**. **M1 A1**

Question 5 [7 marks]

The graph shows three quadratic functions, each with a different discriminant (computed).



Three quadratics with discriminants Δ = 0, 16, and −16 respectively (computed).

(a) Using the graph, identify which curve touches the x-axis at exactly one point, which crosses it at two points, and which does not meet the x-axis at all. Relate each case to the sign of the discriminant. **[2]**

$y=x^2-2x+1$ **touches the x-axis once** ($\Delta=0$, one repeated root); $y=x^2-2x-3$ **crosses it twice** ($\Delta=16>0$, two real roots); $y=x^2-2x+5$ **never meets the x-axis** ($\Delta=-16<0$, no real roots). **A1 A1**

(b) Verify by calculation that the discriminant of $y = x^2-2x+1$ is zero. **[2]**

$\Delta = (-2)^2-4(1)(1) = 4-4 = \mathbf{0}$, confirming this curve has exactly one repeated real root. **M1 A1**

(c) Calculate the discriminant of $y = x^2-2x+5$, and confirm this is consistent with the graph. **[3]**

$\Delta = (-2)^2-4(1)(5) = 4-20 = \mathbf{-16}$. Since $\Delta < 0$, this equation has no real roots, consistent with the curve not crossing the x-axis on the graph. **M1 M1 A1**

2.3 Transformations of Graphs

Question 6 [6 marks]

Transformations allow the graph of a known function $y = f(x)$ to be used to sketch related functions.

(a) Describe the transformation applied to $y = f(x)$ to obtain $y = f(x) + k$ and $y = f(x-k)$, for a positive constant k. **[2]**

$y = f(x) + k$ is a **vertical translation, k units upward**. $y = f(x-k)$ is a **horizontal translation, k units to the right**. **A1 A1**

(b) Describe the transformation applied to $y = f(x)$ to obtain $y = -f(x)$, and to obtain $y = f(-x)$. **[2]**

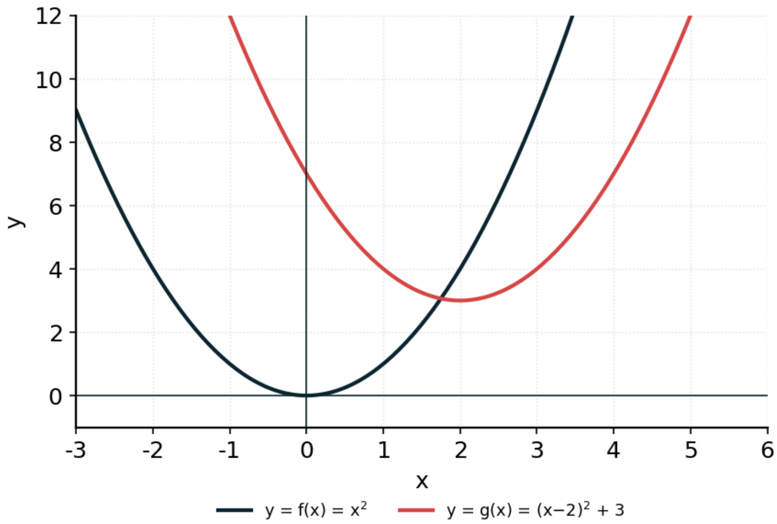
$y = -f(x)$ is a **reflection in the x-axis**. $y = f(-x)$ is a **reflection in the y-axis**. **A1 A1**

(c) Describe the transformation applied to $y = f(x)$ to obtain $y = af(x)$, for a constant $0 < a < 1$. **[2]**

For $0 < a < 1$, $y = af(x)$ is a **vertical compression (stretch)** of $y = f(x)$, by a scale factor of a, towards the x-axis. **A1 R1**

Question 7 [7 marks]

The graph shows $f(x) = x^2$ and $g(x) = (x-2)^2+3$ (computed).



$f(x) = x^2$ and $g(x) = (x-2)^2+3$ (computed).

(a) Using the graph, describe the single transformation that maps $y = f(x)$ onto $y = g(x)$. **[2]**

The graph of $y = g(x)$ is the graph of $y = f(x)$ **translated 2 units to the right and 3 units upward**. **A1 A1**

(b) State the translation vector describing this transformation. **[2]**

The translation vector is (**2, 3**) (2 units in the x-direction, 3 units in the y-direction). **A1 A1**

(c) Using function notation, express g(x) in terms of f(x), and hence confirm this matches the given formula $g(x) = (x-2)^2+3$. **[3]**

A translation of 2 right and 3 up gives $\mathbf{g(x) = f(x-2) + 3}$. Since $f(x) = x^2$, this gives $g(x) = (x-2)^2+3$, which **matches the given formula**. **M1 M1 A1**

Question 8 [6 marks]

Let $f(x) = x^2 - 4x$.

(a) Find expressions for $-f(x)$ and $f(-x)$, and state the transformation each represents. **[2]**

$-f(x) = \mathbf{-x^2+4x}$ (reflection in the x-axis). $f(-x) = \mathbf{x^2+4x}$ (reflection in the y-axis). **A1 A1**

(b) Calculate f(1). **[2]**

$f(1) = 1^2 - 4(1) = 1 - 4 = \mathbf{-3}$. **M1 A1**

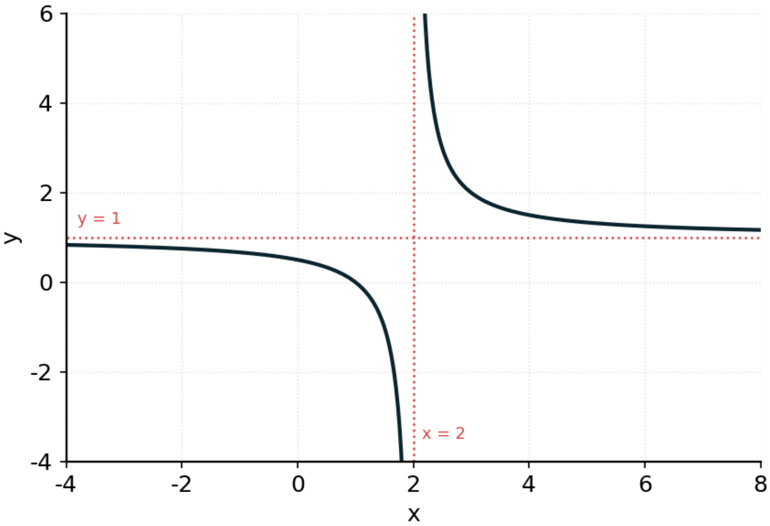
(c) Let g(x) = 2f(x). Calculate g(1). **[2]**

$g(1) = 2f(1) = 2\times(-3) = \mathbf{-6}$. **M1 A1**

2.4 Rational Functions

Question 9 [7 marks]

The graph shows the rational function $y = 1\div(x-2) + 1$ (computed).



$y = 1\div(x-2) + 1$ (computed), showing its two asymptotes.

(a) Using the graph, state the equations of the vertical and horizontal asymptotes. **[2]**

Vertical asymptote: $\mathbf{x = 2}$. Horizontal asymptote: $\mathbf{y = 1}$. **A1 A1**

(b) Using the function, explain algebraically why x = 2 is a vertical asymptote. **[2]**

The function is undefined when the denominator equals zero: $x-2 = 0 \Rightarrow x = 2$. As $x \to 2$, $1\div(x-2) \to \pm\infty$, so **x = 2 is a vertical asymptote**. **R1 R1**

(c) Find the exact y-intercept and x-intercept of the function algebraically. **[3]**

y-intercept (x=0): $y = 1\div(0-2)+1 = -0.5+1 = \mathbf{0.5}$. x-intercept (y=0): $0 = 1\div(x-2)+1 \Rightarrow 1\div(x-2) = -1 \Rightarrow x-2 = -1 \Rightarrow x = \mathbf{1}$. **M1 M1 A1**

Question 10 [6 marks]

Rational functions of the form $y = 1\div(x-a) + b$ have asymptotes determined directly by the constants a and b.

(a) State the general rule for finding the vertical and horizontal asymptotes of $y = 1\div(x-a) + b$. **[2]**

The vertical asymptote is $\mathbf{x = a}$ (where the denominator is zero); the horizontal asymptote is $\mathbf{y = b}$ (the value y approaches as $x \to \pm\infty$). **A1 A1**

(b) State the equations of the asymptotes of $y = 1\div(x+3) - 2$. **[2]**

Writing the function as $1 \div (x-(-3))-2$, the vertical asymptote is $\mathbf{x = -3}$ and the horizontal asymptote is $\mathbf{y = -2}$. **A1 A1**

(c) State the domain of $y = 1 \div (x+3) - 2$. **[2]**

The function is undefined at the vertical asymptote, so the domain is **all real x, except x = −3**. **A1 A1**