IB DP MATHEMATICS

Applications and Interpretation (AI)

Topic 2 Functions

Revision Notes · Standard and Higher Level

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Original notes prepared for the IB Diploma Programme Mathematics AI course (first assessment 2021)

What Topic 2 requires

Topic 2 of Mathematics: Applications and Interpretation is about **modelling with functions**. The examiner is far less interested in abstract algebra than in whether you can pick a sensible model for a real situation, read meaning into its parameters, and use a graphing calculator (GDC) to answer questions about it. Use this checklist before the exam.

You should be able to...	In practice this means
Use function notation and describe domain and range	State $f(x)$, evaluate $f(a)$, and give the sensible domain and range for a model in context.
Choose an appropriate model	Match data or a described situation to a linear, quadratic, exponential, cubic, reciprocal or sinusoidal model.
Interpret parameters in context	Explain what each constant (gradient, intercept, amplitude, growth rate, asymptote) means for the real quantity.
Use a GDC to graph and analyse	Find intercepts, maxima/minima, intersections and solve equations graphically.
Fit models to data	Use regression on the GDC and judge whether the fit is good.
Model further functions (HL)	Work with logarithmic and logistic models, transformations and multi-part (piecewise) models.

> **Exam note:** In AI, technology is assumed throughout. Paper 2 and Paper 3 (HL) are calculator papers; even on Paper 1 many function questions expect a GDC-style method. Always show the equation you solved and quote the GDC result to a sensible accuracy (usually 3 significant figures).

1. The function concept

A **function** is a rule that assigns to each input exactly one output. We write $f(x)$ for the output when the input is x. The set of allowed inputs is the **domain**; the set of outputs that actually occur is the **range**.

Term	Meaning	Example for $f(x) = 2x + 1$
Function notation	$f(x)$ is the output for input x	$f(3) = 2(3) + 1 = 7$
Domain	Set of permitted inputs x	all real x, or a restricted interval in context
Range	Set of outputs $f(x)$ produced	all real values (here a straight line)
Zero / root	input where $f(x) = 0$	$x = -0.5$

Domain and range in context

For a mathematical formula the domain is every input that does not break the rule (no division by zero, no square root of a negative). For a **model** the domain is usually narrower still, because the real quantity has limits: a time t cannot be negative, a population cannot exceed the space available, a length must be positive. Always state the domain that makes physical sense.

Worked example 1 — reading a model with a GDC

The height of water in a tank is modelled by $h(t) = 50 - 3t$ centimetres, where t is time in minutes after a tap is opened. State a sensible domain and range, and find when the tank is empty.

Empty when $h = 0$: $0 = 50 - 3t$, so $t = 50/3 \approx 16.7$ min. On a GDC, graph $y = 50 - 3x$ and read the x-intercept.

Sensible domain: $0 \le t \le 16.7$ (time cannot be negative and the model stops when the tank empties). Range: $0 \le h \le 50$ cm.

2. Choosing and using a model

A mathematical model is a function that captures the important behaviour of a real situation well enough to make predictions. Modelling is a cycle: pick a model, fit it to data, test it, and refine or reject it. Choosing the **family** of function is the first and most examined decision.

If the quantity...	Suitable model	Typical context
changes by a constant amount per step	Linear $y = mx + c$	taxi fare, phone plan, steady filling
rises then falls (or vice versa), symmetric	Quadratic $y = ax^2 + bx + c$	projectile height, area, revenue
changes by a constant percentage per step	Exponential $y = ka^x + c$	growth, decay, cooling, interest
is inversely proportional to another	Reciprocal $y = a/x + b$	time vs speed, pressure vs volume
repeats over a fixed period	Sinusoidal $y = a \sin(b(x-c)) + d$	tides, temperature, daylight
grows then levels off at a limit (HL)	Logistic $L/(1 + Ce^{-kx})$	constrained population, spread

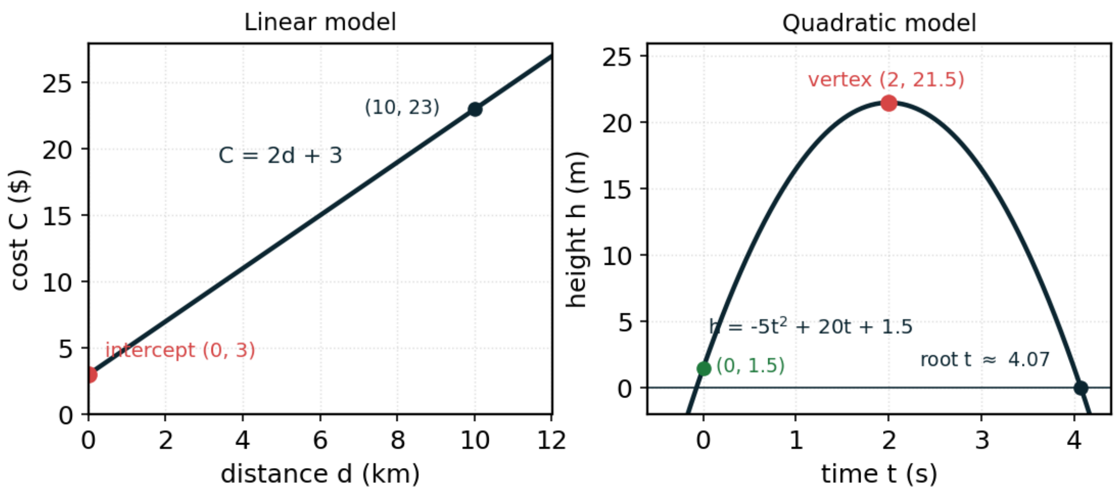

Figure 1. A linear model versus a quadratic model. Left: the taxi cost C = 2d + 3 is a straight line with gradient 2 ($ per km) and vertical intercept 3 ($ fixed fee). Right: the projectile height $h = -5t^2 + 20t + 1.5$ is a parabola with vertex (2, 21.5), y-intercept 1.5, and it lands at the root t ≈ 4.07 s.

Interpreting parameters

The whole point of AI modelling is that every constant *means* something. The gradient of a linear cost model is the price per unit; the vertical intercept is the fixed charge; the base of an exponential is the growth factor per period; the period of a sinusoid is how long one cycle takes. When a question says “interpret” or “what does this value represent”, answer with the real quantity and its units, not with mathematics.

> **Exam technique:** “State the meaning of c in this model” wants a sentence such as “the fixed cost of $8 before any units are used”, not just “the y-intercept”. Units and context earn the mark.

3. Linear models

A linear model has the form $y = mx + c$, giving a straight-line graph. It is the right choice when a quantity changes by the same amount for each equal step in the input.

Parameter	Name	Meaning in a model
m	Gradient (slope)	Rate of change: how much y changes per unit increase in x (with units, e.g. $ per km)
c	Vertical intercept	Value of y when $x = 0$: the starting or fixed amount

The gradient between two points (x_1, y_1) and (x_2, y_2) is $m = (y_2 - y_1) / (x_2 - x_1)$. A positive gradient means increase, negative means decrease, and zero means a constant (horizontal) value.

Worked example 2 — fit and interpret a linear model

A taxi charges a fixed hire fee plus a rate per kilometre. A 4 km trip costs $11.00 and a 10 km trip costs $23.00. Find a linear model $C = md + c$ and interpret both parameters.

Gradient $m = (23 - 11) / (10 - 4) = 12 / 6 = 2$. So the charge rises $2 per km.

Substitute (4, 11): $11 = 2(4) + c$, so $c = 3$. Model: $C = 2d + 3$.

Interpretation: m = $2.00 per kilometre travelled; c = $3.00 fixed hire fee charged before any distance. Check: a 10 km trip gives 2(10) + 3 = $23 ✓.

Piecewise linear models

Many real charges change their rate at a threshold, giving a graph made of joined straight segments. Such a **piecewise linear** model is written with a separate rule for each interval of the domain. For example an electricity tariff might charge one rate up to 200 units and a higher rate beyond:

$$C(u) = 0.15u \text{ for } 0 \le u \le 200, \text{ and } C(u) = 30 + 0.22(u - 200) \text{ for } u > 200.$$

Read the correct piece for the input you are given, and note that a well-designed tariff joins up (is continuous) at the boundary: here both rules give $30 at $u = 200$.

4. Quadratic models

A quadratic model $y = ax^2 + bx + c$ (with $a \neq 0$) has a parabola graph. It fits situations with a single maximum or minimum and symmetric behaviour either side of it — projectile height, cross-sectional area, and profit or revenue.

Feature	How to find it	Meaning in a model
Shape	$a > 0$ opens up (minimum); $a < 0$ opens down (maximum)	whether the turning point is a low or high point
y-intercept	c (value at $x = 0$)	starting value of the quantity
Axis of symmetry	$x = -b / (2a)$	input giving the maximum or minimum
Vertex	substitute the axis value into y	the maximum or minimum value itself
Roots (zeros)	quadratic formula or GDC	inputs where the quantity is zero

Roots and the discriminant

The roots solve $ax^2 + bx + c = 0$. By the quadratic formula $x = (-b \pm \sqrt{b^2 - 4ac}) / (2a)$. The quantity under the root, $\Delta = b^2 - 4ac$, is the **discriminant**:

- $\Delta > 0$: two distinct real roots (graph crosses the axis twice).
- $\Delta = 0$: one repeated root (graph touches the axis at the vertex).
- $\Delta < 0$: no real roots (graph misses the axis entirely).

Worked example 3 — projectile (quadratic model)

A ball is thrown and its height in metres is modelled by $h(t) = -5t^2 + 20t + 1.5$, where t is time in seconds. Find the maximum height and when the ball lands.

Axis of symmetry: $t = -b/(2a) = -20 / (2 \times -5) = 2$ s.

Maximum height: $h(2) = -5(4) + 20(2) + 1.5 = -20 + 40 + 1.5 =$ **21.5 m** at $t = 2$ s.

Lands when $h = 0$. GDC (or formula) on $-5t^2 + 20t + 1.5 = 0$ gives $t = -0.074$ or $t = 4.07$. Time must be positive, so it lands at $\boldsymbol{t \approx 4.07}$ **s**.

Applications: area and revenue

A rectangle of fixed perimeter has an area that is a quadratic function of one side, maximised (a square) at the vertex. Revenue is often price × quantity where quantity falls linearly with price, giving a quadratic in price with a profit-maximising vertex. In both cases the vertex answers the optimisation question.

> **Common pitfall:** The vertex gives the *optimal input* (the x-value) and the *optimal value* (the y-value). Read the question: “what price” wants the x-coordinate; “maximum revenue” wants the y-coordinate.

5. Exponential models

An exponential model changes by a constant **percentage** per step, so it grows or decays ever faster (or slower). Two equivalent forms appear in AI:

$y = ka^x + c$ and $y = ke^{rx} + c$

Parameter	Meaning in the model
a (base)	Growth factor per unit of x: $a > 1$ growth, $0 < a < 1$ decay. E.g. $a = 1.05$ is +5% per step; $a = 0.8$ is −20% per step.
r (rate)	Continuous rate: $r > 0$ growth, $r < 0$ decay. Base and rate link by $a = e^r$.
k	Scales the curve; combined with c it sets the value at $x = 0$ (which is $k + c$).
c	Horizontal asymptote $y = c$: the limiting value the curve approaches but never reaches.

The horizontal asymptote

As $x \to \infty$ (growth) or $x \to \infty$ for decay, the term ka^x tends to 0 for decay and the curve flattens towards $y = c$. In a cooling model c is room temperature; in a decay model with $c = 0$ the quantity approaches zero. Recognising c as the asymptote is a very common exam requirement.

Half-life and doubling time

For decay, the **half-life** is the constant time for the quantity to halve; for growth, the **doubling time** is the constant time to double. Because the change is by a constant factor, these times do not depend on the starting amount. Solve them with a GDC by finding where the model equals half (or twice) the initial value.

Worked example 4 — exponential decay and half-life

A radioactive sample of mass 80 mg decays according to $m(t) = 80(0.90)^t$ mg, where t is in hours. Find the mass after 5 hours and the half-life.

After 5 h: $m(5) = 80(0.90)^5 = 80 \times 0.59049 =$ **47.2 mg** (3 s.f.).

Interpretation: the base 0.90 means the sample loses 10% of its mass each hour, and the asymptote is $m = 0$ (all mass eventually decays).

Half-life: solve $80(0.90)^t = 40$, i.e. $(0.90)^t = 0.5$. GDC (intersection of $y = 80(0.90)^x$ and $y = 40$) gives $t \approx$ **6.58 h**. Check: $80(0.90)^{6.58} \approx 40$ ✓.

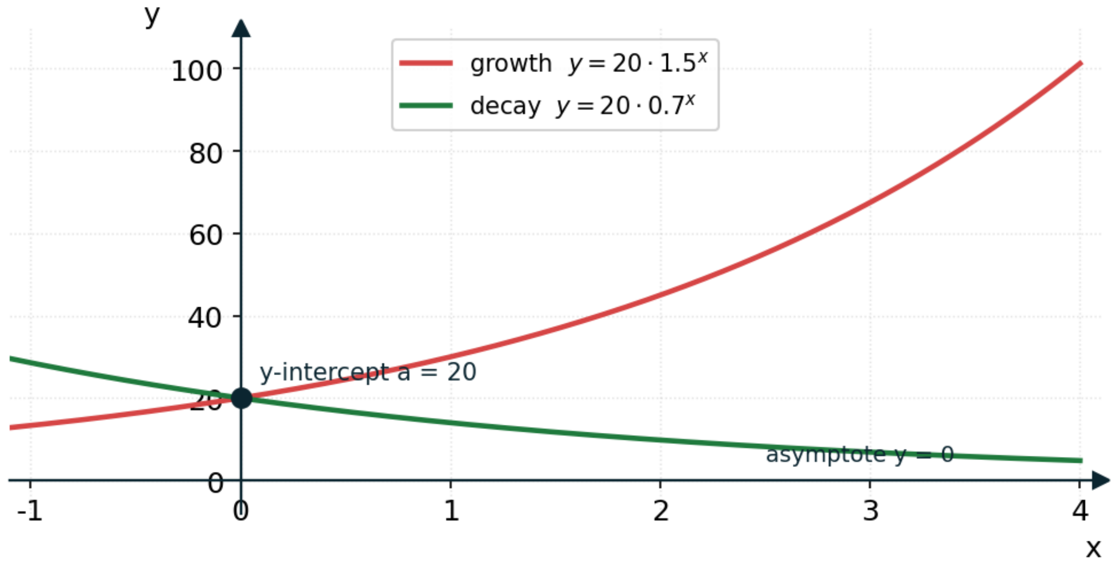

Figure 3. Exponential growth and decay that share the y-intercept a = 20. Growth $y = 20\times1.5^x$ rises by 50% per step; decay $y = 20\times0.7^x$ falls by 30% per step. Both flatten towards the horizontal asymptote y = 0, which the curve approaches but never reaches.

Common pitfall: The asymptote of $y = ka^x + c$ is $y = c$, not $y = 0$. If a cooling drink levels off at 20°C, then $c = 20$ and the model must include it; forgetting c is a frequent error.

6. Variation, cubic and reciprocal models

Direct and inverse variation

Two quantities are in **direct variation** if $y = kx$ (a straight line through the origin): doubling x doubles y. They are in **inverse variation** if $y = k/x$: doubling x halves y. The constant k is found from one known pair of values.

Cubic models

A cubic model $y = ax^3 + bx^2 + cx + d$ can bend twice, so it suits quantities that rise, level and rise again (or the reverse) — for example volume against a linear dimension, or some cost curves. Find its features (intercepts, local maximum and minimum) with a GDC rather than by hand.

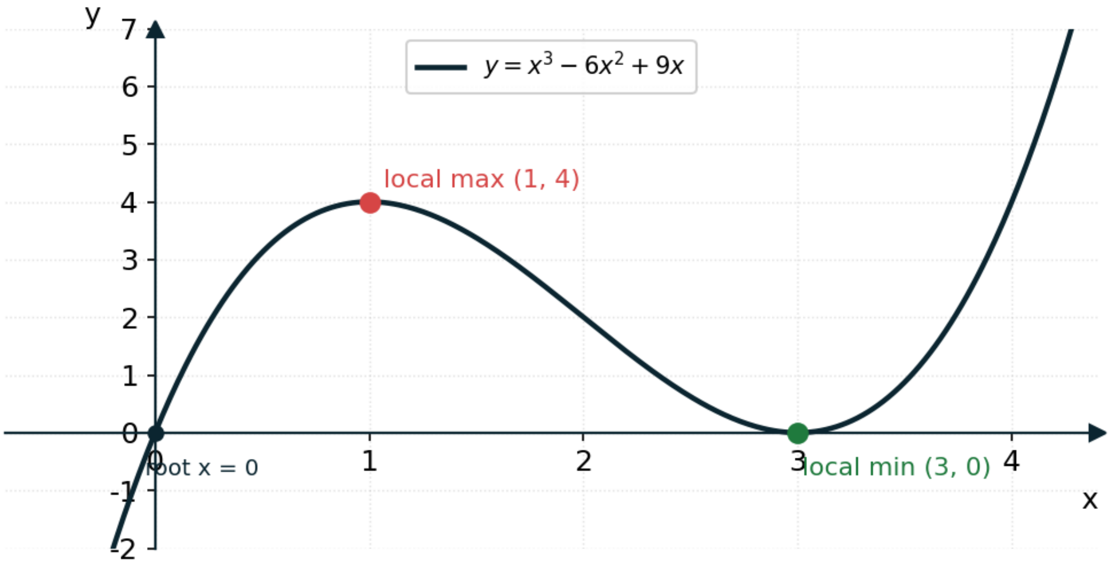

Figure 2. A cubic model $y = x^3 - 6x^2 + 9x$. Solving $f'(x) = 3x^2 - 12x + 9 = 0$ gives $x = 1$ and $x = 3$, so there is a local maximum at (1, 4) and a local minimum at (3, 0); the roots are $x = 0$ and $x = 3$ (repeated), where the curve touches the axis.

The reciprocal / rational model

The model $y = a/x + b$ has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = b$. It fits quantities that fall steeply and then flatten — for example journey time against speed, or average fixed cost per item against the number of items produced.

Worked example 5 — inverse variation

The time t hours to complete a job varies inversely with the number of workers n. With 4 workers the job takes 9 hours. Find the model and the time for 6 workers.

Inverse variation: $t = k/n$. Use (4, 9): $9 = k/4$, so $k = 36$. Model: $t = 36/n$.

For 6 workers: $t = 36/6 =$ **6 hours**. Interpretation: more workers, less time — the product $n \times t =$ 36 worker-hours is constant.

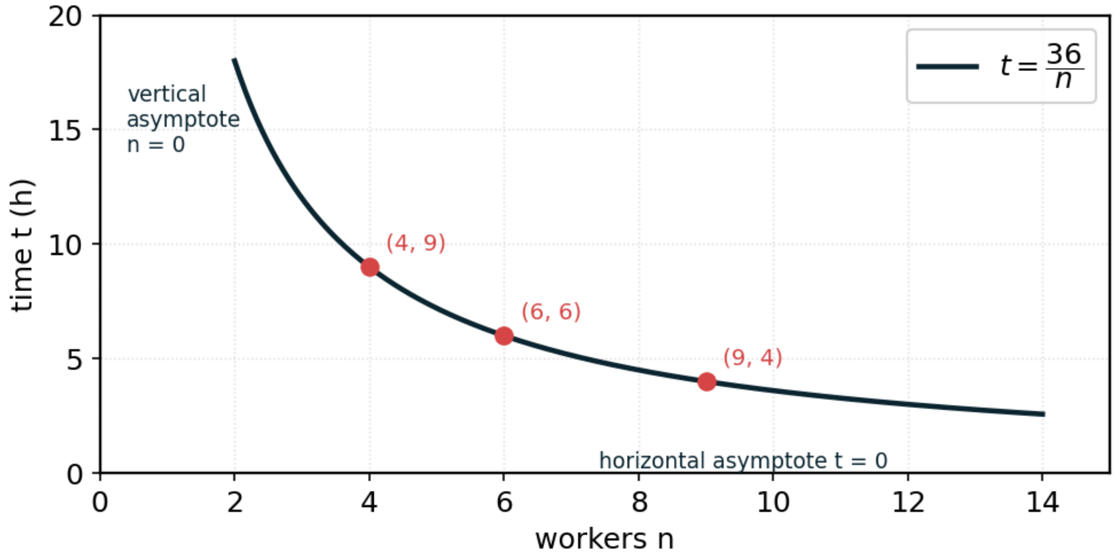

Figure 6. An inverse-variation model t = 36/n (hours to finish a job with n workers). The constant k = 36, so the product n×t = 36 worker-hours stays fixed: the points (4, 9), (6, 6) and (9, 4) all lie on the curve. There is a vertical asymptote at n = 0 and a horizontal asymptote at t = 0.

7. Sinusoidal models

Periodic phenomena — tides, temperature over a day or year, hours of daylight, the height of a point on a rotating wheel — repeat over a fixed interval and are modelled with sine or cosine:

$$y = a\sin(b(x - c)) + d \text{ or } y = a\cos(b(x - c)) + d$$

Parameter	Name	Meaning / how to find it
a	Amplitude	Half the distance from maximum to minimum: $a = (\max - \min)/2$
d	Principal axis	Midline value the curve oscillates about: $d = (\max + \min)/2$
b	Controls period	Period = $2\pi/b$ (radians) or $360°/b$ (degrees)
c	Horizontal shift (phase)	Moves the curve left/right; sets where a peak or the midline occurs

So the maximum value is $d + a$, the minimum is $d - a$, and one complete cycle takes the period. Choose sine or cosine to make c convenient: a cosine peaks at $x = c$, while a sine crosses its midline going upward there.

Worked example 6 — tide / temperature (sinusoidal model)

The depth of water in a harbour varies with the tide between a low of 2.0 m and a high of 8.0 m, with successive high tides 12 hours apart. High tide occurs at $t = 3$ hours. Model the depth and find the depth at $t = 7$ hours.

Amplitude $a = (8.0 - 2.0)/2 = 3.0$ m. Principal axis $d = (8.0 + 2.0)/2 = 5.0$ m.

Period 12 h, so $b = 360°/12 = 30°$ per hour (degree mode). A cosine peaks at $t = c$, and the peak is at $t = 3$, so $c = 3$.

Model: $D(t) = 3.0\cos(30(t - 3)) + 5.0$.

At $t = 7$: $30(7 - 3) = 120°$, $\cos 120° = -0.5$, so $D = 3.0(-0.5) + 5.0 =$ **3.5 m**.

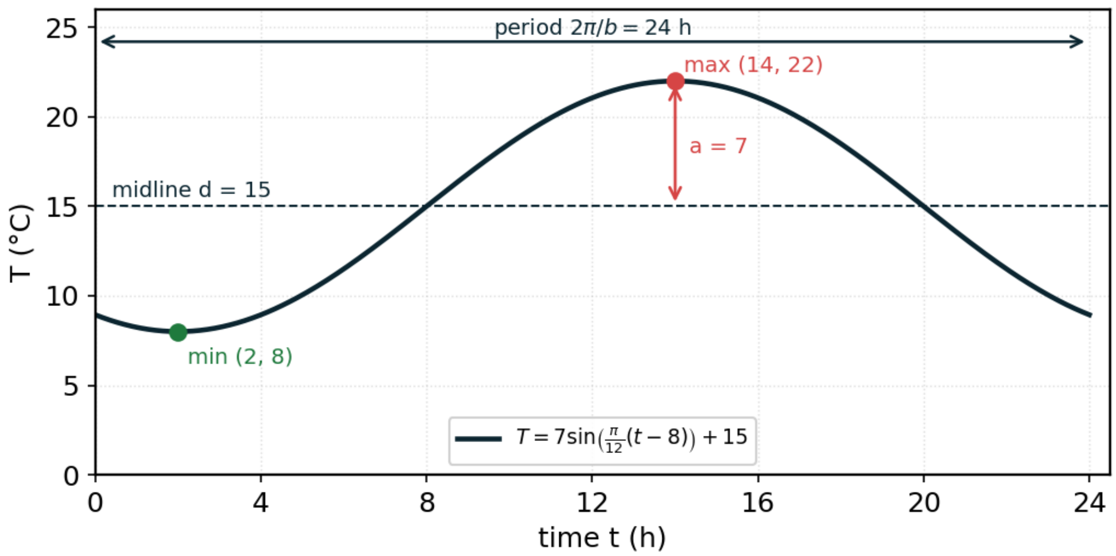

Figure 4. A sinusoidal daily-temperature model $T = 7\sin(\pi/12\,(t - 8)) + 15$ (radian mode). Amplitude $a = 7$, midline $d = 15$, and period $2\pi/b = 24$ h (verified: $b = \pi/12$). The maximum 22°C occurs at $t = 14$ h and the minimum 8°C at $t = 2$ h.

Common pitfall: Match your calculator mode to b. If the period is given in ordinary units and you set $b = 360/\text{period}$, work in **degrees**; if you set $b = 2\pi/\text{period}$, work in **radians**. Mixing them is the most common sinusoidal error.

8. Using models: interpolation, extrapolation and fit

Once a model is chosen and fitted, it is used to make predictions. Two words describe this:

- **Interpolation**: predicting *within* the range of the data used. Usually reliable.
- **Extrapolation**: predicting *beyond* that range. Risky, because the pattern may not continue.

Evaluating fit

Judge a model by how close its predictions are to the data and whether it respects the real behaviour (for example, a population model should not go negative). On the GDC, regression reports a correlation or R^2 value: the closer to 1, the better the linear fit — but a good number never overrides common sense about whether the model shape is appropriate.

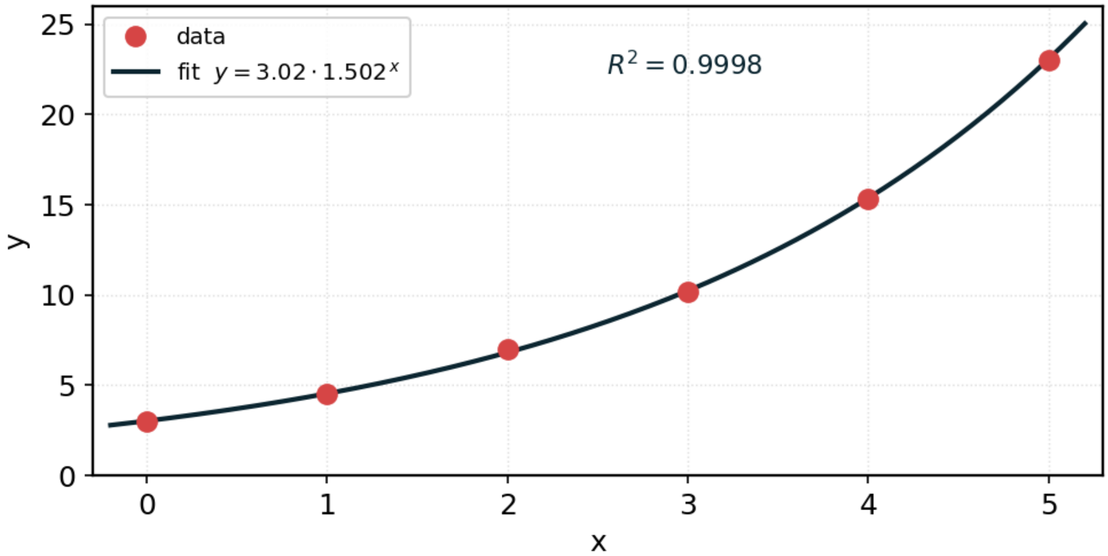

Figure 5. An exponential model fitted to data by regression. A least-squares fit of $y = AB^x$ to the six points gives $A \approx 3.02$ and $B \approx 1.502$ ($R^2 \approx 0.9998$); the curve drawn is exactly $y = 3.02 \times 1.502^x$ using those fitted values.

Solving equations graphically with a GDC

To answer "when does the quantity reach a given value", graph the model and the target value as two functions and use the calculator's intersection tool; to find a zero, use the root/zero tool. This graphical method works for every model in this topic, including ones with no neat algebraic solution such as exponentials and sinusoids.

> **Common pitfall:** Extrapolation can give absurd answers — a linear model of a child's height predicts a giant adult, and an unbounded exponential growth model eventually exceeds any real limit. Always ask whether the prediction is physically sensible before trusting it.

9. Further modelling functions (HL)

Logarithmic models (HL)

A logarithmic model $y = a + b \ln x$ rises quickly at first and then ever more slowly, with a vertical asymptote at $x = 0$. It suits quantities showing diminishing returns — perceived loudness or brightness against physical intensity, or a response that grows without a clear ceiling but with slowing gains. It is, in effect, the mirror image of an exponential.

The logistic model (HL)

Real growth is rarely unlimited: food, space or a market run out. The **logistic model** captures growth that starts nearly exponential and then levels off at a ceiling called the **carrying capacity**:

$$P(t) = L / (1 + Ce^{-kt})$$

Parameter	Meaning in the model
L	Carrying capacity: the upper limit (horizontal asymptote) the quantity approaches as $t \to \infty$.

Parameter	Meaning in the model
k	Controls how fast the growth happens (steepness of the rise).
C	Sets the starting value: at $t = 0$, $P = L/(1 + C)$.

Worked example 7 — logistic model (HL)

A population of fish in a lake is modelled by $P(t) = 5000 / (1 + 24e^{-0.3t})$, with t in years. Find the initial population, the carrying capacity, and the population after 10 years.

Initial ($t = 0$): $P = 5000/(1 + 24) = 5000/25 =$ **200 fish**.

Carrying capacity: as $t \to \infty$, $e^{-0.3t} \to 0$, so $P \to$ **5000 fish** (the asymptote L).

After 10 years: $e^{-0.3(10)} = e^{-3} = 0.0498$, so $P = 5000/(1 + 24 \times 0.0498) = 5000/2.195 \approx$ **2278 fish** (GDC).

Exam technique: For logistic models, read L straight off as the carrying capacity, get the initial value from $L/(1 + C)$, and use the GDC for any value in between — do not attempt to rearrange by hand under exam pressure.

10. Piecewise models and scaling (HL)

A **piecewise model** uses different functions on different parts of the domain, which is ideal when a real process changes behaviour. A journey might accelerate (quadratic), then cruise (linear), then stop; a heating system might warm exponentially and then hold constant. Define each piece with its interval and check the pieces meet sensibly at the boundaries.

Scaling adjusts a model to different units or sizes without changing its shape. Multiplying the output rescales the vertical axis (for example converting a model from thousands to units of currency), while multiplying the input rescales the horizontal axis (converting years to months). Recognising scaling lets you reuse a fitted model in new units and connects directly to the transformations in the next section.

Exam technique: For a piecewise model, always confirm which interval your input falls in *before* substituting, and use the boundary values to test that the pieces join (continuity) if the context requires a smooth quantity.

11. Transformations of graphs (HL)

Transformations let you build a model by adjusting a known parent function. Applied to $y = f(x)$:

Transformation	Effect on the graph	Modelling use
$f(x) + d$	Vertical translation by d	adds a baseline, e.g. the asymptote or principal axis
$f(x - c)$	Horizontal translation by c	shifts a starting time or phase
$a\,f(x)$	Vertical stretch, factor a	sets amplitude or scales the output units

Transformation	Effect on the graph	Modelling use
$f(bx)$	Horizontal stretch, factor $1/b$	changes a period or compresses a timescale
$-f(x)$	Reflection in the x-axis	turns growth into decay, a peak into a trough

This is exactly why the sinusoid $a \sin(b(x - c)) + d$ has the meanings it does: a is a vertical stretch (amplitude), b a horizontal stretch (period), c a horizontal translation (phase), and d a vertical translation (principal axis). **Composite transformations** combine several of these; apply stretches and reflections before translations to keep the order straight.

> **Exam technique:** Note the sign convention: $f(x - c)$ shifts the graph *right* by c (a positive c moves it in the positive direction), and $f(bx)$ with $b > 1$ *compresses* horizontally. These catch out many candidates.

12. Modelling with more than one function (HL)

Complex situations may need several functions used together, or compared to choose the best. Two ideas matter here.

Combining and comparing models

You might add or subtract models (total cost = fixed model + variable model), or graph two candidate models against the same data and pick the one that fits better and behaves sensibly beyond the data. Intersection points found on a GDC answer “when is option A cheaper than option B” type questions.

Least-squares fit (conceptually)

When a GDC fits a model to data by regression, it chooses the parameters that make the model as close as possible to the points. “As close as possible” is measured by the **sum of the squared vertical distances** (residuals) between the data points and the model; the best fit is the one that makes this sum least — hence **least squares**. You are not asked to compute it by hand, but you should understand that this is what “line of best fit” or a regression model means, and that a single large outlier can pull the fit noticeably because its residual is squared.

Assessing appropriateness

A high R^2 is not proof of a good model. Always check that the **shape** is right (does the quantity really level off, oscillate, or grow without limit?), that predictions stay physically possible, and that extrapolation is treated with caution. The best model balances closeness of fit against sensible behaviour.

13. Quick-reference: models at a glance

Model	Form	Key parameters / features
Linear	$y = mx + c$	m rate of change; c starting/fixed value
Quadratic	$y = ax^2 + bx + c$	vertex at $x = -b/2a$; $\Delta = b^2 - 4ac$
Cubic	$y = ax^3+bx^2+cx+d$	up to two turning points; find on GDC

Model	Form	Key parameters / features
Exponential	$y = ka^x + c$ or $ke^{rx} + c$	constant %; asymptote $y = c$; half-life/doubling
Reciprocal	$y = a/x + b$	asymptotes $x = 0$ and $y = b$
Sinusoidal	$y = a \sin(b(x-c)) + d$	amplitude a; period $2\pi/b$ or $360°/b$; axis d
Logarithmic (HL)	$y = a + b \ln x$	diminishing returns; asymptote $x = 0$
Logistic (HL)	$y = L/(1 + Ce^{-kx})$	carrying capacity L; start $L/(1+C)$

14. Practice questions

Attempt each question with a GDC where useful, then check against the worked answers that follow. Questions marked (HL) are Higher Level only.

1. A gym charges a $20 joining fee plus $15 per month. Write a linear model for the total cost C after n months and interpret both parameters.

2. A firework's height is $h(t) = -5t^2 + 30t$ metres. Find the maximum height and the time in the air.

3. A cup of coffee cools according to $T(t) = 20 + 70(0.85)^t$ with T in °C and t in minutes. State the room temperature and find the temperature after 6 minutes.

4. Using the model in Q3, find how long until the coffee reaches 40°C.

5. The number of bacteria doubles every 3 hours from an initial 500. Write an exponential model and find the number after 12 hours.

6. A daily temperature is modelled by $T(t) = 6 \sin(15(t - 9)) + 18$ in °C, with t in hours (degree mode). State the maximum and minimum temperatures and the period.

7. The time t to fill a pool varies inversely with the pump rate r. At 20 L/min it takes 90 minutes. Find the model and the time at 30 L/min.

8. A phone plan A costs $30 flat; plan B costs $10 + $0.05 per minute. Using a GDC, find how many minutes make the two plans cost the same.

9. (HL) A rumour spreads by $N(t) = 800/(1 + 15e^{-0.4t})$ people after t hours. State the carrying capacity and the initial number who knew it.

10. (HL) The graph of $y = f(x)$ is transformed to $y = 3f(x - 2) + 5$. Describe the transformations in words and their likely modelling meaning.

Worked answers

1. $C = 15n + 20$. Gradient 15 = monthly charge of $15; intercept 20 = one-off $20 joining fee (cost at $n = 0$).

2. Axis $t = -30/(2\times-5) = 3$ s; max height $h(3) = -5(9) + 30(3) = -45 + 90 =$ **45 m**. Lands when $-5t^2 + 30t = 0$, i.e. $t(30 - 5t) = 0$, so $t =$ **6 s** in the air.

3. Room temperature is the asymptote $c =$ **20°C**. After 6 min: $T = 20 + 70(0.85)^6 = 20 + 70(0.3771) = 20 + 26.4 =$ **46.4°C** (3 s.f.).

4. Solve $20 + 70(0.85)^t = 40$, so $(0.85)^t = 20/70 = 0.2857$. GDC intersection gives $t \approx$ **7.71 min**. Check: $20 + 70(0.85)^{7.71} \approx 40$ ✓.

5. $N = 500(2)^{t/3}$. After 12 h: $t/3 = 4$, so $N = 500(2)^4 = 500 \times 16 =$ **8000 bacteria**.

6. Amplitude 6, axis 18, so maximum = 18 + 6 = **24°C**, minimum = 18 − 6 = **12°C**. Period = 360°/15 = **24 hours** (one day, as expected).

7. $t = k/r$; using (20, 90): $k = 20 \times 90 = 1800$, so $t = 1800/r$. At $r = 30$: $t = 1800/30 =$ **60 minutes**.

8. Set $30 = 10 + 0.05m$, so $0.05m = 20$, $m =$ **400 minutes**. The GDC intersection of $y = 30$ and $y = 10 + 0.05x$ confirms $x = 400$.

9. (HL) Carrying capacity $L =$ **800 people** (the asymptote). Initial ($t = 0$): $N = 800/(1 + 15) = 800/16 =$ **50 people**.

10. (HL) Vertical stretch factor 3 (e.g. tripling the amplitude or output scale), horizontal translation 2 to the right (a 2-unit delay in the input), and vertical translation 5 up (raising the baseline / asymptote by 5).