Name:

Section:

Number Sequences Worksheet

1 The table below shows the number of ways John can pay when parking for various times.

Time (hours)	1	2	3	4	5	6	7		n	$n + 1$	$n + 2$
Number of ways	1	2	3	5	8	a	b		x	y	z

(i) Find the values of a and b. [3]

(ii) Write down an equation connecting x, y and z. [1]

2

The diagram shows the first four rows of a pattern of numbers.

Row 1 1 2 1

Row 2 2 3 2 3 2

Row 3 3 4 3 4 3 4 3

Row 4 4 5 4 5 4 5 4 5 4

The table shows some results obtained from this pattern.

Row number	1	2	3	4	5		*n*
Number of numbers in the row	3	5	7	9	*p*		*x*
Product of the first two numbers in the row	2	6	12	20	*q*		*y*
Sum of all the numbers in the row	4	12	24	40	*r*		*z*
Middle number in the row	2	2	4	4	*s*		

(a) Find the values of *p*, *q*, *r* and *s*. [2]

(b) Find expressions, in terms of *n*, for *x*, *y* and *z*. [3]

(c) Write down the middle number in Row 101. [1]

3 These are the first four patterns in a sequence made using counters.

(a) Complete the table for the patterns in this sequence.

Pattern number	1	2	3	4	5
Number of counters	3	8	15		

[1]

(b) Find an expression, in terms of *n*, for the number of counters in Pattern *n*.

..... [2]

(c) Ken has a bag containing 1358 counters.
He makes the largest possible pattern in the sequence, Pattern p, using these counters.

(i) Find the value of p.

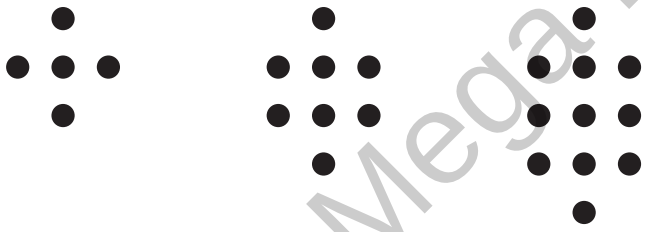
$p =$ ..... [3]

(ii) He uses all of the remaining counters to make another pattern in the sequence, Pattern q.

Find the value of q.

$q =$ ..... [2]

4 Shani makes a sequence of patterns using counters.



Pattern 1 Pattern 2 Pattern 3

(a) Complete the table.

Pattern number	1	2	3	4	5
Number of counters	5	8	11		

[1]

(b) Find an expression, in terms of n, for the number of counters in Pattern n.

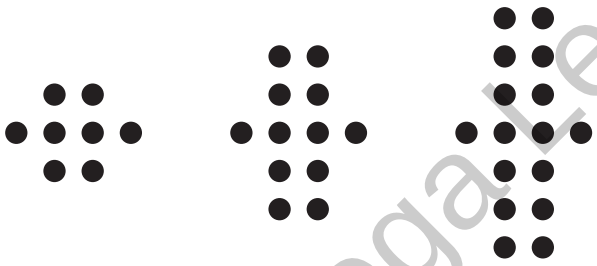
..... [2]

(c) Shani has 100 counters.
She uses some of the counters to make Pattern 20.
She uses all the remaining counters to make Pattern k.

Find the value of k.

$k =$ ..... [3]

5 **(a)** Here are the first three patterns in a sequence made from counters.



Pattern 1 Pattern 2 Pattern 3

(i) Complete the table for the patterns in this sequence.

Pattern number	1	2	3	4	5
Number of counters	8	12	16		

[1]

(ii) Find an expression, in terms of n, for the number of counters in Pattern n.

..... [2]

(iii) Jamal has 150 counters.
He uses these counters to make the largest pattern possible, Pattern p.

Find the value of p.

$p =$ ..... [2]

(b) The 4th term in a different sequence is 26.
This sequence is linear and the 8th term is 2.

(i) Find the first term of this sequence.

..... [2]

(ii) Find an expression, in terms of n, for the nth term of this sequence.

..... [2]

6 Here are the first four terms of a sequence.

$$\frac{12}{16} \quad \frac{17}{25} \quad \frac{22}{36} \quad \frac{27}{49}$$

Find an expression for the nth term of the sequence.

..... [4]

7 Here are the first four terms of a number sequence.

$T_1 = 1^2 + 3 \ = 4$

$T_2 = 2^2 + 8 \ = 12$

$T_3 = 3^2 + 13 = 22$

$T_4 = 4^2 + 18 = 34$

(a) Find T_5.

T_5 = ... [1]

(b) Find an expression, in terms of n, for T_n.

T_n = ... [3]

8 Here are the first three patterns in a sequence made using dots and lines.

Pattern 1 Pattern 2 Pattern 3

(a) Complete the table for the first five patterns in this sequence.

Pattern number	1	2	3	4	5
Number of dots	3	6			
Number of lines	2	7			

[2]

(b) Find an expression, in terms of n, for the number of lines in Pattern n.

..... [2]

(c) Anwar makes one of these patterns using 92 lines.

Find the number of dots in Anwar's pattern.

..... [2]

9 Here are the first five terms of a sequence.

$$\frac{3}{4} \quad \frac{7}{8} \quad \frac{11}{12} \quad \frac{15}{16} \quad \frac{19}{20}$$

(a) Write down the next two terms.

Answer ..... , ... [1]

(b) The kth term is $\frac{1199}{1200}$.

Find k.

Answer $k =$ ..... [1]

(c) Find an expression, in terms of n, for the nth term.

Answer ..... [2]

10 The first four terms, u_1, u_2, u_3 and u_4, in a sequence of numbers are given below.

$$u_1 = 1 \times 3 + 2^2 = 7$$

$$u_2 = 2 \times 4 + 3^2 = 17$$

$$u_3 = 3 \times 5 + 4^2 = 31$$

$$u_4 = 4 \times 6 + 5^2 = 49$$

(a) Evaluate u_5 .

Answer ..... [1]

(b) The nth term of the sequence, u_n, is of the form $n(n + p) + (n + q)^2$.

Write down the value of p and the value of q.

Answer $p =$ .....

$q =$ ..... [1]

(c) u_n can also be written in the form $An^2 + Bn + C$.

Find the values of A, B and C.

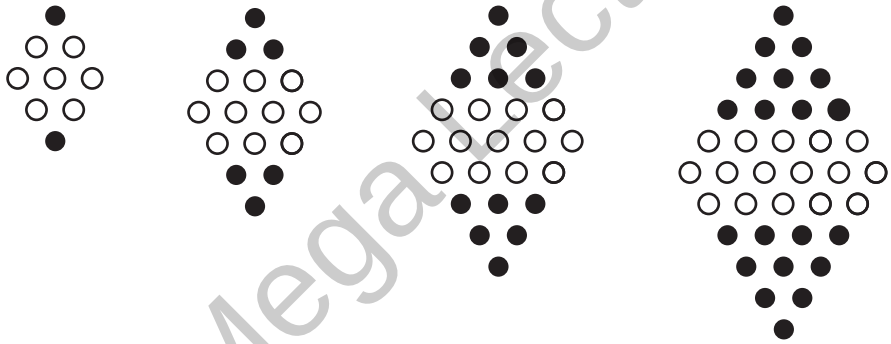
Answer $A =$ .....

$B =$ .....

$C =$ ..... [2]

11 The sequence of diagrams shows patterns made from some black beads and some white beads.
Each diagram has two rows more than the previous diagram.

Diagram 1 Diagram 2 Diagram 3 Diagram 4



(a) Complete the table for Diagram 5.

Diagram number	1	2	3	4	5
Total number of beads	9	16	25	36	
Number of white beads	7	10	13	16	
Number of black beads	2	6	12	20	

[1]

(b) Write down an expression, in terms of n, for

(i) the number of white beads in Diagram n,

Answer ..... [1]

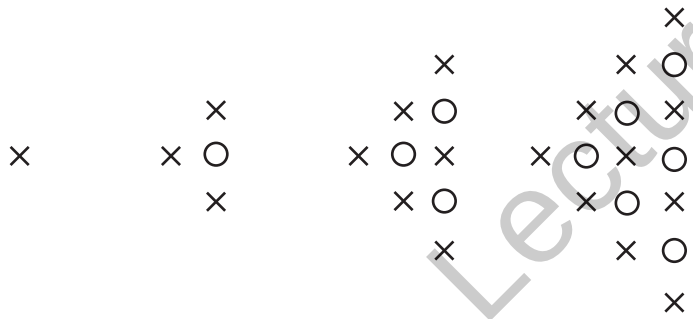
(ii) the total number of beads in Diagram n.

Answer ..... [1]

(c) Find an expression, in terms of n, for the number of black beads in Diagram n.
Give your answer in its simplest form.

Answer ..... [2]

12



Pattern 1 Pattern 2 Pattern 3 Pattern 4 Pattern 5

The diagrams show patterns made from crosses (×) and circles (○).

(a) Draw pattern 5 above. [1]

The table shows the number of crosses and circles in each pattern.

Pattern number (n)	1	2	3	4	5	6
Number of crosses	1	3	6	10		
Number of circles	0	1	3	6		
Total number of crosses and circles	1	4	9	16	25	36

(b) Complete the table. [2]

(c) Find an expression, in terms of n, for the total number of crosses and circles in pattern n.

..... [1]

(d) An expression, in terms of n, for the number of crosses in pattern n is $\frac{1}{2}n^2 + \frac{1}{2}n$.

How many crosses are there in pattern 30?

..... [1]

(e) Show that the number of circles in pattern n is $\frac{1}{2}n^2 - \frac{1}{2}n$.

[1]

(f) The number of **crosses** in pattern m is equal to $5m$.

Find m.

Answer $m =$ ..... [3]

13 Two sequences have 1, 3, 5 as their first three terms.

(a) In the first sequence, each term is 2 more than the term before it.

(i) Find an expression, in terms of n, for the nth term.

Answer ..... [1]

(ii) The kth term of this sequence is 841.

Find the value of k.

Answer $k =$ ..... [1]

(b) The nth term of the second sequence is

$$2^{n-1} - \frac{(n-1)(n-4)}{2}.$$

(i) Find the fourth term of this sequence.

Answer ..... [1]

(ii) Find the fifth term of this sequence.

Answer ..... [1]

14 (a) The nth term of a sequence is given by $n^2 - 5n$.

(i) Find the 2nd term in the sequence.

Answer ..... [1]

(ii) The pth term in the sequence is 150.

Find the value of p.

Answer $p =$ ..... [2]

(b) The nth term of another sequence is given by $3n^2 - kn$.
The 5th term in this sequence is 55.

Find the value of k.

Answer $k =$ ..... [2]

15 The first term of a sequence is 13.
The following terms are found by alternately adding 4 and 6 to the previous term.
The first six terms are

13 17 23 27 33 37

(a) Write down the next two terms of the sequence.

Answer ..... [1]

(b) Write down the **value** of the term that is closest to 999.

Answer ..... [1]

(c) Write down the difference between the values of the 91st and 93rd terms.

Answer ..... [1]

(d) Find the 80th term.

Answer ..... [1]

(e) The nth term is 203.

Find n.

Answer $n =$ ..... [1]

16 (a) The first four terms of a sequence, S, are 89, 83, 77, 71.

(i) Find an expression for S_n, the nth term of this sequence.

Answer $S_n =$ ..... [2]

(ii) Find the smallest value of n for which $S_n < 0$.

Answer $n =$ ..... [1]

(b) The *n*th term of a different sequence, *T*, is given by $T_n = n^2 - 4n$.

(i) Find and simplify an expression for $T_{n+1} - T_n$.

Answer ..... [2]

(ii) The difference between T_{p+1} and T_p is 75.

Find the value of *p*.

Answer $p =$ ..... [1]

17 The sequence of diagrams below shows small black and small white squares in an arrangement to form large squares.

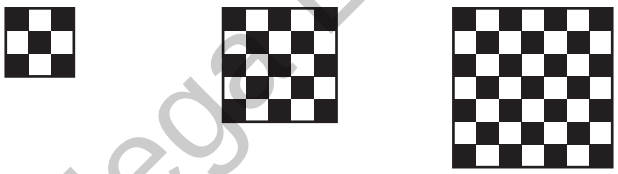


Diagram 1 Diagram 2 Diagram 3

The table below shows the numbers of black and white squares in each diagram.

Diagram (*n*)	1	2	3	4
Black squares	5	13	25	
White squares	4	12	24	
Total number of black and white squares	9	25	49	

(a) For each diagram, how many more black squares are there than white squares?

Answer ..... [1]

(b) On the table, complete the column for Diagram 4. [1]

(c) Write down an expression, in terms of n, for the total number of black and white squares in Diagram n.

Answer ..... [1]

18 **(a)** The first five terms of a sequence are 17, 11, 5, −1, −7.

Find, in terms of n, an expression for the nth term of this sequence.

Answer ..... [2]

(b) The nth term, S_n, of a different sequence is found using the formula $S_n = n^2 + 3n$.

(i) Work out the first four terms of this sequence.

Answer ....., ..., ..., ... [2]

(ii) The nth term, T_n, of another sequence is found using the formula $T_n = 5n - 12$.

There are two values of n for which $\frac{S_n}{T_n} = 6$.

Form and solve an equation in n to find these two values.

Answer n = ... and ..... [4]

19 The first four lines of a pattern of numbers are shown below.

1st line	3^2	−	1^2	=	8×1
2nd line	5^2	−	1^2	=	$8 \times (1 + 2)$
3rd line	7^2	−	1^2	=	$8 \times (1 + 2 + 3)$
4th line	9^2	−	1^2	=	$8 \times (1 + 2 + 3 + 4)$

(a) Write down the 7th line of the pattern.

Answer ..... [1]

(b) Write down an expression, in terms of n, to complete the nth line of the pattern.

Answer ..... $= 8 \times (1 + 2 + 3 + 4 + \ \ldots + n)$ [1]

(c) **Using the *n*th line of the pattern**, show that $1 + 2 + 3 + 4 + \ldots + n = \dfrac{n(n + 1)}{2}$.

[2]

20 Small triangles are formed by placing rods between dots as shown in the diagrams.

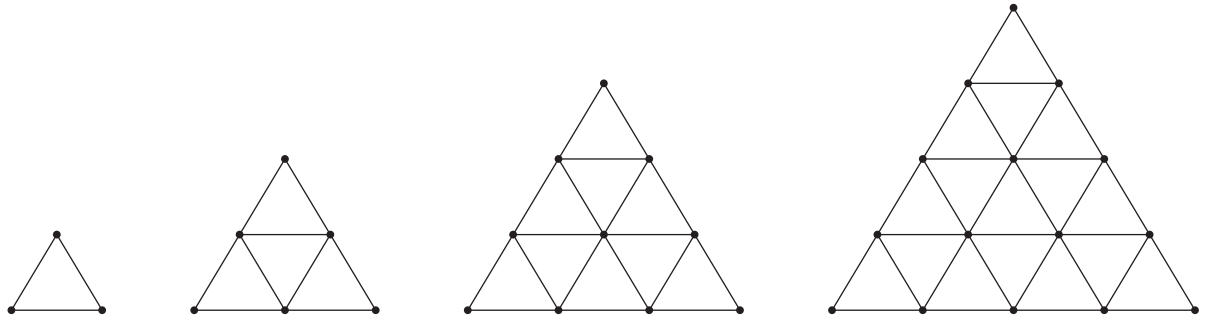


Diagram 1 Diagram 2 Diagram 3 Diagram 4

(a) Complete the table.

Diagram n	1	2	3	4	5
Number of small triangles (T)	1	4	9	16	
Number of dots (D)	3	6	10	15	
Number of rods (R)	3	9	18	30	

[2]

(b) Find an expression, in terms of n, for the number of small triangles (T) formed in Diagram n.

Answer .....[1]

(c) Given that $R = D + T - 1$, find the value of n when $D = 561$ and $R = 1584$.

Answer $n =$ .....[2]

21 These are the first five terms of a sequence.

4 8 16 32 64

(a) Find the next number in the sequence.

..... [1]

(b) The nth term of the sequence above is 2^{n+1} .

Write down an expression, in terms of n, for the nth term of these sequences.

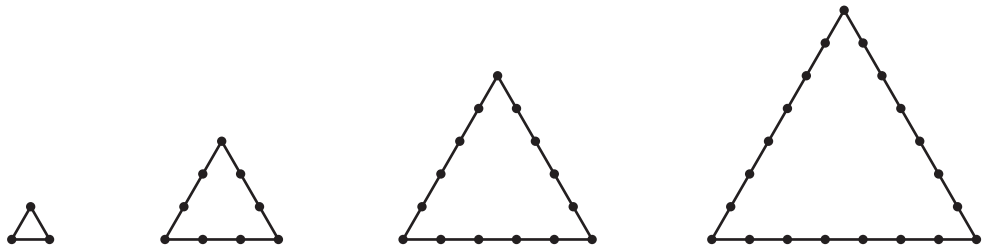
(i) 1 5 13 29 61 …

..... [1]

(ii) 10 19 32 53 90 …

..... [2]

22



Triangle 1 Triangle 2 Triangle 3 Triangle 4

The diagrams show a sequence of triangles made up of identical sticks.
Each triangle has two more sticks on each edge than its previous triangle.
The table shows information relating to this sequence.

The diagrams show a sequence of triangles made up of identical sticks.
Each triangle has two more sticks on each edge than its previous triangle.
The table shows information relating to this sequence.

Triangle number	1	2	3	4		*n*
Number of sticks on each side	1	3	5			*x*
Number of sticks in the triangle	3	9	15			*y*

(a) Complete the column for triangle 4. [1]

(b) Find an expression, in terms of n, for x.

Answer $x =$ ..... [1]

(c) Find an expression, in terms of n, for y.

Answer $y =$ ..... [1]

(d) The total number of sticks in the first triangle = 3
The total number of sticks in the first two triangles = 12
The total number of sticks in the first three triangles = 27

(i) Write down the total number of sticks in the first four triangles.

Answer ..... [1]

(ii) Find an expression, in terms of n, for the total number of sticks in the first n triangles.

Answer ..... [2]

23 The nth term of a sequence is $9n + 4$.

(a) Calculate the value of the term that is closest to 2012.

Answer .....[2]

(b) Calculate the difference between the 10th term and the 6th term.

Answer ..... [1]

(c) **(i)** Find an expression, in terms of x and y, for the difference between the xth term and the yth term.

Answer ..... [1]

(ii) **Hence** explain why it is not possible for any two terms of this sequence to differ by 123.

Answer .....

..... [1]

24 It is given that $N = 87 \times 132$.

(a) Complete the statements in the answer space.

Answer (a) $88 \times 132 = N +$ ..... [1]

$87 \times 131 = N -$ ..... [1]

(b) Hence evaluate $88 \times 132 - 87 \times 131$.

Answer (b) ..... [1]