IB DP MATHEMATICS

AA Topic 1 · Number and Algebra

MARK SCHEME

1.1 Arithmetic Sequences · 1.2 Geometric Sequences

1.3 Exponents, Logs & Binomial Thm · 1.4 Financial Applications

Standard and Higher Level · Syllabus 2025

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Original exam-style questions written for the IB Diploma Programme Mathematics course (first assessment 2025)

Section A — Multiple Choice: Answer Key (15 marks)

1. Answer: **B** — For an arithmetic sequence, each term is found by adding the common difference d a total of (n−1) times to the first term, giving $u_n = u_1 + (n-1)d$.

2. Answer: **C** — $u_{10} = 5 + (10-1)\times 3 = 5 + 27 = 32$.

3. Answer: **B** — The sum of the first n terms of an arithmetic sequence is $S_n = n\div 2 \times (2u_1 + (n-1)d)$, obtained by pairing the first and last terms of the series.

4. Answer: **C** — The sequence 3, 7, 11, 15, ... has a constant common difference of 4 between consecutive terms ($7-3 = 11-7 = 15-11 = 4$), so it is arithmetic; the other sequences do not have a constant difference.

5. Answer: **B** — For a geometric sequence, each term is found by multiplying the previous term by the common ratio r; after (n−1) multiplications from the first term, $u_n = u_1 r^{n-1}$.

6. Answer: **C** — $u_5 = 4\times 2^{5-1} = 4\times 16 = 64$.

7. Answer: **C** — An infinite geometric series only converges to a finite sum when $|r| < 1$ (i.e. $-1 < r < 1$); otherwise the terms do not shrink towards zero and the series diverges.

8. Answer: **A** — For an infinite geometric series with $|r| < 1$, the sum to infinity is $S\infty = u_1 \div (1-r)$, obtained by taking the limit of S_n as $n \to \infty$.

9. Answer: **B** — The addition law of logarithms states $\log_a(x) + \log_a(y) = \log_a(xy)$.

10. Answer: **B** — Since $8 = 2^3$, and the bases are equal, the exponents must be equal, so $x = 3$.

11. Answer: **B** — The general term in the expansion of $(x+y)^n$ is ${}^nC_r\, x^{n-r}y^r$, where ${}^nC_r$ is the binomial coefficient.

12. Answer: **C** — Since $a^0 = 1$ for any valid base a, it follows that $\log_a(1) = 0$.

13. Answer: **B** — Compound interest is calculated on the accumulated balance (principal plus previously earned interest) each period, giving the exponential formula $FV = PV(1 + r)^n$, where r is the rate per period as a decimal.

14. Answer: **C** — $FV = 1000\times(1.05)^2 = 1000\times 1.1025 = \$1102.50$.

15. Answer: **C** — Compound interest earns 'interest on interest', so the investment value grows exponentially over time, whereas simple interest grows only linearly (interest is always calculated on the fixed original principal); over a long period, compound interest therefore produces a greater final value.

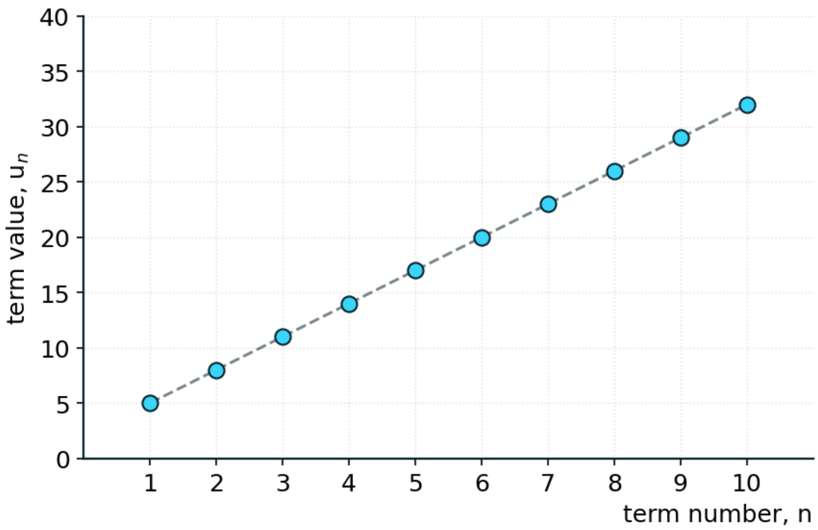
Section B — Structured Questions: Worked Solutions (65 marks)

Mark-scheme notation: M = method mark, A = accuracy mark (dependent on the preceding M mark), R = reasoning mark.

1.1 Arithmetic Sequences

Question 1 [7 marks]

The graph shows the first 10 terms of an arithmetic sequence (computed).



Terms u_n of an arithmetic sequence for n = 1 to 10 (computed).

(a) Using the graph, write down the value of the first term u_1 and the common difference d of the sequence. **[2]**

From the graph, u_1 = **5** and the common difference is **d = 3** (each term is 3 greater than the previous term). **A1 A1**

(b) Using $u_n = u_1 + (n-1)d$, calculate u_{10}, the 10th term, and confirm this matches the graph. **[2]**

$u_{10} = 5 + (10-1)\times3 = 5 + 27 =$ **32**, which matches the value shown on the graph at n = 10. **M1 A1**

(c) The sequence is continued beyond the terms shown on the graph. Using the formula for the nth term, predict u_{15}, the 15th term. **[3]**

$u_{15} = u_1 + (15-1)d = 5 + 14\times3 = 5 + 42 =$ **47**. **M1 M1 A1**

Question 2 [6 marks]

Arithmetic sequences and series can be described using standard formulas relating u_1, d, n, and S_n.

(a) State the formula for the nth term, u_n, of an arithmetic sequence, defining each symbol used. **[2]**

$\mathbf{u_n = u_1 + (n-1)d}$, where u_1 is the first term, d is the common difference, and n is the term number. **A1 A1**

(b) State the formula for the sum of the first n terms, S_n, of an arithmetic sequence. **[2]**

$\mathbf{S_n = n\div2 \times (2u_1 + (n-1)d)}$ (equivalently, $S_n = n\div2 \times(u_1+u_n)$). **A1 A1**

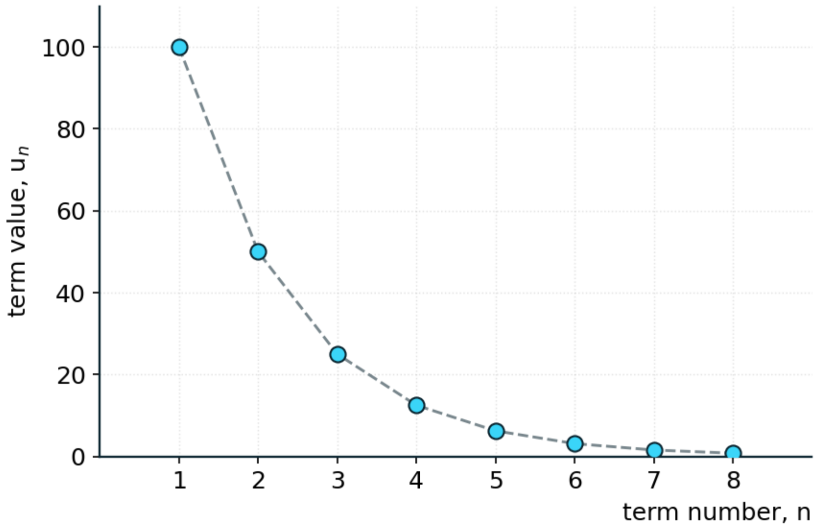
(c) An arithmetic sequence has $u_1 = 8$ and $u_5 = 24$. Find the common difference, d. **[2]**

$u_5 = u_1 + 4d \Rightarrow 24 = 8 + 4d \Rightarrow 4d = 16 \Rightarrow d = \mathbf{4}$. **M1 A1**

1.2 Geometric Sequences

Question 3 [7 marks]

The graph shows the first 8 terms of a geometric sequence (computed).



Terms u_n of a geometric sequence for n = 1 to 8 (computed).

(a) Using the graph, write down the value of u_1, and use two consecutive terms to determine the common ratio, r. **[2]**

$u_1 = \mathbf{100}$. Using $u_2 \div u_1 = 50 \div 100$, the common ratio is $\mathbf{r = 0.5}$. **A1 A1**

(b) Using $u_n = u_1 r^{n-1}$, verify the value of u_4 shown on the graph. **[2]**

$u_4 = 100 \times (0.5)^3 = 100 \times 0.125 = \mathbf{12.5}$, which matches the value shown on the graph at n = 4. **M1 A1**

(c) Calculate u_9, the 9th term of the sequence (beyond the terms shown on the graph), giving your answer to 3 significant figures. **[3]**

$u_9 = 100 \times (0.5)^8 = 100 \times 0.00390625 = \mathbf{0.391}$ (3 s.f.). **M1 M1 A1**

Question 4 [6 marks]

Geometric sequences and series can also be described using standard formulas relating u_1, r, n, and S_n.

(a) State the formula for the nth term, u_n, of a geometric sequence, defining each symbol used. **[2]**

$\mathbf{u_n = u_1 r^{n-1}}$, where u_1 is the first term, r is the common ratio, and n is the term number. **A1 A1**

(b) State the formula for the sum of the first n terms, S_n, of a geometric sequence (for $r \neq 1$). **[2]**

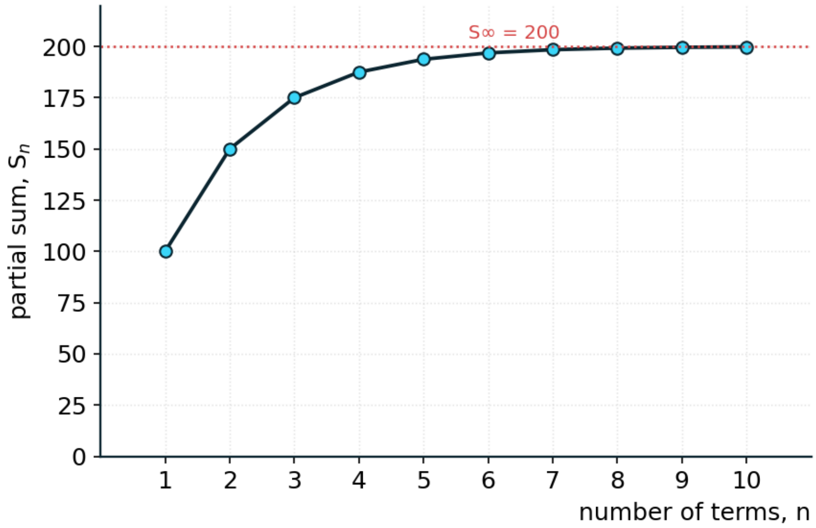
$\mathbf{S_n = u_1(1-r^n) \div (1-r)}$. **A1 A1**

(c) A geometric sequence has $u_1 = 6$ and $u_4 = 48$. Find the common ratio, r. **[2]**

$u_4 = u_1 r^3 \Rightarrow 48 = 6r^3 \Rightarrow r^3 = 8 \Rightarrow r = \mathbf{2}$. **M1 A1**

Question 5 [7 marks]

The graph shows the partial sum S_n (the sum of the first n terms) of the same geometric sequence, $u_1 = 100$, $r = 0.5$, plotted against n (computed).



Partial sums S_n of a geometric series, $u_1 = 100$, $r = 0.5$ (computed).

(a) Using the graph, describe what happens to S_n as n increases, and estimate the value that S_n approaches. **[2]**

As n increases, S_n **increases but levels off**, approaching a horizontal asymptote at approximately **$S_\infty \approx 200$**. **A1 A1**

(b) State the condition on r required for an infinite geometric series to have a finite sum to infinity, and state whether this condition is satisfied here. **[2]**

A finite sum to infinity requires $|r| < 1$. Here, $r = 0.5$, so $|r| < 1$ is **satisfied**, and a sum to infinity exists. **A1 R1**

(c) Using $S_\infty = u_1 \div (1 - r)$, calculate the exact value of S_∞, and confirm it matches your estimate from the graph. **[3]**

$S_\infty = 100 \div (1 - 0.5) = 100 \div 0.5 = \mathbf{200}$, which confirms the asymptote read from the graph in part (a). **M1 M1 A1**

1.3 Exponents & Logarithms

Question 6 [6 marks]

The laws of logarithms allow expressions involving logarithms to be simplified or combined.

(a) State the law of logarithms for $\log_a(x) + \log_a(y)$, and for $\log_a(x) - \log_a(y)$. **[2]**

$\log_a(x) + \log_a(y) = \mathbf{\log_a(xy)}$; $\log_a(x) - \log_a(y) = \mathbf{\log_a(x \div y)}$. **A1 A1**

(b) Simplify $\log_a(x^3)$ in terms of $\log_a(x)$. **[2]**

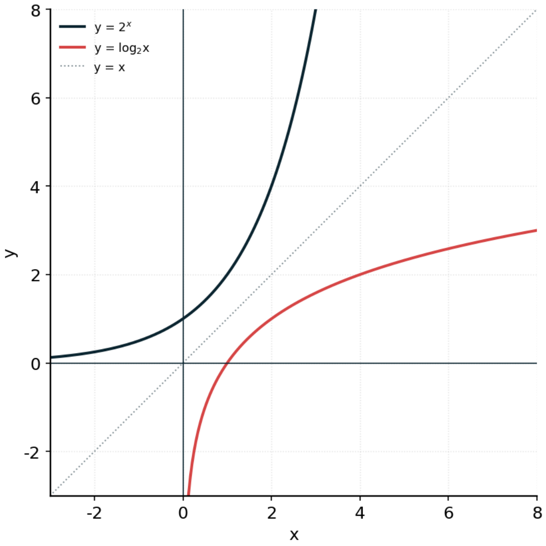
Using the power law of logarithms, $\log_a(x^3)$ = **$3 \log_a(x)$**. **A1 A1**

(c) Solve for x: $\log_2(x) = 5$. **[2]**

$\log_2(x) = 5 \Rightarrow x = 2^5 =$ **32**. **M1 A1**

Question 7 [7 marks]

The graph shows $y = 2^x$ and $y = \log_2 x$ plotted together with the line $y = x$ (computed).



$y = 2^x$ and $y = \log_2 x$, shown with the line $y = x$ (computed).

(a) Using the graph, describe the relationship between the graphs of $y = 2^x$ and $y = \log_2 x$. **[2]**

The two graphs are **reflections of each other in the line y = x**; this is because $y = \log_2 x$ is the **inverse function** of $y = 2^x$. **A1 R1**

(b) Using the graph, estimate the solution to the equation $2^x = 6$. **[2]**

Reading across from $y = 6$ to the curve $y = 2^x$ and down to the x-axis gives approximately **$x \approx 2.6$**. **M1 A1**

(c) Calculate $\log_2(6)$ algebraically, using the change-of-base formula, giving your answer to 3 significant figures. Confirm this is consistent with your estimate in part (b). **[3]**

$\log_2(6) = \ln(6) \div \ln(2) =$ **2.58** (3 s.f.), consistent with the graphical estimate of $x \approx 2.6$ in part (b). **M1 M1 A1**

Question 8 [6 marks]

The binomial theorem allows expressions of the form $(x+y)^n$ to be expanded without repeated multiplication.

(a) State the general term in the binomial expansion of $(x+y)^n$, in terms of n, r, x, and y. **[2]**

The general term is **$^nC_r\, x^{n-r}y^r$**, where ^nC_r is the binomial coefficient. **A1 A1**

(b) Write out the full binomial expansion of $(x+2)^3$. **[2]**

$(x+2)^3 = {}^3C_0x^3 + {}^3C_1x^2(2) + {}^3C_2x(2)^2 + {}^3C_3(2)^3 =$ **$x^3 + 6x^2 + 12x + 8$**. **M1 A1**

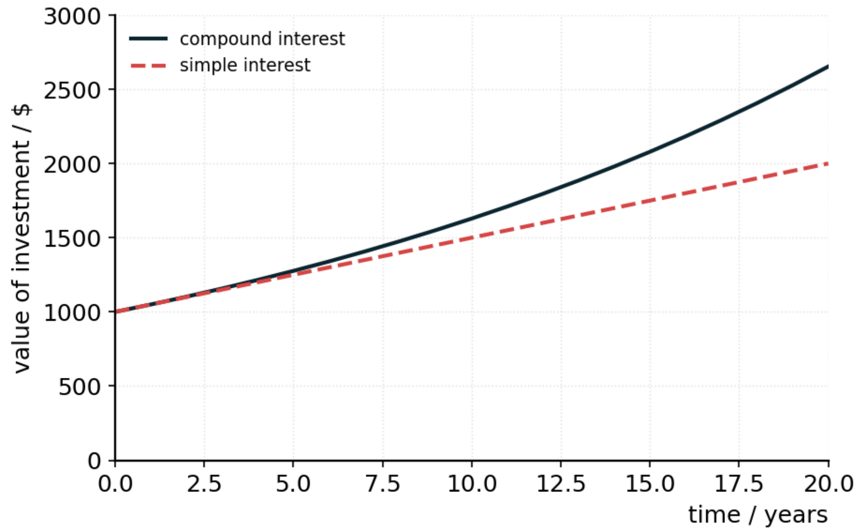
(c) Hence state the coefficient of x^2 in this expansion. **[2]**

From part (b), the coefficient of x^2 is **6**. **A1 A1**

1.4 Financial Applications

Question 9 [7 marks]

The graph shows the value of a $1000 investment over 20 years, earning 5% per annum, under compound interest and under simple interest (computed).



Value of a $1000 investment at 5% per annum: compound interest vs simple interest (computed).

(a) Using the graph, compare how the value of the compound-interest investment changes over time with the value of the simple-interest investment. **[2]**

Both investments start at the same value ($1000) and grow at first at a similar rate, but the **compound-interest curve grows exponentially**, increasing ever more steeply, while the **simple-interest line grows linearly** (at a constant rate); the compound-interest investment therefore becomes increasingly larger than the simple-interest investment as time passes. **A1 A1**

(b) Using $FV = PV(1+r)^n$, calculate the value of the compound-interest investment after 20 years, confirming it matches the graph. **[2]**

$FV = 1000\times(1.05)^{20}$ = **$2653.30**, matching the value shown on the graph at 20 years. **M1 A1**

(c) Calculate the value of the simple-interest investment after 20 years, and hence find the difference between the two investments at that time. **[3]**

Simple interest: $FV = 1000 + (1000\times0.05\times20) = 1000 + 1000$ = **$2000.00**. Difference = 2653.30 − 2000.00 = **$653.30**. **M1 M1 A1**

Question 10 [6 marks]

Compound interest is widely used in real-world financial applications, such as savings accounts, loans, and investments.

(a) Define 'compound interest', distinguishing it from simple interest. **[2]**

Compound interest is interest calculated on the accumulated balance (the original principal plus any interest already earned), so the amount of interest earned grows each period. **Simple interest**, by contrast, is calculated only on the **original principal** each period, so the interest earned is constant. **A1 A1**

(b) Calculate the value of a $2000 investment after 3 years at 4% per annum compound interest. **[2]**

FV = $2000\times(1.04)^3 = 2000\times1.124864 =$ **$2249.73**. **M1 A1**

(c) Explain why compound interest results in a greater final value than simple interest over long time periods, for the same principal and rate. **[2]**

With compound interest, each period's interest is calculated on a **growing balance** (since previously earned interest is added to the principal), so the investment grows **exponentially**. With simple interest, interest is always calculated on the **fixed original principal** only, so the investment grows **linearly**. Exponential growth eventually outpaces linear growth, so compound interest gives a greater final value over long time periods. **R1 R1**