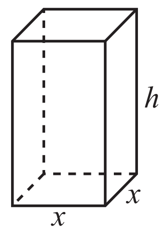
Name:

Section:

Graphs of Functions Worksheet

1



A cuboid has height h cm and a square base of edge x cm.
The volume of the cuboid is $60\,\text{cm}^3$.

(a) Show that the surface area, $A\,\text{cm}^2$, of the cuboid is given by $A = 2x^2 + \dfrac{240}{x}$.

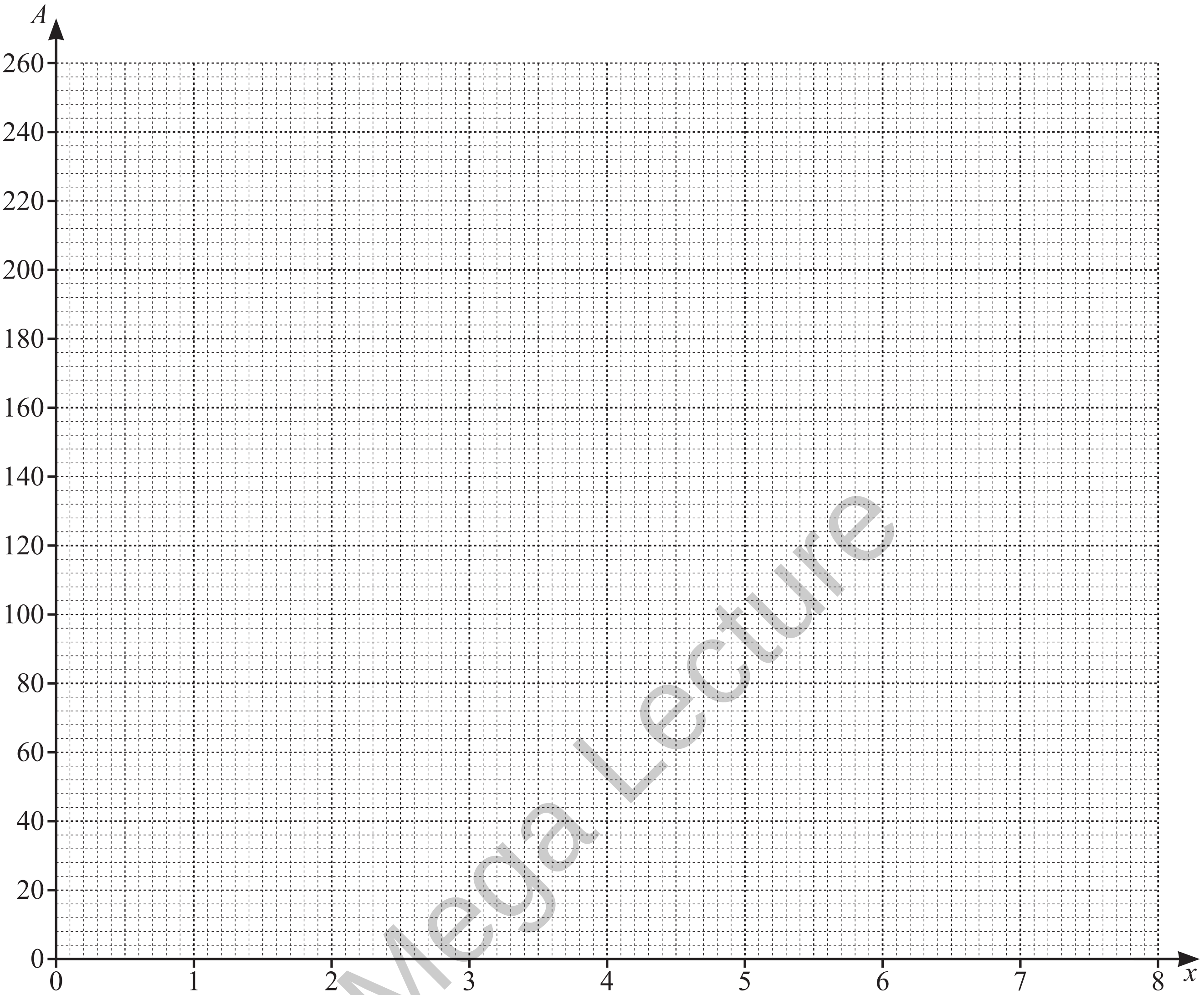
[2]

(b) Complete the table for $A = 2x^2 + \dfrac{240}{x}$.

x	1	2	3	4	5	6	7	8
A	242	128	98	92			132	158

[2]

(c) On the grid, draw the graph of $A = 2x^2 + \dfrac{240}{x}$ for $1 \leqslant x \leqslant 8$.



[3]

(d) Find the minimum possible surface area of the cuboid.

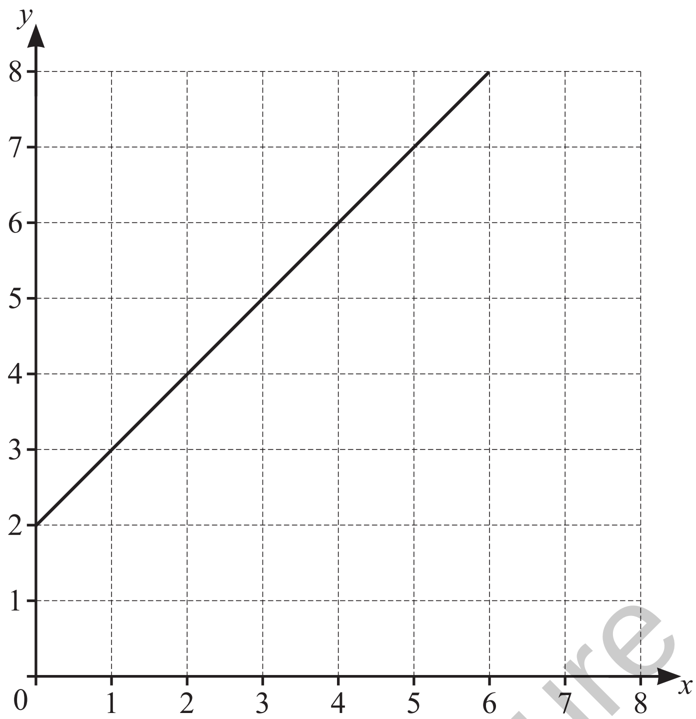
.... cm^2 [1]

(e) The cuboid has a surface area of $120\,cm^2$.
The height of the cuboid is greater than the length of the edge of its base.

Find the dimensions of the cuboid.

.... cm by .... cm by .... cm [3]

2



The line $y = x + 2$ is drawn on the grid.

(a) On the grid, draw the line $x + 2y = 7$. [2]

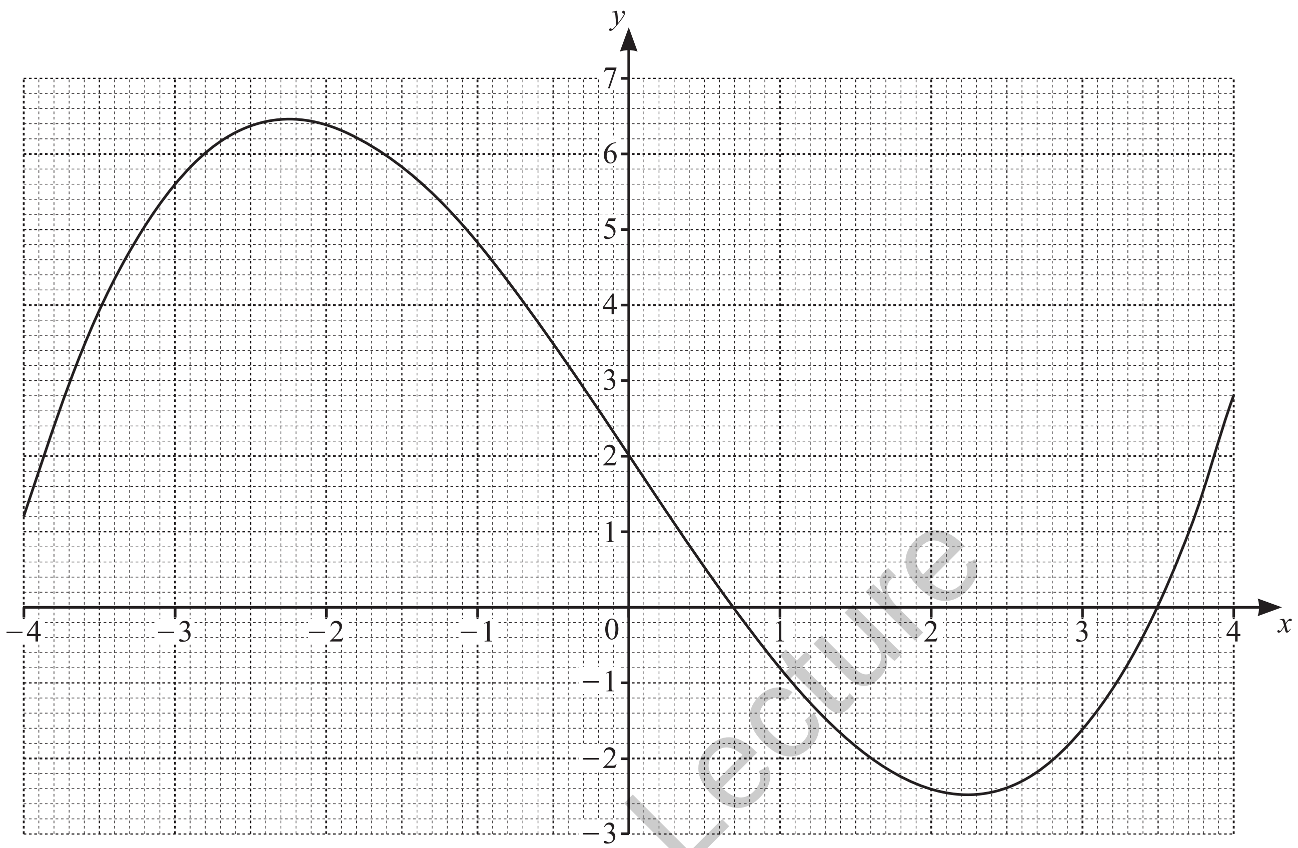
(b) Use your graph to find the solution of these simultaneous equations.

$$y = x + 2$$
$$x + 2y = 7$$

$x =$ ....

$y =$ .... [1]

3 The graph of $y = \frac{x^3}{5} - 3x + 2$ is drawn on the grid.



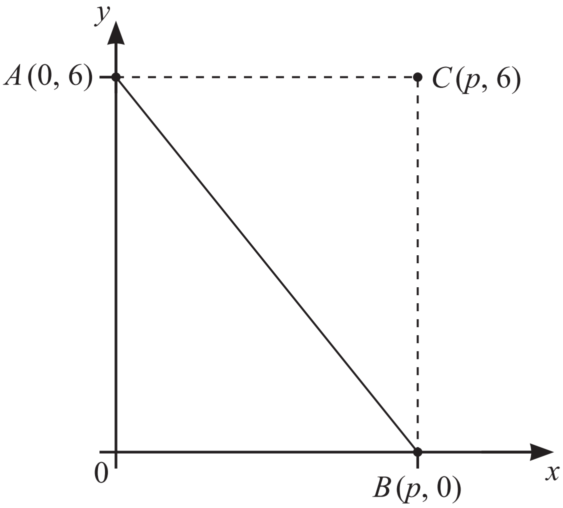
(a) By drawing a tangent, estimate the gradient of the curve at $x = -1$.

.... [2]

(b) By drawing a suitable straight line on the graph, find the solutions of the equation $\frac{x^3}{5} - 3x = 0$.

.... [3]

4



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The diagram shows the points $A(0, 6)$, $B(p, 0)$ and $C(p, 6)$.
The equation of the line AB is $3y + 4x = 18$.

(a) Find the value of p.

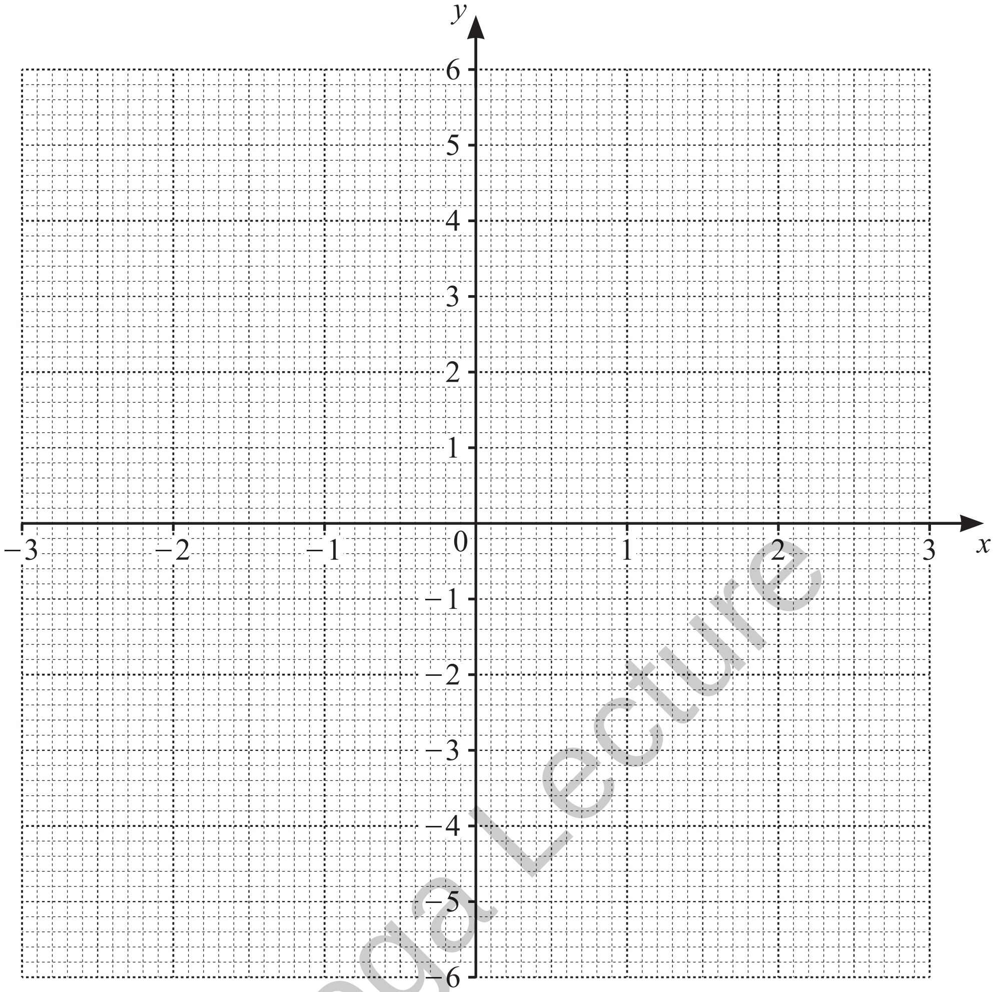
$p =$ .... [1]

5 **(a)** Complete the table for $y = \frac{x^3}{2} - 3x - 1$.

x	−3	−2	−1	0	1	2	3
y		1	1.5	−1	−3.5	−3	3.5

[1]

(b) On the grid, draw the graph of $y = \frac{x^3}{2} - 3x - 1$ for $-3 \leqslant x \leqslant 3$.



[3]

(c) Use your graph to explain why $x^3 - 6x - 2 = 6$ has only one solution.

.... [2]

(d) Line L passes through the points (1, 1) and (−2, −1).

(i) On the grid, draw line L. [1]

(ii) Work out the gradient of line L.

.... [2]

(iii) Find the x-coordinates of the points where line L intersects the curve $y = \frac{x^3}{2} - 3x - 1$.

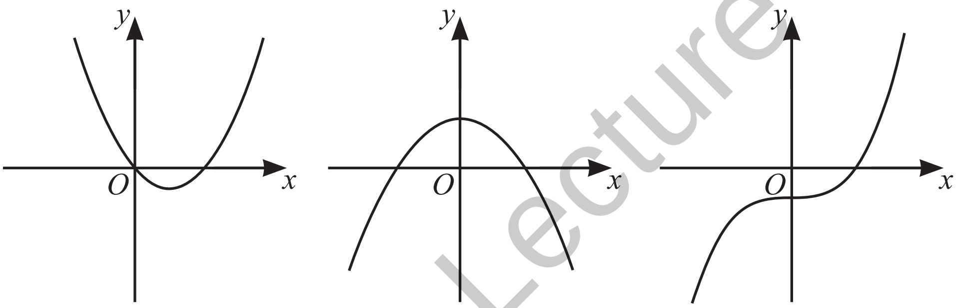
$x =$ .... , $x =$ .... , $x =$ .... [2]

6 Here are the equations of five curves.

$y = 2 - x^2$ $y = x^3 - 2$ $y = x^2 + 2x - 8$ $y = x^3 - 3x$ $y = x^2 - 3x$

Sketches of three of these curves are drawn below.

Write the correct equation underneath each sketch.



.... [3]

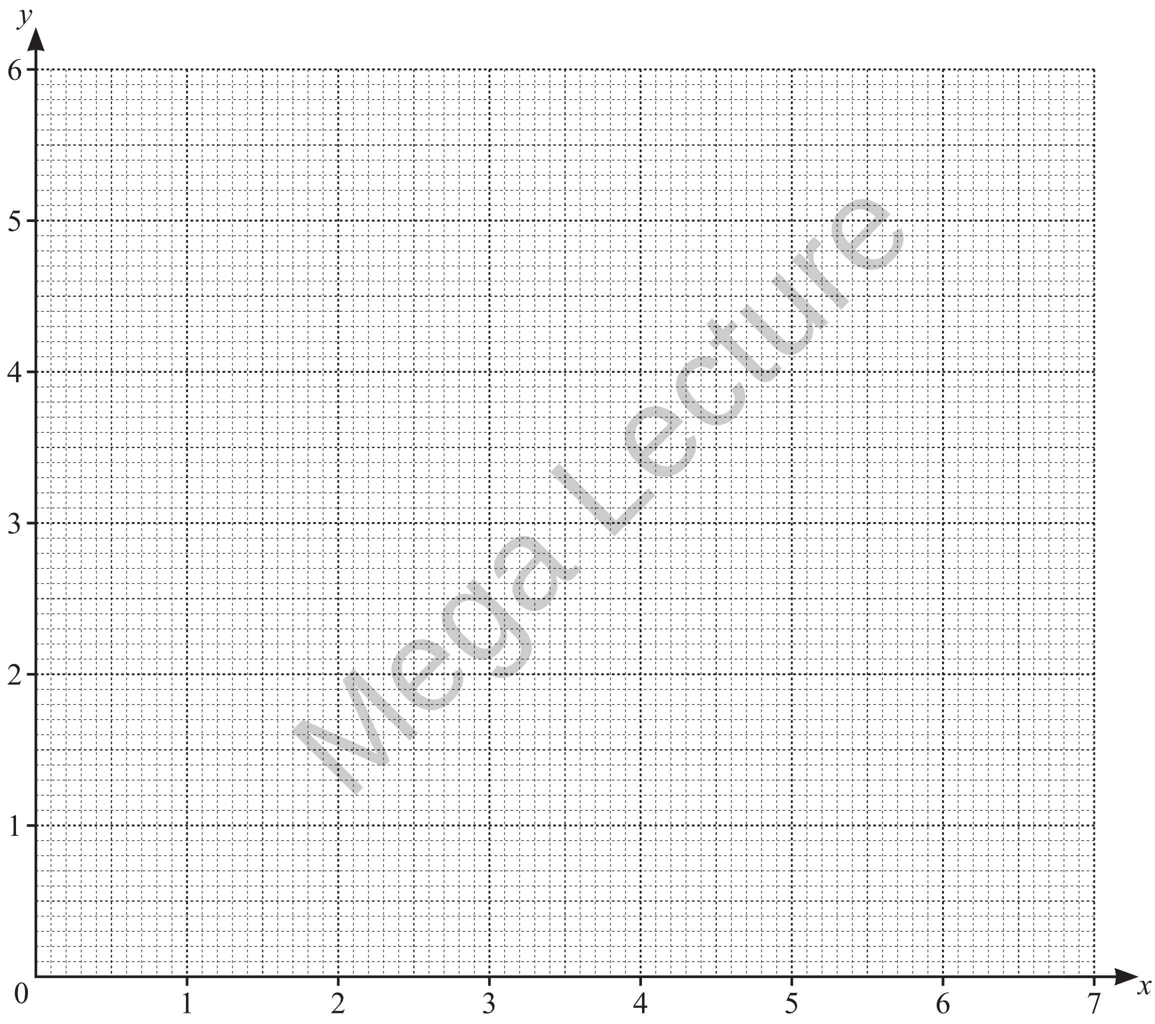
7 **(a)** Complete the table for $y = \frac{x}{4} + \frac{2}{x}$.

The values of y are given correct to 2 decimal places where appropriate.

x	0.5	1	1.5	2	3	4	5	6	7
y	4.13	2.25	1.71	1.5	1.42	1.5	1.65	1.83	

[1]

(b) On the grid, draw the graph of $y = \frac{x}{4} + \frac{2}{x}$ for $0.5 \leqslant x \leqslant 7$.



[3]

(c) By drawing a tangent, estimate the gradient of $y = \frac{x}{4} + \frac{2}{x}$ when $x = 1$.

.... [2]

(d) **(i)** On the grid, draw the graph of $2y + x = 6$.

[2]

(ii) Write down the x-coordinates of the points of intersection of the graphs of $2y + x = 6$ and $y = \frac{x}{4} + \frac{2}{x}$.

$x =$ .... and $x =$ .... [2]

(iii) These x-coordinates are the solutions of the equation $3x^2 + Ax + B = 0$.

Use $2y + x = 6$ and $y = \frac{x}{4} + \frac{2}{x}$ to find the value of A and the value of B.

$A =$ ....

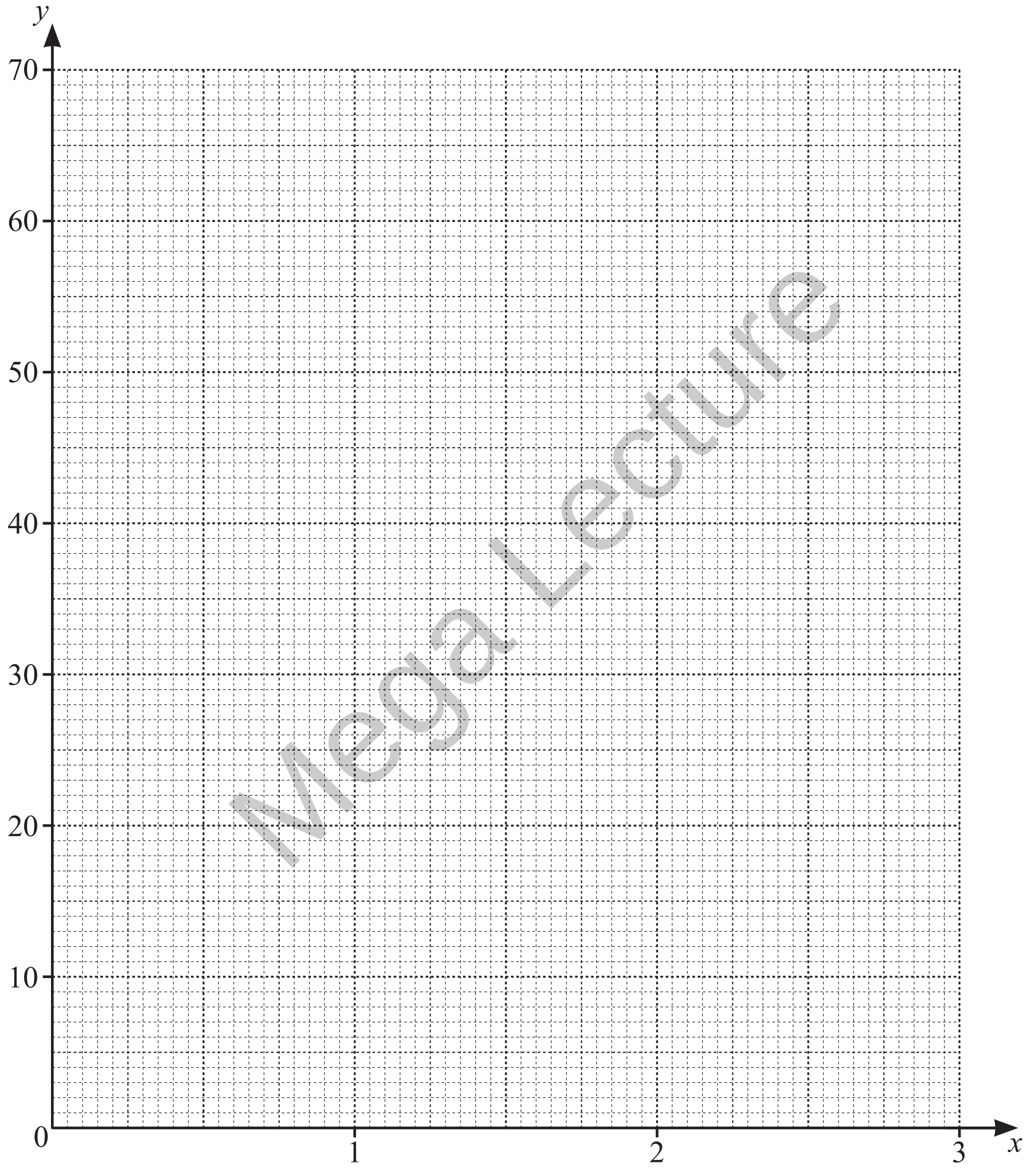
$B =$ .... [3]

8 **(a)** The table shows some values for $y = 4^x$.

x	0	0.5	1	1.5	2	2.5	3
y			4	8	16	32	64

(i) Complete the table. [1]

(ii) Draw the graph of $y = 4^x$ for $0 \leqslant x \leqslant 3$.



[3]

(iii) By drawing a tangent, estimate the gradient of the curve when $x = 2$.

.... [2]

(iv) The solutions of the equation $3(4^x) + ax + b = 0$ can be found from the points of intersection of $y = 4^x$ and $y = 20x - 12$.

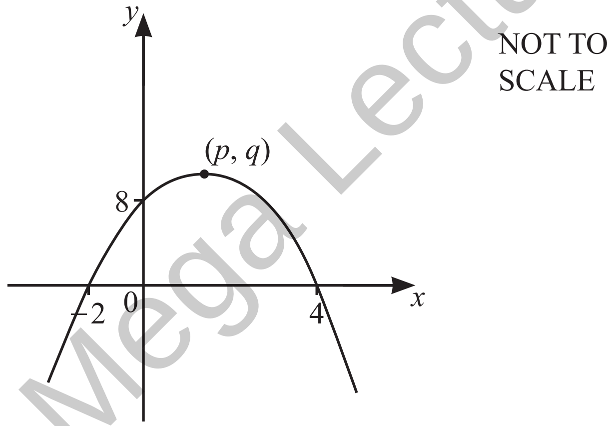
(a) Find the value of a and the value of b.

$a =$ .... $b =$ .... [2]

(b) By drawing the line $y = 20x - 12$ on the grid opposite, find all the solutions of $3(4^x) + ax + b = 0$.

.... [3]

(b) Here is a sketch of the graph of a quadratic function.



The curve has a maximum point (p, q).

Find the value of p and the value of q.

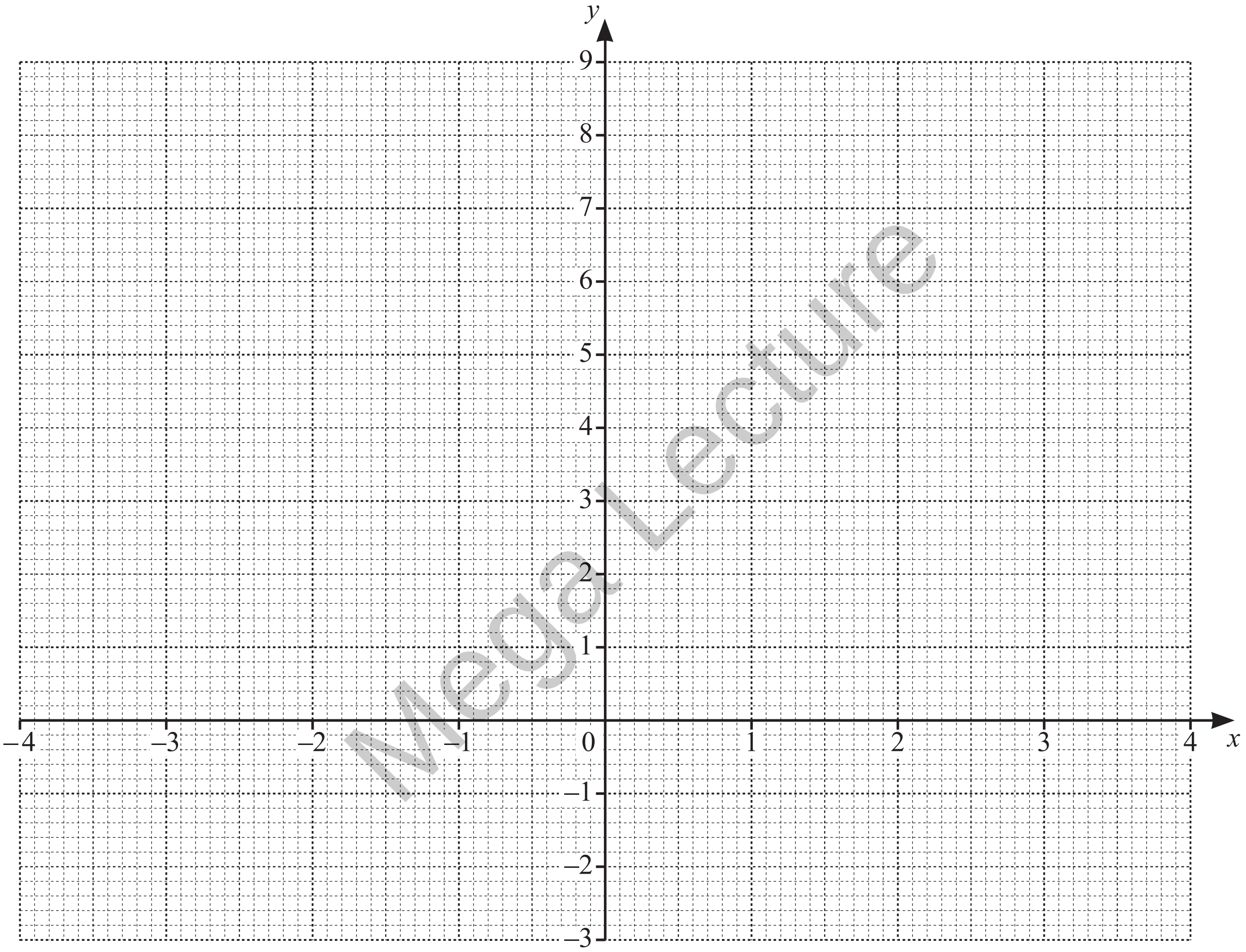
$p =$ .... $q =$ .... [3]

9 **(a)** Complete the table for $y = 3 + 2x - \frac{x^3}{5}$.

x	−4	−3	−2	−1	0	1	2	3	4
y	7.8	2.4	0.6	1.2	3	4.8	5.4	3.6	

[1]

(b) Draw the graph of $y = 3 + 2x - \frac{x^3}{5}$ for $-4 \leqslant x \leqslant 4$.



[3]

(c) By drawing a tangent, estimate the gradient of the graph of $y = 3 + 2x - \frac{x^3}{5}$ at (1, 4.8).

.... [2]

(d) **(i)** On the grid, draw the line $2y + x = 8$.

[2]

(ii) Write down the x-coordinates of the points where the line intersects the graph of $y = 3 + 2x - \frac{x^3}{5}$.

.... [2]

(iii) These x-coordinates are the solutions of the equation $2x^3 + Ax + B = 0$.

Find the value of A and the value of B.

A = ...

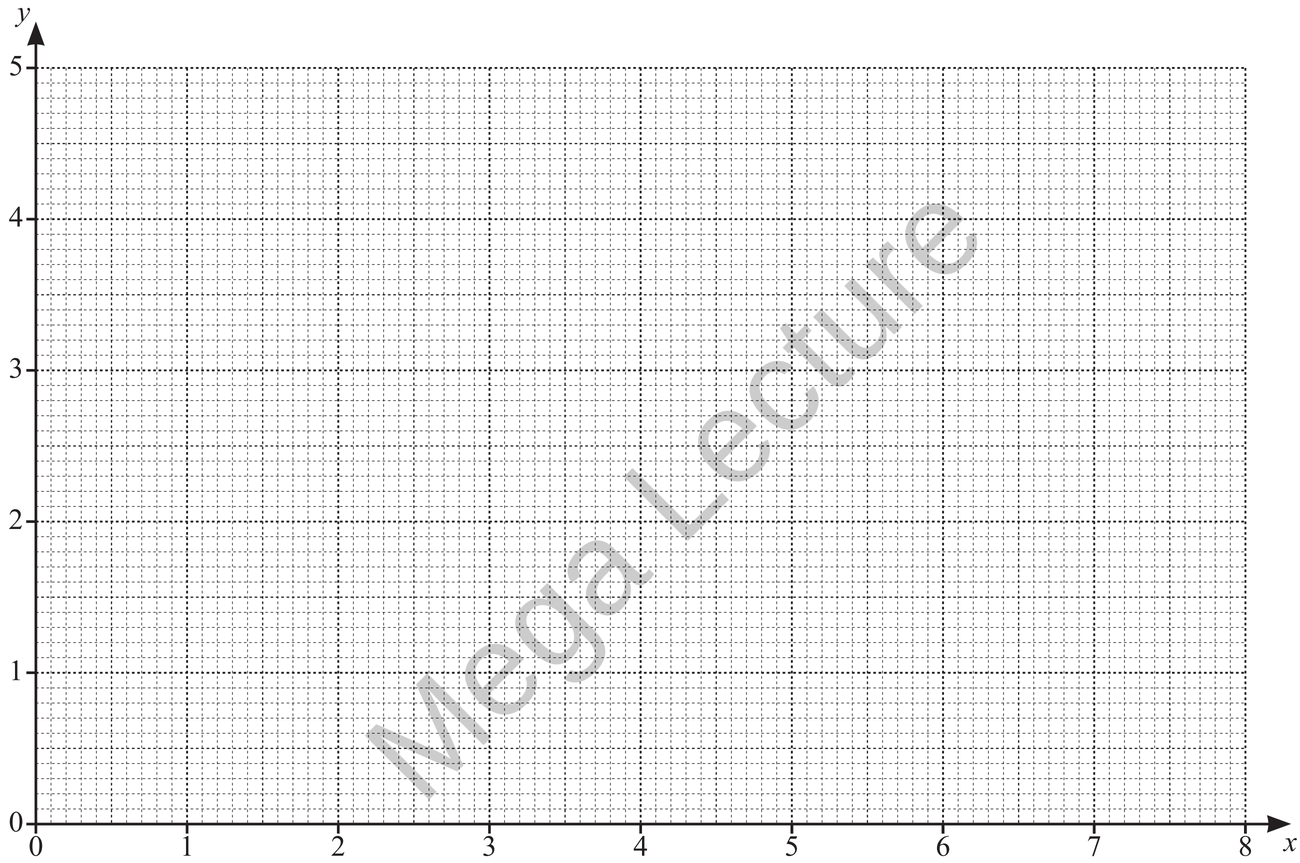
B = ... [3]

10 The table shows some values for $y = 1 + \frac{2}{x}$, given correct to 2 decimal places where appropriate.

x	0.5	1	2	3	4	5	6	7	8
y	5	3	2	1.67	1.5	1.4	1.33	1.29	

(a) Complete the table. [1]

(b) Draw the graph of $y = 1 + \frac{2}{x}$ for $0.5 \leqslant x \leqslant 8$.



[2]

(c) The line L crosses the graph of $y = 1 + \frac{2}{x}$ at $x = 2$ and $x = 5$.

Find the equation of L.

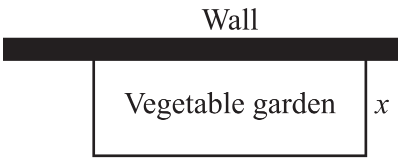
.... [3]

(d) A line with gradient $-\frac{1}{3}$ crosses the graph of $y = 1 + \frac{2}{x}$ when $x = 1$ and when $x = k$.

By drawing a suitable line on your grid, find k.

$k =$ .... [2]

11 Zara fences off a piece of land next to a wall to make a vegetable garden.



The garden is a rectangle with the wall as one side of the rectangle.
The area of the garden is 18 square metres.
The width of the garden is x metres.

(a) The total length of fencing required for the garden is y metres.

Show that $y = 2x + \frac{18}{x}$.

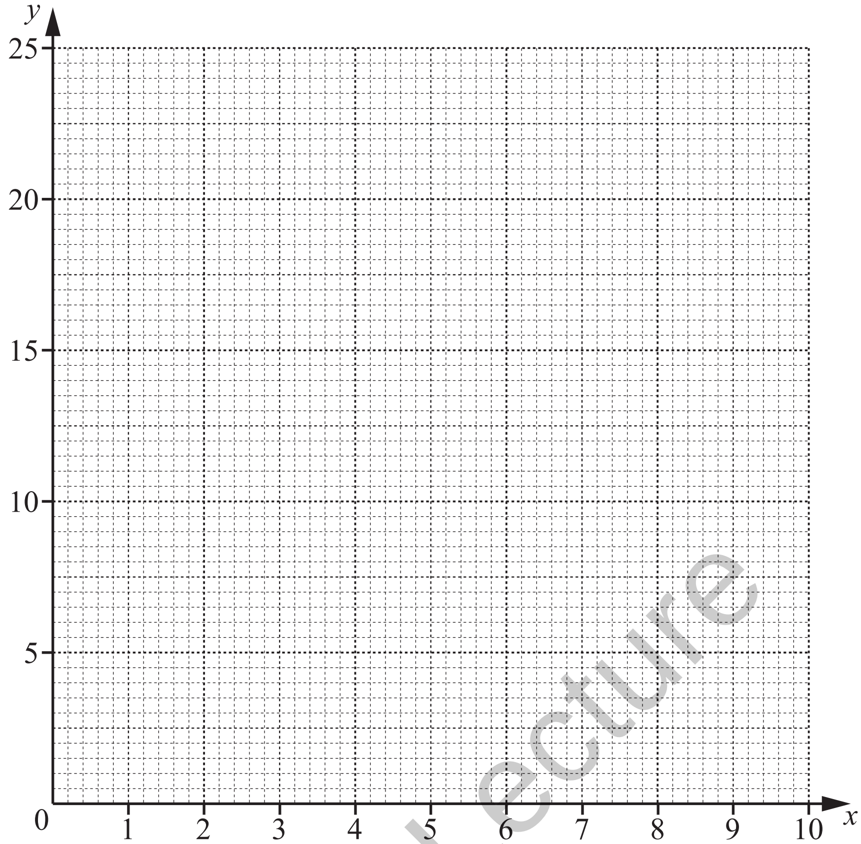
[1]

(b) **(i)** Complete the table for $y = 2x + \frac{18}{x}$.

x	1	2	3	4	5	6	7	8	9
y			12	12.5	13.6	15	16.6	18.3	

[2]

(ii) On the grid, draw the graph of $y = 2x + \dfrac{18}{x}$ for $1 \leqslant x \leqslant 9$.



[3]

(c) Use your graph to find the two possible widths of the garden if 14 metres of fencing is used.

Answer .... m or .... m [2]

(d) The fencing costs $20 per metre.

(i) Find the minimum amount it will cost Zara to build the fence.

$ .... [2]

(ii) Zara wants to spend no more than $350 on the fence.

Find the greatest possible width of the garden Zara can make.

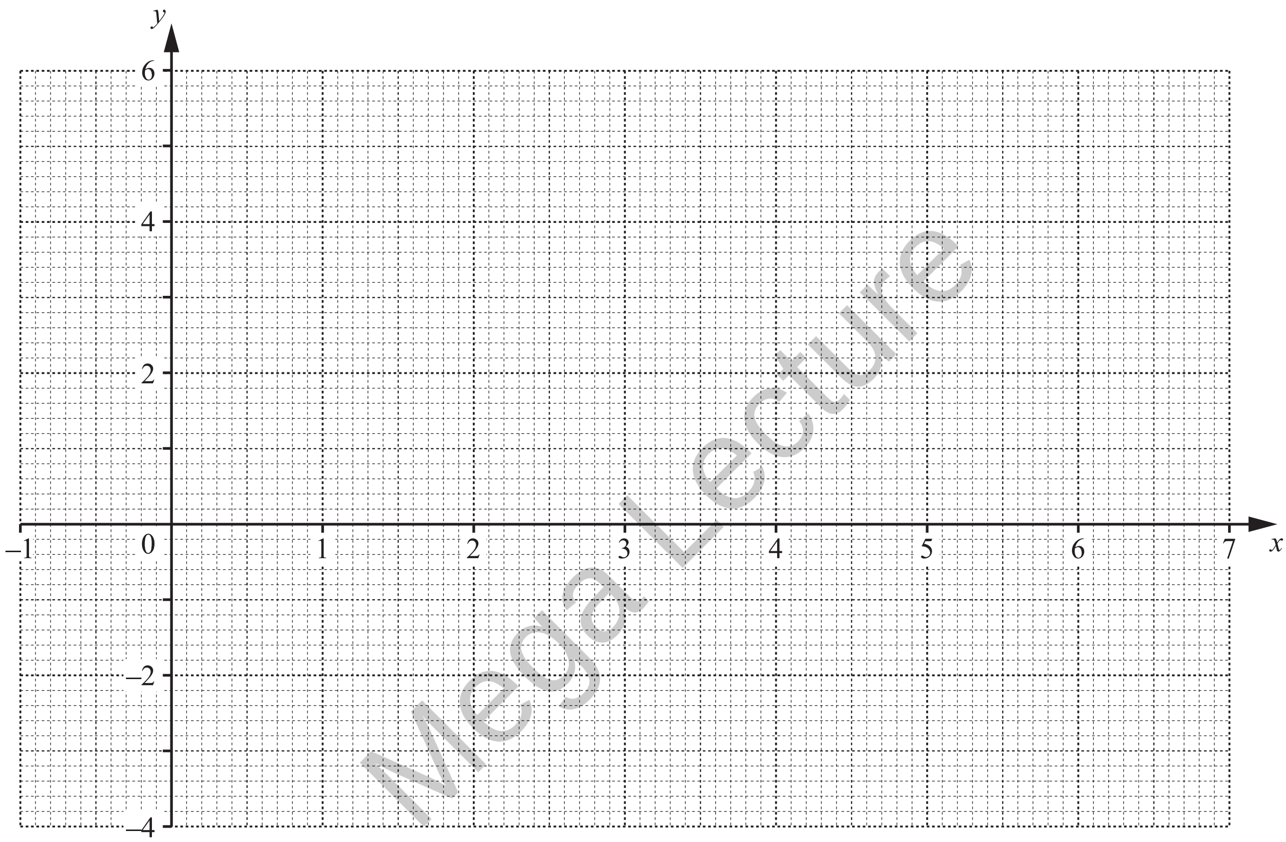
Answer .... m [2]

12 **(a)** Complete the table for $y = \frac{x^2}{2} - 3x + 2$.

x	-1	0	1	2	3	4	5	6	7
y		2	–0.5	–2	–2.5	–2	–0.5	2	

[1]

(b) Draw the graph of $y = \frac{x^2}{2} - 3x + 2$ for $-1 \leqslant x \leqslant 7$.



[3]

(c) By drawing a tangent, estimate the gradient of the curve at $x = 1.5$.

Answer .... [2]

(d) Complete these inequalities to describe the range of values of x where $y \geqslant 0$.

Answer $x \leqslant$ ....

$x \geqslant$ .... [2]

(e) **(i)** On the same grid, draw the line $4y + 3x = 12$. [2]

(ii) The x-coordinates of the points of intersection of this line and the curve are the solutions of the equation $2x^2 + Ax + B = 0$.

Find the value of A and the value of B.

Answer $A =$ ....

$B =$ .... [2]

13 **(a)** The variables x and y are connected by the equation $y = 3 + x - \frac{x^2}{2}$.

Some corresponding values of x and y are given in the table below.

x	−3	−2	−1	0	1	2	3	4	5
y		−1	1.5	3	3.5	3	1.5	−1	

(i) Complete the table. [1]

(ii) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-3 \leqslant x \leqslant 5$.
Using a scale of 1 cm to 1 unit, draw a vertical y-axis for $-5 \leqslant y \leqslant 5$.

Draw the graph of $y = 3 + x - \frac{x^2}{2}$ for $-3 \leqslant x \leqslant 5$.

[3]

(iii) By drawing a tangent, estimate the gradient of the curve at (3, 1.5).

Answer .... [2]

(iv) The points of intersection of the graph of $y = 3 + x - \frac{x^2}{2}$ and the line $y = k$ are the solutions of the equation $10 + 2x - x^2 = 0$.

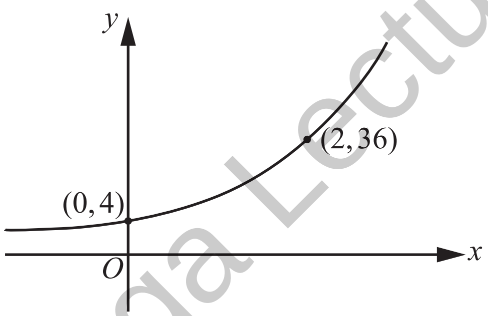
(a) Find the value of k.

Answer .... [1]

(b) By drawing the line $y = k$ on your graph, find the solutions of the equation $10 + 2x - x^2 = 0$.

Answer .... [2]

(b) This is a sketch of the graph of $y = pa^x$, where $a > 0$.
The graph passes through the points (0, 4) and (2, 36).



(i) Write down the value of p.

Answer .... [1]

(ii) Find the value of a.

Answer .... [1]

(iii) The graph passes through the point (4, q).

Find the value of q.

Answer .... [1]

14 A random number, x, is generated, where x is any real number.

(a) Manuel adds 2 to x.
He subtracts x from 10.
Manuel then multiplies these two results to give his number, y.

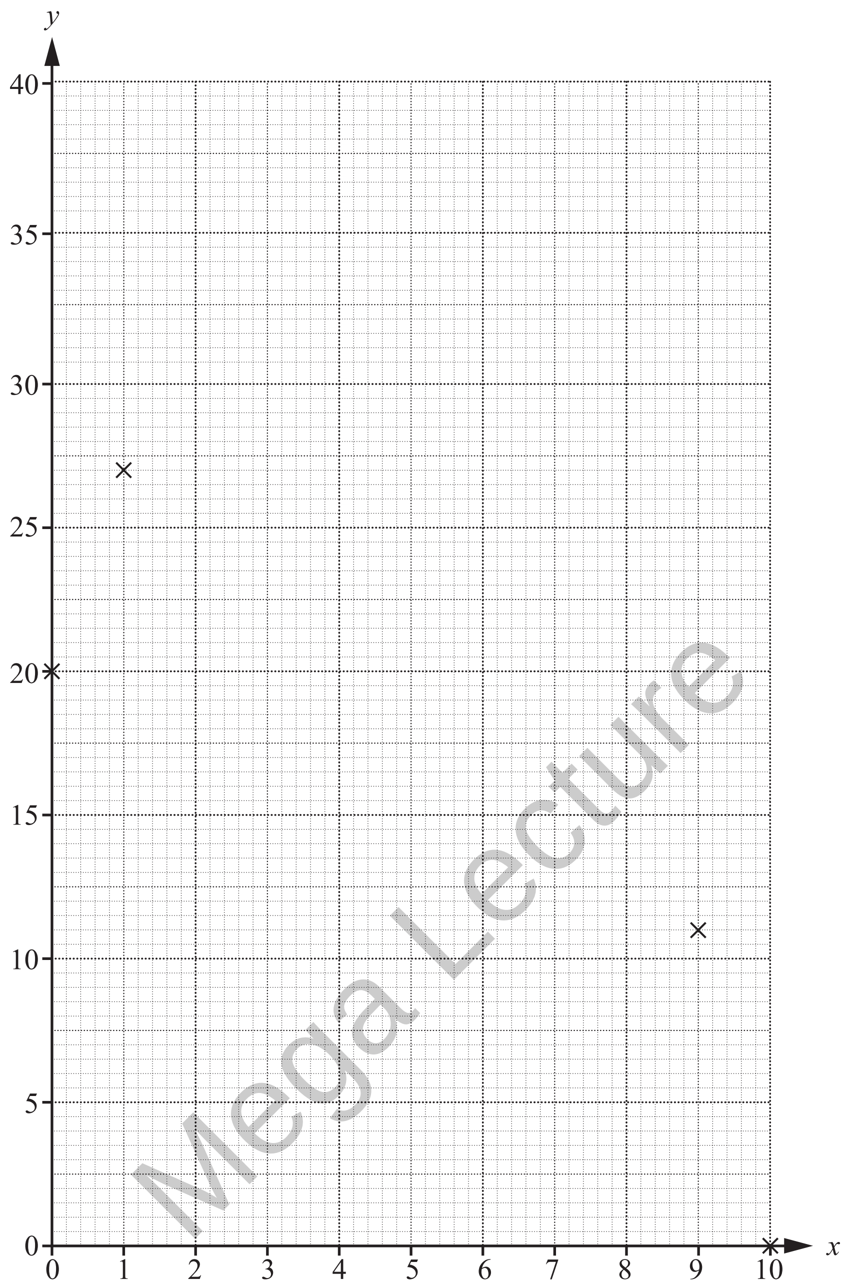
Show that $y = 20 + 8x - x^2$.

[2]

(b) On the grid opposite, draw the graph of $y = 20 + 8x - x^2$ for $0 \leqslant x \leqslant 10$.
Four points have been plotted for you. [4]

(c) On the same grid, draw a suitable line to find the value of Manuel's number, y, when it is the same as the random number, x.

.... [2]



(d) Jolene multiplies the random number, x, by 5 and then adds 2 to give her number, z.

Calculate the possible values of x when Manuel's number, y, and Jolene's number, z, are the same.

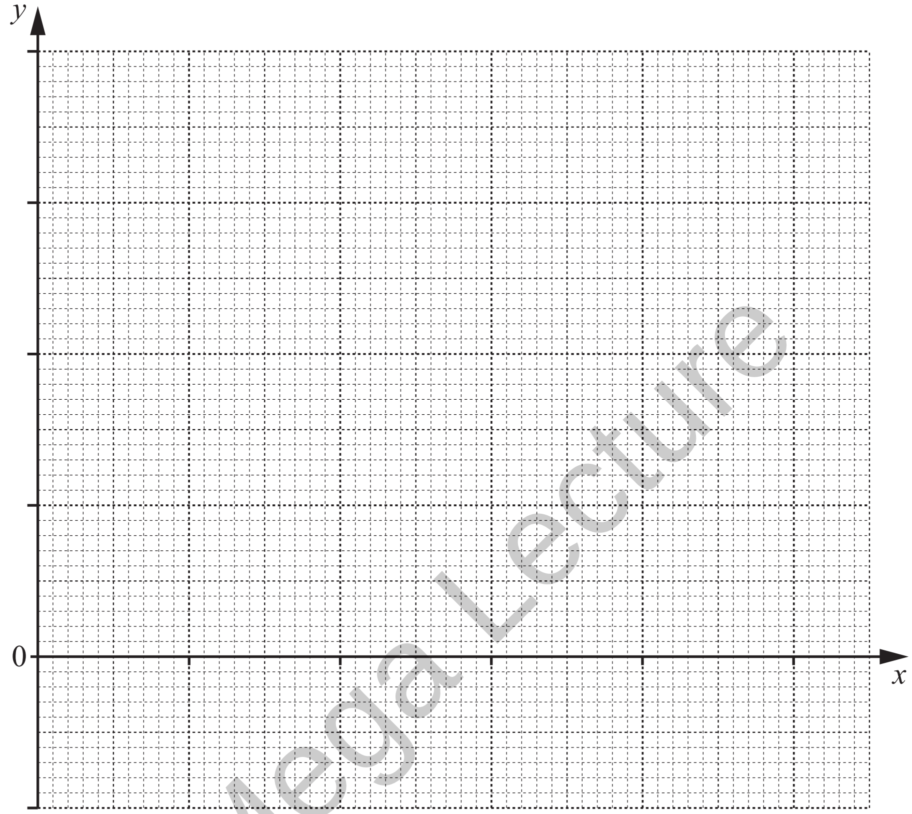
Answer $x =$ .... or .... [4]

15 **(a)** Complete the table of values for $y = \frac{x}{20}(x^2 - 10)$.

x	0	1	2	3	4	5
y	0	−0.45	−0.6	−0.15	1.2	

[1]

(b) Using a scale of 2 cm to 1 unit on both axes, draw the graph of $y = \frac{x}{20}(x^2 - 10)$ for $0 \leqslant x \leqslant 5$.



[2]

(c) By drawing a tangent, estimate the gradient of the curve at the point where $x = 2.5$.

Answer .... [2]

(d) Use your graph to solve the equation $\frac{x}{20}(x^2 - 10) = 0$ for $0 \leqslant x \leqslant 5$.

Answer $x =$ .... or .... [2]

(e) The graph of $y = \frac{x}{20}(x^2 - 10)$, together with the graph of a straight line L, can be used to solve the equation $x^3 + 10x - 80 = 0$ for $0 \leqslant x \leqslant 5$.

(i) Find the equation of line L.

Answer .... [2]

(ii) Draw the graph of line L on the grid. [1]

(iii) Hence solve the equation $x^3 + 10x - 80 = 0$ for $0 \leqslant x \leqslant 5$.

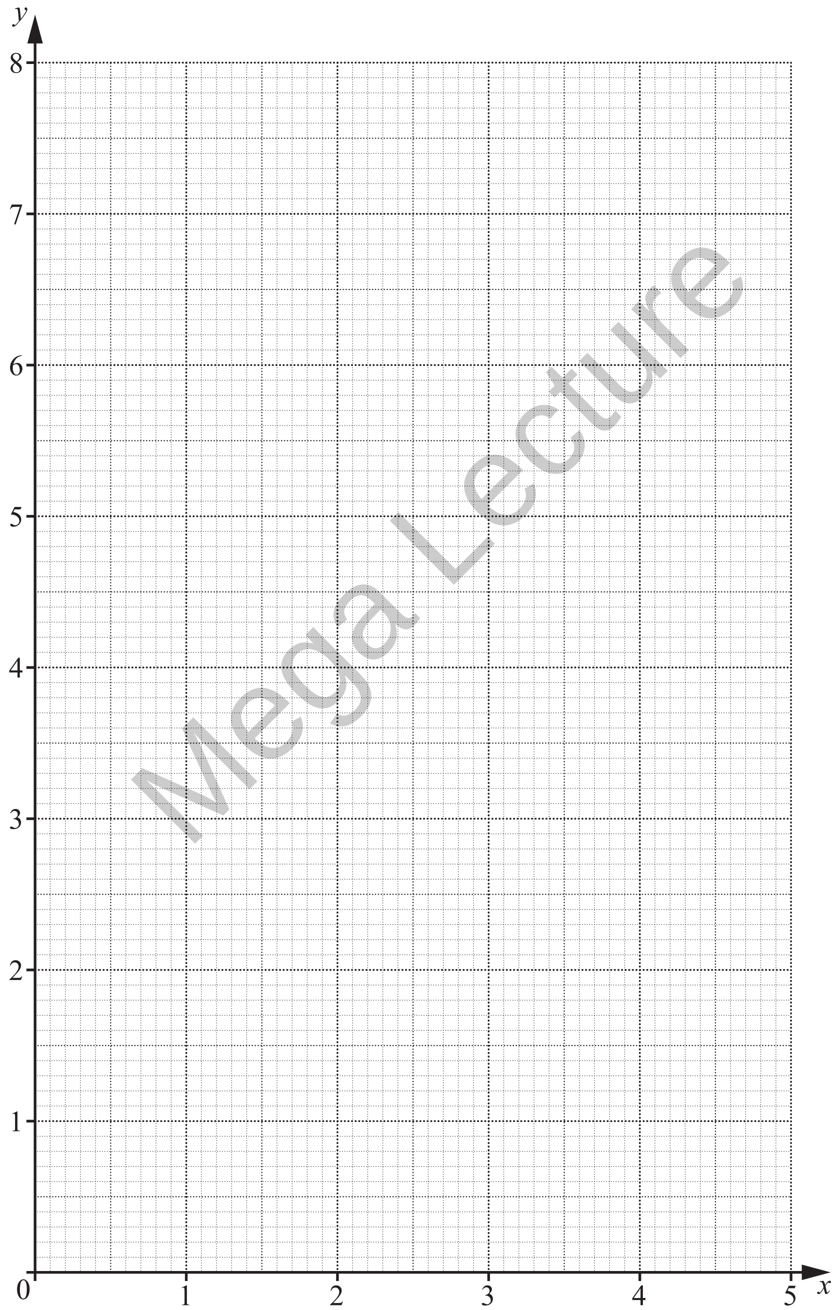
Answer $x =$ .... [1]

16 The table below shows some values of x and the corresponding values of y for $y = \frac{1}{4} \times 2^x$.

x	0	1	2	3	4	5
y	$\frac{1}{4}$		1	2	4	8

(a) Complete the table. [1]

(b) On the grid below, draw the graph of $y = \frac{1}{4} \times 2^x$.



[2]

(c) By drawing a suitable line, find the gradient of your graph where $x = 4$.

Answer .... [2]

(d) **(i)** Show that the line $2x + y = 6$, together with the graph of $y = \frac{1}{4} \times 2^x$, can be used to solve the equation

$$2^x + 8x - 24 = 0 .$$

[1]

(ii) Hence solve $2^x + 8x - 24 = 0$.

Answer $x =$ .... [2]

(e) The points P and Q are (2, 3) and (5, 4) respectively.

(i) Find the gradient of PQ .

Answer .... [1]

(ii) On the grid, draw the line l, parallel to PQ, that touches the curve $y = \frac{1}{4} \times 2^x$. [1]

(iii) Write down the equation of l.

Answer .... [2]

17 The distance, *d* metres, of a moving object from an observer after *t* minutes is given by

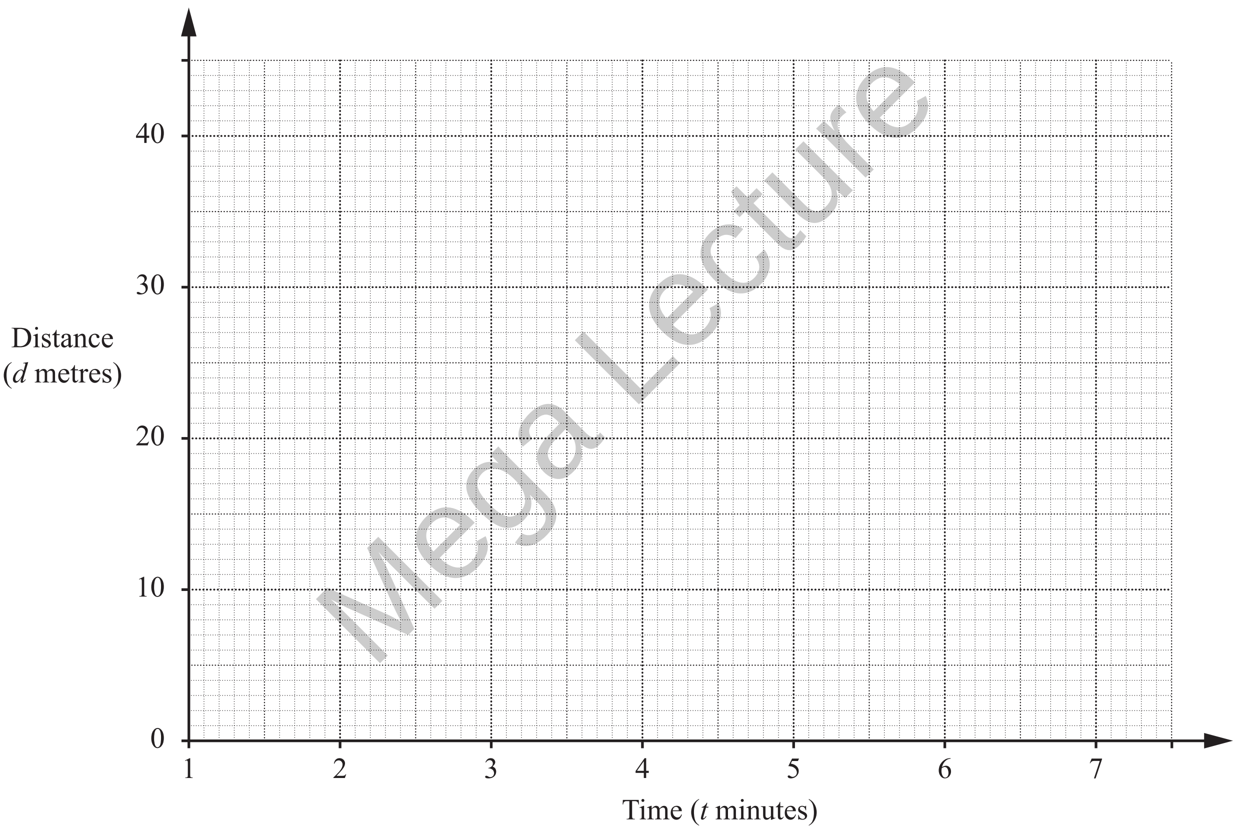
$$d = t^2 + \frac{48}{t} - 20.$$

(a) Some values of *t* and *d* are given in the table.
The values of *d* are given to the nearest whole number where appropriate.

t	1	1.5	2	2.5	3	3.5	4	4.5	5	6	7
d	29	14	8	5	5	6	8	11	15	24	

Complete the table. [1]

(b) On the grid, plot the points given in the table and join them with a smooth curve.



[2]

(c) **(i)** By drawing a tangent, calculate the gradient of the curve when $t = 4$.

Answer .... [2]

(ii) Explain what this gradient represents.

Answer .... [1]

(d) For how long is the object less than 10 metres from the observer?

Answer .... minutes [2]

(e) **(i)** Using your graph, write down the two values of t when the object is 12 metres from the observer.
For each value of t, state whether the object is moving towards or away from the observer.

Answer When t = ..., the object is moving .... the observer.

When t = ..., the object is moving .... the observer. [2]

(ii) Write down the equation that gives the values of t when the object is 12 metres from the observer.

Answer .... [1]

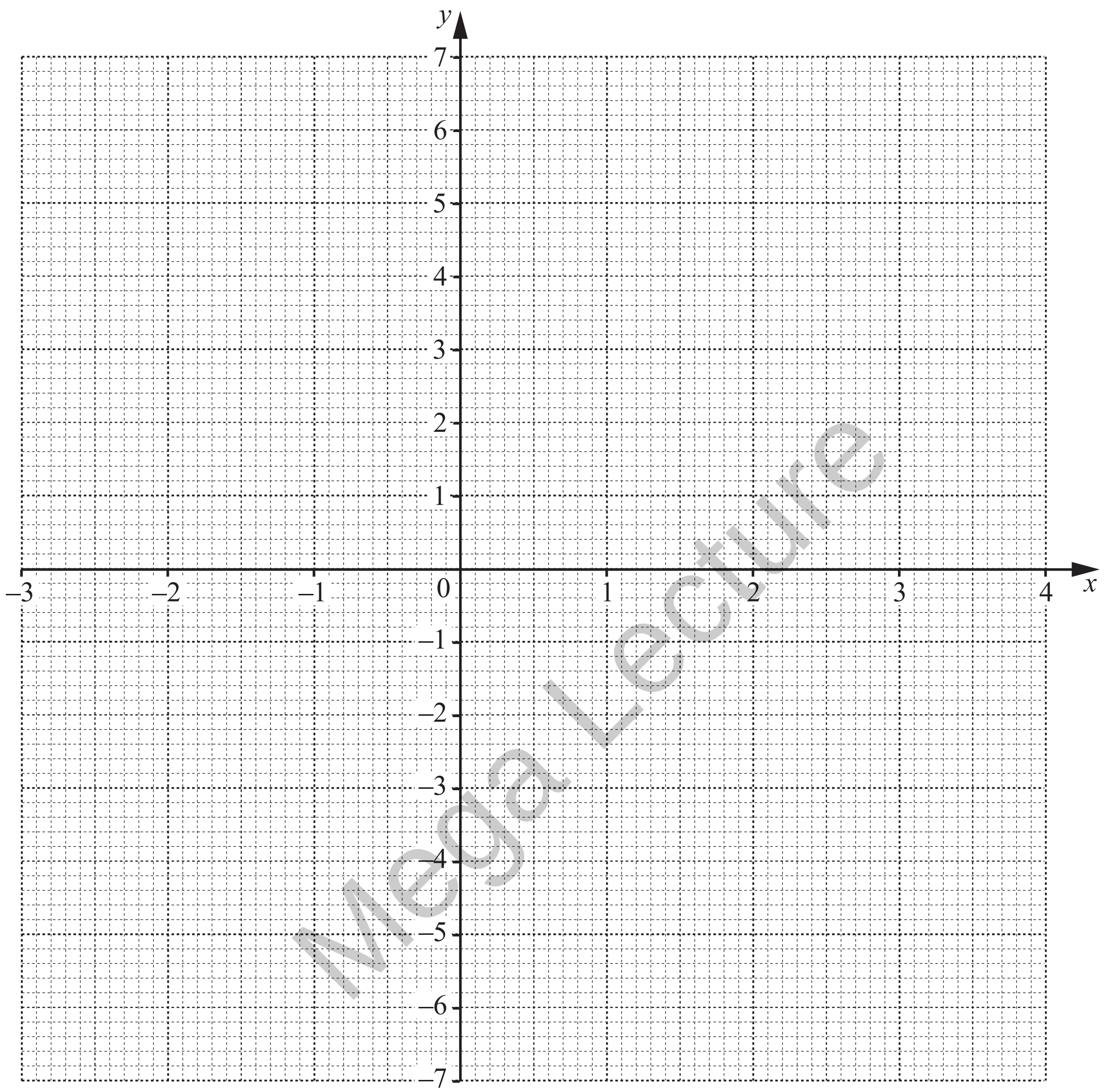
(iii) This equation is equivalent to $t^3 + At + 48 = 0$.

Find A.

Answer A = ... [1]

18 **(i)** Complete the table of values for $y = 6 + x - x^2$, and hence draw the graph of $y = 6 + x - x^2$ on the grid opposite.

x	-3	-2	-1	0	1	2	3	4
y	-6	0		6	6		0	-6



[3]

(ii) Use your graph to estimate the maximum value of $6 + x - x^2$.

Answer .... [1]

(iii) By drawing the line $x + y = 4$, find the approximate solutions to the equation

$$2 + 2x - x^2 = 0.$$

Answer $x =$ .... or .... [2]

(iv) The equation $x - x^2 = k$ has a solution $x = 3.5$.

By drawing a suitable line on the grid, find the other solution.
Label your line with the letter *L*.

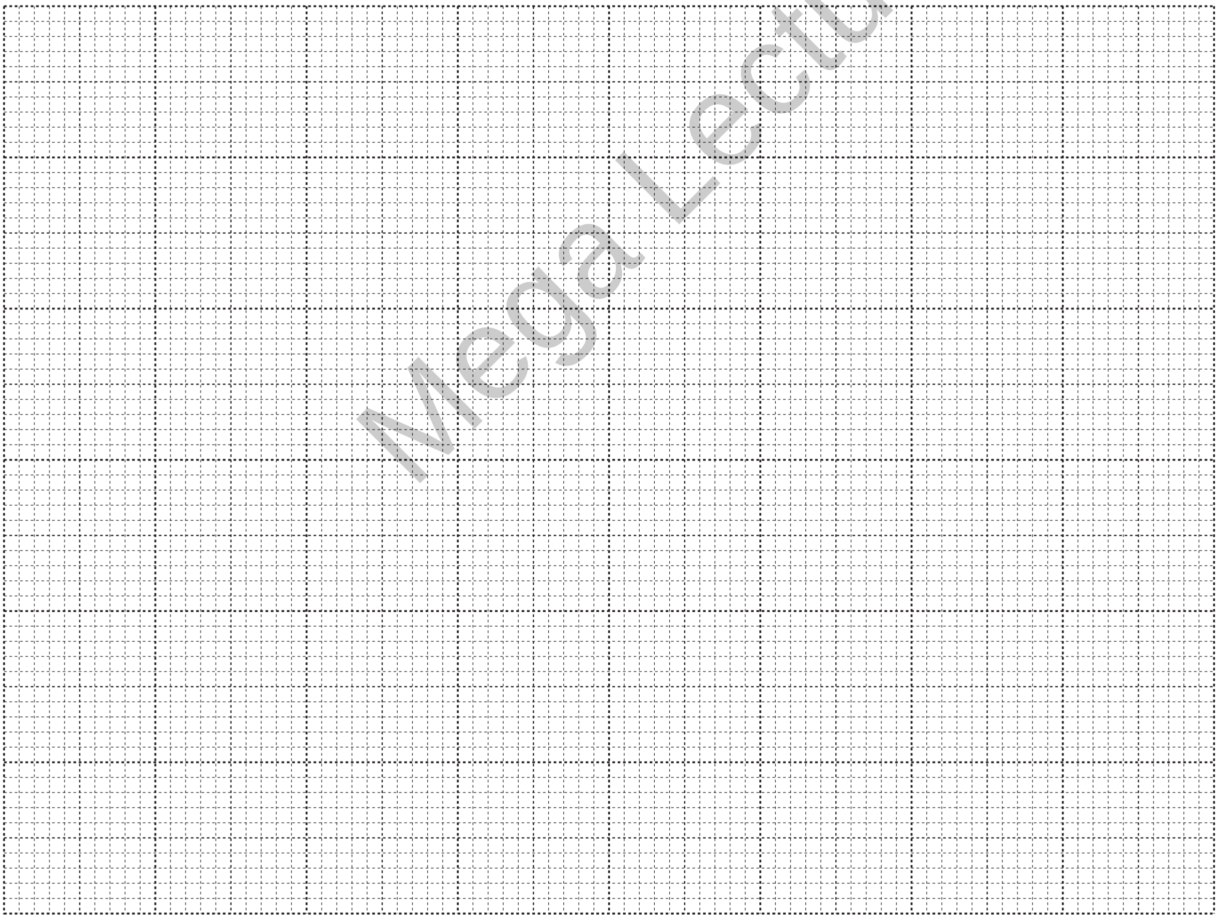
Answer .... [2]

19 The table below is for $y = x^2 - 4x - 1$.

x	–2	–1	0	1	2	3	4	5	6
y		4	–1	–4	–5	–4	–1	4	

(a) Complete the table. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \leqslant x \leqslant 6$.
Using a scale of 2 cm to 5 units, draw a vertical y-axis for $-10 \leqslant y \leqslant 15$.
Plot the points from the table and join them with a smooth curve.



[3]

(c) By drawing a tangent, estimate the gradient of the curve at $x = 3$.

Answer .... [2]

(d) **(i)** Find the least value of y.

Answer .... [1]

(ii) $y \leqslant 4$ for $a \leqslant x \leqslant b$.

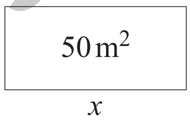
Find the least possible value of a and the greatest possible value of b.

Answer a ....

b .... [2]

(e) **Use your graph** to solve the equation $x^2 - 4x + 2 = 0$.
Show your working to explain how you used your graph.

20 Adil wants to fence off some land as an enclosure for his chickens.
The enclosure will be a rectangle with an area of $50\,\text{m}^2$.



(a) The enclosure is x m long.

Show that the total length of fencing, L m, required for the enclosure is given by

$$L = 2x + \frac{100}{x}.$$

[2]

(b) The table below shows some values of x and the corresponding values of L, correct to one decimal place where appropriate, for $L = 2x + \frac{100}{x}$.

x	2	4	6	8	10	12	14	16	18	20
L	54	33	28.7	28.5	30	32.3	35.1	38.3		

Complete the table. [2]

(c) On the grid opposite
draw a horizontal x-axis for $0 \leqslant x \leqslant 20$ using a scale of 1 cm to represent 2 m
and a vertical L-axis for $0 \leqslant L \leqslant 60$ using a scale of 2 cm to represent 10 m.

On the grid, plot the points given in the table and join them with a smooth curve. [3]

(d) Adil only has 40 m of fencing.

Use your graph to find the range of values of x that he can choose.

Answer .... $\leqslant x \leqslant$ .... [2]

(e) **(i)** Find the minimum length of fencing Adil could use for the enclosure.

Answer .... m [1]

(ii) Find the length and width of the enclosure using this minimum length of fencing.
Give your answers correct to the nearest metre.

Answer Length = ... m Width = ... m [1]

(f) Suggest a suitable length and width for an enclosure of area 100 m^2, that uses the minimum possible length of fencing.

Answer Length = ... m Width = ... m [1]