## NEW-Syllabus Revision Notes



## 2013-2022

## AS-Level



## A N Chowhan

author of best-selling book on A-level Physics Paper 5 (best-seller in 18 countries)

## Important Notes

- The formulas/equations with borders (rectangles) drawn around them are very important from the examination point of view, but the formula sheets of paper 1 (MCQ paper) and paper 2 (theory paper) do not contain these formulas; so students are advised to memorise them.
- Keywords have been underlined throughout the text. Definitions/statements lacking keywords are not awarded full marks (as indicated, in blue, at the end of the definitions/statements); so students are advised to pay special attention to the keywords (as they memorise the definitions/statements).
- Bracketed information only serves as an additional detail that is not required (in order for the examiner to award the intended marks).


## 1 SI Base Quantities and their Units

Base quantities are fundamental physical quantities. All other quantities may be expressed in terms of base quantities. In SI (System International or International System of Units), seven quantities have been selected as base quantities.

| base quantity | unit |
| :--- | :--- |
| length | metre (m) |
| mass | kilogram (kg) |
| time | second (s) |
| temperature | kelvin (K) |
| current | ampere (A) |
| amount of substance | mole (mol) |
| luminous intensity | candela (cd) |

Figure 1.1.1

## Notes

- All physical quantities consist of a numerical magnitude and a unit (e.g. mass of a body $=2 \mathrm{~kg}$ ).
- Luminous intensity is not included in A-level Physics syllabus.


## Now it's your turn

Do, in your workbook, the following questions (of chapter 1.1) in the same order:
10, 4

## (Q1/21/O/N/15)

10 (a) State two SI base quantities other than mass, length and time.

1. ..temperature.
2. current

## (Q1/22/M/J/17)

4 (a) State two SI base units other than kilogram, metre and second.

1. Kelvin
2. ampere

## 2 Determining SI Base Units of Derived Quantities

All physical quantities other than base quantities (e.g. speed) are called derived quantities. These quantities may be expressed in terms of base quantities, and hence their units may be expressed as products or quotients of base units (e.g. SI base units of speed are ' $\mathrm{m} \mathrm{s}^{-1}$ ).

## Notes

- The only thing needed to determine the SI base units of a derived quantity is its expression in terms of base quantities.
- The correct definition of a physical quantity uses other quantities, NOT their units. For example, it is technically incorrect to define density as the mass per cubic metre; rather, it should be defined as the mass per unit volume.


## Example 1

Determine SI base units of force $(F)$.

## Solution

$$
F=m a
$$

So:
SI base units of $F=\mathrm{kg} \mathrm{m} \mathrm{s}^{-2}$

## Example 2

Determine SI base units of pressure $(P)$.

## Solution

$$
P=\frac{F}{A}=\frac{m a}{A}
$$

So:
SI base units of $P=\frac{\mathrm{kg} \cdot \mathrm{m} \mathrm{s}^{-2}}{\mathrm{~m}^{2}}=\mathrm{kg} \mathrm{m}^{-1} \mathrm{~s}^{-2}$

## Example 3

Determine SI base units of work (w).

## Solution

$$
w=F s=m a . s
$$

So:
SI base units of $w=\mathrm{kg} \cdot \mathrm{m} \mathrm{s}^{-2} \cdot \mathrm{~m}=\mathrm{kg} \mathrm{m}^{2} \mathrm{~s}^{-2}$

## Note

As work and energy have the same SI unit (joule), so SI base units of work can also be determined by using the expression for kinetic energy $\left(E_{\mathrm{k}}\right)$ or gravitational potential energy ( $E_{\mathrm{p}}$ ) as done below:

$$
E_{\mathrm{k}}=\frac{1}{2} m v^{2}
$$

So:

$$
\text { SI base units of } E_{\mathrm{k}}=\mathrm{kg} \cdot\left(\mathrm{~m} \mathrm{~s}^{-1}\right)^{2}=\mathrm{kg} \mathrm{~m}^{2} \mathrm{~s}^{-2}
$$

Likewise as:
$E_{\mathrm{p}}=m g h$
So:
SI base units of $E_{\mathrm{p}}=\mathrm{kg} \cdot \mathrm{m} \mathrm{s}^{-2} \cdot \mathrm{~m}=\mathrm{kg} \mathrm{m}^{2} \mathrm{~s}^{-2}$

## 3 Checking the Homogeneity of Physical Equations

## Example 1

Check whether or not the following equation is homogenous:

$$
\begin{equation*}
s=u t+\frac{1}{2} a t^{2} \tag{i}
\end{equation*}
$$

where $s$ is displacement, $u$ is initial velocity, $t$ is time and $a$ is acceleration.

## Solution

SI base unit of $s=m$
SI base unit of $u t=\mathrm{m} \mathrm{s}^{-1} \cdot \mathrm{~s}=\mathrm{m}$
SI base unit of $\frac{1}{2} a t^{2}=\mathrm{m} \mathrm{s}^{-2} \cdot \mathrm{~s}^{2}=\mathrm{m}$
As all the terms in Eq. (i) have the same SI base unit, so it is a homogenous equation.

## Example 2

Check whether or not the following equation is homogenous:

$$
\begin{equation*}
s=u t^{2}+\frac{1}{2} a t \tag{ii}
\end{equation*}
$$

where $s$ is displacement, $u$ is initial velocity, $t$ is time and $a$ is acceleration.

## Solution

SI base unit of $s=m$
SI base unit of $u t^{2}=\mathrm{m} \mathrm{s}^{-1}$. $\mathrm{s}^{2}=\mathrm{m} \mathrm{s}$
SI base unit of $\frac{1}{2} a t=\mathrm{m} \mathrm{s}^{-2} \cdot \mathrm{~s}=\mathrm{m} \mathrm{s}^{-1}$
As different terms in Eq. (ii) have different SI base units, so it is a non-homogenous equation.

## Notes

- A correct physical equation (e.g. $P=\rho g h$ ) is always homogenous.
- A non-homogenous equation is always incorrect.
- Rearranging an equation does not affect its homogeneity.


## Now it's your turn

Do, in your workbook, the following questions (of chapter 1.1) in the same order:
2, 7, 22, 3, 17, 13, 14, 8, 16

## Important Note

In all solved past-paper questions throughout the text, things (such as text/formula/drawing) written/drawn in red serve as additional explanations only, so students are not required to write/draw them in order to score the intended marks.

## (Q1/21/O/N/17)

2 (a) The drag force $F_{\mathrm{D}}$ acting on a sphere moving through a fluid is given by the expression

$$
F_{\mathrm{D}}=K \rho v^{2}
$$

where $K$ is a constant, $\rho$ is the density of the fluid
and $\quad v$ is the speed of the sphere.
Determine the SI base units of $K$.

$$
\begin{aligned}
K & =\frac{F_{D}}{\rho v^{2}} \left\lvert\, \begin{array}{c}
\text { So: } \\
\text { SI base units }
\end{array}\right. \\
& =\frac{k g \cdot}{\rho v^{2}}\left|\begin{array}{cc}
\text { of } K
\end{array}\right| \begin{aligned}
& k m^{-2} \\
&=\frac{m s^{-2}}{m m^{-3} \cdot m^{2} s^{-2}}=m^{2}
\end{aligned} \\
&
\end{aligned}
$$

## (Q1/22/0/N/16)

7 (a) (i) Define pressure
..pressure $=\frac{\text { force }}{\text { area }}$
[1]
(ii) Show that the SI base units of pressure are $\mathrm{kgm}^{-1} \mathrm{~s}^{-2}$.

$$
\begin{align*}
P & =\frac{F}{A} \\
& =\frac{m a}{A}
\end{aligned} \begin{aligned}
\text { So: SI base units } & =\frac{k g . m s^{-2}}{m^{2}}  \tag{1}\\
& =k g \mathrm{~m}^{-1} \mathrm{~s}^{-2}
\end{align*}
$$

(b) Gas flows through the narrow end (nozzle) of a pipe. Under certain conditions, the mass $m$ of gas that flows through the nozzle in a short time $t$ is given by

$$
\frac{m}{t}=k C \sqrt{\rho P}
$$

where $k$ is a constant with no units, $C$ is a quantity that depends on the nozzle size, $\rho$ is the density of the gas arriving at the nozzle, $P$ is the pressure of the gas arriving at the nozzle.

Determine the base units of $C$.
$C=\frac{m}{k t \sqrt{\rho P}}$
So: base

$$
\begin{align*}
& \text { units } \left.=\frac{\mathrm{kg}}{\mathrm{~s} \cdot(\mathrm{~kg} \mathrm{~m}}{ }^{-3} \cdot \mathrm{~kg}_{\mathrm{m}} \mathrm{~m}^{-2}\right)^{1 / 2} \\
& =\frac{k s}{s \cdot\left(\mathrm{~kg}^{2} \mathrm{~m}^{-4} \mathrm{~s}^{-2}\right)^{1 / 2}} \\
& =\frac{\mathrm{kg}}{\mathrm{~s} \cdot \mathrm{~kg} \mathrm{~m}^{-2} \mathrm{~s}^{-1}}=\mathrm{m}^{2} \\
& \text { base units }  \tag{3}\\
& m^{2}
\end{align*}
$$

## (Q2/23/0/N/21)

22 The SI base units of energy are $\mathrm{kg} \mathrm{m}^{2} \mathrm{~s}^{-2}$.
(c) A metal rod is heated at one end so that thermal energy flows to the other end.

The thermal energy $E$ that flows through the rod in time $t$ is given by

$$
E=\frac{c A\left(T_{1}-T_{2}\right) t}{L}
$$

where $A$ is the cross-sectional area of the rod,
$T_{1}$ and $T_{2}$ are the temperatures of the ends of the rod, $L$ is the length of the rod,
and $c$ is a constant.
Determine the SI base units of $c$.

$$
c=\frac{\epsilon L}{A\left(T_{1}-T_{2}\right) t}
$$

So:

$$
\begin{aligned}
\text { SI base units } & =\frac{\mathrm{kg} \mathrm{~m}^{2} \mathrm{~s}^{-2} \cdot m}{\mathrm{~m}^{2} \cdot \mathrm{k} \cdot \mathrm{~s}} \quad \begin{aligned}
\text { * Note that SI base unit of } \\
\text { of } c
\end{aligned} \\
& =k g T_{1}-T_{2}^{\prime} \text { is } K \text {, NOT ' } o \text { '. }
\end{aligned}
$$

## (Q1/23/0/N/17)

(a) (i) Define power. $P=\omega / t$

$$
\text { power }=\frac{\text { work done }}{\text { time taken }}
$$

(ii) Show that the SI base units of power are $\mathrm{kgm}^{2} \mathrm{~s}^{-3}$.

$$
\begin{aligned}
P & =\frac{w}{t} \\
& =\frac{F s}{t}
\end{aligned} \begin{aligned}
\text { So: SI base units } & =\frac{\mathrm{kg} \cdot \mathrm{~ms}^{-2} \cdot \mathrm{~m}}{\mathrm{~s}} \\
& =\mathrm{kg} \mathrm{~m}^{2} \mathrm{~s}^{-3}
\end{aligned}
$$

(b) All bodies radiate energy. The power $P$ radiated by a body is given by

$$
P=k A T^{4}
$$

where $T$ is the thermodynamic temperature of the body,
$A$ is the surface area of the body
and $k$ is a constant.
(i) Determine the SI base units of $k$.
$k=\frac{P}{A T^{4}}$
So:
$\begin{aligned} & \begin{aligned} & \text { SI base units }=\frac{k g m^{2} s^{-3}}{\text { of } k} \\ & m^{2} \cdot k^{4}\end{aligned} \\ &=k g s^{-3} k^{-4}\end{aligned}$
base units .......................................
(ii) On Fig. 1.1, sketch the variation with $T^{2}$ of $P$. The quantity $A$ remains constant.


Fig. 1.1
(ii) $9 f$ :

then:

Now as:

$$
\begin{aligned}
& P=k A T^{4} \\
\Rightarrow & P \propto T^{4} \\
\Rightarrow & P \propto\left(T^{2}\right)^{2}
\end{aligned}
$$

So $P$ us. $T^{2}$ graph is also parabolic
(just like $y$ vs. $x$ graph).

## (Q1/23/M/J/13)

17 (b) Two wires each of length $l$ are placed parallel to each other a distance $x$ apart, as shown in Fig. 1.1.


Fig. 1.1
Each wire carries a current $I$. The currents give rise to a force $F$ on each wire given by

$$
F=\frac{K I^{2} l}{x}
$$

where $K$ is a constant.
(ii) On Fig. 1.2, sketch the variation with $x$ of $F$. The quantities $I$ and $l$ remain constant.

i.e. $F \propto \frac{1}{x}$

Fig. 1.2
(iii) The current $I$ in both of the wires is varied.

On Fig. 1.3, sketch the variation with $I$ of $F$. The quantities $x$ and $l$ remain constant.


Fig. 1.3

## (Q1/21/M/J/15)

13 (a) Use the definition of power to show that the SI base units of power are $\mathrm{kgm}^{2} \mathrm{~s}^{-3}$.

$$
\begin{aligned}
\text { Power } & =\frac{\text { work }}{\text { time }} \\
& =\frac{\text { Es }}{t} \\
& =\frac{\text { mas }}{t}
\end{aligned} \left\lvert\, \begin{aligned}
\text { SI base units } & =\frac{k g . \mathrm{ms}^{-2} \cdot m}{\mathrm{~s}} \\
& =k \mathrm{~km}^{2} \mathrm{~s}^{-3}
\end{aligned}\right.
$$

(b) Use an expression for electrical power to determine the SI base units of potential difference.

$$
\begin{array}{r|rl}
P & =V I & \text { So: SI base units }
\end{array}=\frac{k g \mathrm{~m}^{2} \mathrm{~s}^{-3}}{A}
$$

units

$A^{-}$

## (Q1/22/M/J/15)

14 (a) Use the definition of work done to show that the SI base units of energy are $\mathrm{kgm}^{2} \mathrm{~s}^{-2}$.
work $=$ force $x$ displacement $\mid$ So:

$$
=m a \cdot s
$$

$\rightarrow v=w / q$
SI base units = SI base units
of energy of work

$$
=\mathrm{kg} \cdot \mathrm{~ms}^{-2} \cdot \mathrm{~m}
$$

$$
\begin{equation*}
=k g m^{2} s^{-2} \tag{2}
\end{equation*}
$$

(b) Define potential difference.
$\frac{\text { Puda.m. energy transformed from electrical to other forms }}{\text { charge }}$
(c) Determine the SI base units of resistance. Show your working.

$$
\begin{align*}
& R=\frac{V}{I} \\
&=\frac{W / Q}{I} \\
&=\frac{w / I t}{I} \left\lvert\, \begin{aligned}
\text { So: bI base units } & =\frac{k g m^{2} s^{-2}}{A^{2} \cdot s} \\
& =k g m^{2} s^{-3} A^{-2} \\
& =\frac{w}{I^{2} t}
\end{aligned} \quad \begin{aligned}
\text { resistance, } R
\end{aligned}\right. \\
&
\end{align*}
$$

## (Q4/22/M/J/16)

8 (b) The intensity of a sound wave passing through air is given by

$$
I=K v \rho f^{2} A^{2}
$$

where $I$ is the intensity (power per unit area),
$K$ is a constant without units, $v$ is the speed of sound, $\rho$ is the density of air, $f$ is the frequency of the wave and $A$ is the amplitude of the wave.

Show that both sides of the equation have the same SI base units.

$$
\begin{aligned}
& \text { LbS = I } \\
& =\frac{P}{A} \\
& =\frac{\epsilon / t}{A} \\
& =\frac{m g h / t}{A} \\
& =\frac{m g h}{A t} \\
& \text { So: } \\
& \begin{array}{l}
\text { SI base units }=\frac{\mathrm{kg} \cdot \mathrm{~ms}^{-2} \cdot \mathrm{~m}}{\mathrm{~m}^{2} \cdot \mathrm{~s}} \text { of LHS }
\end{array} \\
& =k \mathrm{~kg}^{-3} \\
& \text { Now: } \\
& \text { SI base units }=\mathrm{ms}^{-1} \cdot \mathrm{~kg} \mathrm{~m}^{-3} \cdot\left(s^{-1}\right)^{2} \cdot \mathrm{~m}^{2} \\
& \text { of RHS } \\
& \text { Hence: } \\
& =k g s^{-3} \\
& \text { SI base units = SI base units } \\
& \text { of LHS of RHS }
\end{aligned}
$$

## (Q1/21/O/N/14)

16 (a) Mass, length and time are SI base quantities. State two other base quantities.

1. temperature $\qquad$
2. ..curncemt
(b) A mass $m$ is placed on the end of a spring that is hanging vertically, as shown in Fig.1.1.


Fig. 1.1
The mass is made to oscillate vertically. The time period of the oscillations of the mass is $T$. The period $T$ is given by

$$
T=C \sqrt{\frac{m}{k}}
$$

where $C$ is a constant and $k$ is the spring constant. $\rightarrow f=k \boldsymbol{k} \quad$ (Hooke's law)
Show that $C$ has no units.

$$
\Rightarrow k=f / x
$$

$$
\begin{aligned}
& c=T . \sqrt{\frac{k}{m}} \\
& =T \cdot \sqrt{\frac{F / K}{m}} \\
& =T \cdot \sqrt{\frac{m a / x}{m}} \\
& =T \cdot \sqrt{\frac{a}{x}} \\
& \text { So: } \\
& \text { units of } C=S . \sqrt{\frac{m s^{-2}}{m}} \\
& =s \cdot s^{-1}=1 \\
& \text { Hence } C \text { has no units. }
\end{aligned}
$$

4 Prefixes

| prefix | symbol | value |
| :--- | :---: | :---: |
| pico | p | $10^{-12}$ |
| nano | n | $10^{-9}$ |
| micro | $\mu$ | $10^{-6}$ |
| milli | m | $10^{-3}$ |
| centi | c | $10^{-2}$ |
| deci | d | $10^{-1}$ |
| kilo | k | $10^{3}$ |
| mega | M | $10^{6}$ |
| giga | G | $10^{9}$ |
| tera | T | $10^{12}$ |

Figure 1.1.2

## Notes

- Prefixes are used to indicate decimal submultiples or multiples of both base and derived units. For example: ' 1 cm ' is a decimal submultiple of metre ( m ).
- The symbol of prefix is always written before the symbol of unit (e.g. 1 mm for one millimetre).
- Two prefixes cannot be combined. For example, $1 \times 10^{6} \times 10^{3} \mathrm{~m}$ should not be written as: 1 Mkm ; rather, it should be written as: 1 Gm (which is equal to $1 \times 10^{9} \mathrm{~m}$ ).
- The use of prefixes makes the process of converting a unit into a larger or a smaller unit very simple. For example:
- $2.4 \mathrm{~cm}=2.4 \times 10^{-2} \mathrm{~m}$
- $2.4 \mathrm{~cm}^{2}=2.4(\mathrm{~cm})^{2}=2.4 \times\left(10^{-2} \mathrm{~m}\right)^{2}=2.4 \times 10^{-4} \mathrm{~m}^{2}$
- $2.4 \mathrm{~cm}^{3}=2.4(\mathrm{~cm})^{3}=2.4 \times\left(10^{-2} \mathrm{~m}\right)^{3}==2.4 \times 10^{-6} \mathrm{~m}^{3}$


## Now it's your turn

Do, in your workbook, the following questions (of chapter 1.1) in the same order:
21, 12

## (Q1/22/0/N/21)

21 (a) A unit may be stated with a prefix that represents a power-of-ten multiple or submultiple.
Complete Table 1.1 to show the name and symbol of each prefix and the corresponding power-of-ten multiple or submultiple.

Table 1.1

| prefix | power-of-ten multiple <br> or submultiple |
| :---: | :---: |
| kilo (k) | $10^{3}$ |
| tera (T) | $\mathbf{1 0 ^ { 1 2 }}$ |
| pico (p) | $10^{-12}$ |

## (Q1/23/0/N/15)

12 (a) The intensity of a progressive wave is defined as the average power transmitted through a surface per unit area.

$$
\rightarrow I
$$

Show that the SI base units of intensity are $\mathrm{kgs}^{-3}$.

$$
\begin{aligned}
I & =\frac{P}{A} \\
& =\frac{\epsilon / t}{A} \\
& =\frac{m g h / t}{A} \\
& =\frac{m g h}{A T} \\
& =\frac{\mathrm{kg} ~ \text { of } I}{\mathrm{~m}^{-3}}
\end{aligned}
$$

(b) (i) The intensity $I$ of a sound wave is related to the amplitude $x_{0}$ of the wave by

$$
I=K \rho c f^{2} x_{0}^{2}
$$

where $\rho$ is the density of the medium through which the sound is passing, $c$ is the speed of the sound wave,
$f$ is the frequency of the sound wave
and $K$ is a constant.
Show that $K$ has no units.
$K=\frac{I}{\rho C f^{2} x_{0}^{2}}$

[2]

$$
\rightarrow 10^{-12} \mathrm{Wm}^{-2}
$$

(ii) Calculate the intensity, in $\mathrm{pWm}^{-2}$, of a sound wave where
intensity = 30.8 $\mathrm{pWm}^{-2}[3]$

$$
\begin{aligned}
& K=20 \text {, } \\
& \rho=1.2 \text { in SI base units, } \\
& c=330 \text { in SI base units, } \\
& f=260 \text { in SI base units } \\
& \text { and } x_{0}=0.24 \mathrm{~nm} . \rightarrow 0.24 \times 10^{-9} \mathrm{~m} \\
& I=k \rho c f^{2} x_{0}^{2} \\
& =(20)(1.2)(330)(260)^{2}\left(0.24 \times 10^{-9}\right)^{2} \\
& =3.084 \times 10^{-11} \mathrm{Wm}^{-2} \\
& =30.84 \times 10^{-12} \mathrm{Wm}^{-2} \\
& =30.84 \mathrm{pWm}^{-2}
\end{aligned}
$$

## 5 Conventions for Labelling Graph Axes and Table Columns

Let us imagine a car moving along a straight road with uniform acceleration. The table in Fig. 1.1.3 (a) shows the values of velocity $(v)$ of the car at different instants of time $(t)$, and the graph in Fig. 1.1.3 (b) shows the variation with time of the velocity of the car.

| $t / 10^{2} \mathrm{~s}$ | $v / \mathrm{m} \mathrm{s}^{-1}$ |
| :---: | :---: |
| 0 | 0 |
| 1 | 5 |
| 2 | 10 |
| 3 | 15 |
| 4 | 20 |
| 5 | 25 |

Figure 1.1.3 (a)


Figure 1.1.3 (b)

## Note

The headings of table columns (and likewise the labels of graph axes) should contain both: the quantity and its unit, with the distinguishing mark ' $/$ ' between them (such as $t / 10^{2} \mathrm{~s}$ ).

## 6 Making Reasonable Estimates of Physical Quantities

## Example 1

Make reasonable estimate of the mass, in grams (g), of an apple.

## Solution

(It may be assumed that there are on average 5 apples in 1 kg (or 1000 g ). So, the mass of an apple may be estimated to be:)
200 g
(According to the mark scheme, any value from 50 g to 500 g is acceptable for the reasonable estimate of the mass of an apple)

## Example 2

Make reasonable estimate of the volume, in $\mathrm{cm}^{3}$, of the head of an adult person.

## Solution

(It may be assumed that the volume of the head of an adult person is almost equal to the volume of a sphere of diameter 20 cm or radius 10 cm . Now, the volume of a sphere of radius 10 cm is calculated as:

$$
V=\frac{4}{3} \pi r^{3}=\frac{4}{3} \cdot \pi \cdot(10)^{3}=4189 \mathrm{~cm}^{3}
$$

So, the volume of the head of an adult person may be estimated to be:)
$4000 \mathrm{~cm}^{3}$
(According to the mark scheme, any value from $2000 \mathrm{~cm}^{3}$ to $9000 \mathrm{~cm}^{3}$ is acceptable for the reasonable estimate of the volume of the head of an adult person)

## Notes

- When making an estimate, DO NOT try to state the whole range of acceptable values.
- State your estimate to no more than 2 significant figures.

[^0]
## (Q1/21/0/N/19)

19 (a) Make estimates of:
(i) the mass, in g , of a new pencil

$$
\begin{align*}
& \text { no of pencils in } 1 \mathrm{~kg} \approx 100 \\
& \Rightarrow \text { mass of } 1 \text { pencil } \approx \frac{1000 \mathrm{~g}}{100}=10 \mathrm{~g} \\
& \text { mass }= \\
& \begin{array}{l}
\text { according to mark scheme, } \\
\\
\\
\\
\\
\\
\text { range of acceptable values } \\
\text { is } 1 \mathrm{~g} \text { to } 20 \mathrm{~g} .
\end{array} \tag{1}
\end{align*}
$$

## 7 Scalars and Vectors

Scalar quantities only have magnitude (e.g. mass); whereas vector quantities have both: magnitude and direction (e.g. force).

## Notes

- scalar $\times$ scalar $=$ scalar
(e.g. speed $\times$ time $=$ distance travelled)
- scalar $\times$ vector $=$ vector
- vector $\times$ vector $=$ scalar or vector
- Square of a vector is always a scalar.
(e.g. mass $\times$ velocity $=$ momentum)
(e.g. force $\times$ displacement $=$ work)
(also: force $\times$ displacement $=$ moment of a force)
(e.g. velocity ${ }^{2}$ is a scalar)


## Representation of a Vector

Let us imagine a force $(F)$ of 25 N acting to the right on a body. This force can be represented graphically by a line segment, with an arrowhead drawn at the right end (or in the middle of the line), as shown in Fig. 1.1.4.

$$
\begin{aligned}
& \begin{array}{l}
\text { scale: } \\
1 \mathrm{~cm}=5 \mathrm{~N}
\end{array} \\
& \hline
\end{aligned} \begin{gathered}
F=25 \mathrm{~N} \\
(5 \mathrm{~cm})
\end{gathered}
$$

Figure 1.1.4

## Notes

- The length of line segment indicates the magnitude of the vector (according to the scale).
- The arrowhead indicates the direction of the vector.


## Negative of a Vector

The negative of a vector $(A)$ is a vector having the same magnitude (as $A$ ) but opposite direction, as shown in Fig. 1.1.5.


Figure 1.1.5

## Physical Significance of ' + ' and ' - ' Signs for Scalars and Vectors

- If, when evaluated, a vector turns out to be negative (e.g. velocity after collision $=-2 \mathrm{~m} \mathrm{~s}^{-1}$ ), it means that its direction is opposite to the direction taken as positive, and vice versa.
- If however a scalar turns out to be negative (e.g. temperature $=-10^{\circ} \mathrm{C}$ ), it does not mean that it has opposite direction (as scalars don't have direction); rather, it means that it is even less than 0 , and vice versa.
- If change in a scalar turns out to be negative (e.g. change in $K E=$ final $K E-$ initial $K E=-15 \mathrm{~J}$ ), it means that the scalar decreases, and vice versa.
- If however change in a vector turns out to be negative (e.g. change in velocity $=-10 \mathrm{~m} \mathrm{~s}^{-1}$ ), it does not mean that the vector decreases; rather, it means that the change in the vector takes place in the direction opposite to the direction taken as positive.


## 8 Vector Addition and Subtraction

## Example 1 (vector addition)

If:


Figure 1.1.6 (NOT TO SCALE)
then determine the magnitude and direction of the resultant of the two forces acting at point $X$. [3 or 4]
Method 1 (head-to-tail method)
Scale: $1 \mathrm{~cm}=2 \mathrm{~N}$


Now as:
length of resultant force vector $=11 \mathrm{~cm}$
(where $1 \mathrm{~cm}=2 \mathrm{~N}$ )
so:
magnitude of the resultant force $=11 \times 2$

$$
=22 \mathrm{~N}
$$

Likewise
direction of the resultant force $=37^{\circ}$ above the horizontal

## Steps of Working

1 Choose appropriate scale.
2 Draw any one of the two vectors according to the scale, and in the right direction.
3 Starting from the head of the first vector draw the second vector (also according to the scale and in the right direction).

4 Join the tail of the first vector to the head of the second (or final) vector by a straight line and draw two arrowhead symbols (pointing from the tail of the first vector to the head of the second vector) in the middle of the line. The length of this line indicates the magnitude of the resultant vector, and its direction indicates the direction of the resultant vector.

Method 2 (parallelogram method)
Scale: $1 \mathrm{~cm}=2 \mathrm{~N}$


Now as:
length of resultant force vector $=11 \mathrm{~cm}$
(where $1 \mathrm{~cm}=2 \mathrm{~N}$ )
so:
magnitude of the resultant force $=11 \times 2$

$$
=22 \mathrm{~N}
$$

Likewise:
direction of the resultant force $=37^{\circ}$ above the horizontal

## Steps of Working

1 Choose appropriate scale.
2 Starting from the same point draw both vectors (according to the scale and in the right directions).
3 Draw a line parallel to one vector and passing through the head of the other vector.
4 Repeat step 3 for the other vector.
5 Join the tail point of both vectors to the point of intersection of the two lines drawn in steps 3 and 4 by a straight line, and draw two arrowhead symbols (pointing from the tail point of the vectors to the point of intersection of the lines) in the middle of the line. The length of this line indicates the magnitude of the resultant vector, and its direction indicates the direction of the resultant vector.

## Example 2 (vector subtraction)

If:

then determine the magnitude and direction of $F_{1}-F_{2}$.

## Solution

As:

$$
F_{1}-F_{2}=F_{1}+\left(-F_{2}\right)
$$

So (the magnitude and direction of) ' $F_{1}-F_{2}$ ' can be determined by adding $F_{1}$ to ' $-F_{2}$ ' as done below:
Scale: $1 \mathrm{~cm}=2 \mathrm{~N}$


Now as:
length of $F_{1}-F_{2}$ vector $=6.5 \mathrm{~cm}$
(where $1 \mathrm{~cm}=2 \mathrm{~N}$ )
so:
magnitude of $F_{1}-F_{2}=6.5 \times 2$
$=13 \mathrm{~N}$
Likewise:
direction of $F_{1}-F_{2}=79^{\circ}$ below the horizontal

## Triangle Sum Theorem, Sine Rule and Cosine Rule

If:


Figure 1.1.8 (NOT TO SCALE)
then by triangle sum theorem:

$$
A+B+C=180^{\circ}
$$

by sine rule:

$$
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}
$$

and by cosine rule:

$$
a^{2}=b^{2}+c^{2}-2 b c \cdot \cos A
$$

## Note

If the magnitudes of any two angles and one side (or one angle and two sides) in a triangle are known, then the magnitudes of the remaining angles and sides can be determined by simply using the above theorem and rules (i.e. without having to draw the scaled vector triangle).

## Now it's your turn

Do, in your workbook, the following questions (of chapter 1.1) in the same order:
$1,6,11,15,9,5,18$

## (Q1/22/F/M/18)

1 (a) Complete Fig. 1.1 to indicate whether each of the quantities is a vector or a scalar.

| quantity | vector or scalar |
| :---: | :---: |
| acceleration | vector |
| speed | scalar |
| power | scalar |

Fig. 1.1

## (Q1/22/F/M/17)

6 (a) Complete Fig. 1.1 by putting a tick $(\checkmark)$ in the appropriate column to indicate whether the listed quantities are scalars or vectors.

| quantity | scalar | vector |
| :---: | :---: | :---: |
| acceleration |  |  |
| force |  |  |
| kinetic energy |  |  |
| momentum |  |  |
| power |  |  |
| work |  |  |

Fig. 1.1

## (Q1/22/0/N/15)

$$
\rightarrow V=3 \times 10^{\circ} \mathrm{ms}^{-}
$$

11 (a) The frequency of an $X$-ray wave is $4.6 \times 10^{20} \mathrm{~Hz} \rightarrow f$
Calculate the wavelength in pm. $\rightarrow 10^{-12} \mathrm{~m}$

$$
\Rightarrow=f \lambda \quad \begin{aligned}
v \times 10^{8}=\left(4.6 \times 10^{20}\right) . \lambda \left\lvert\, \begin{aligned}
\lambda & =6.522 \times 10^{-13} \mathrm{~m} \\
& =0.6522 \times 10^{-12} \mathrm{~m} \\
& =0.6522 \mathrm{pm}
\end{aligned}\right. \\
\text { wavelength }=\ldots
\end{aligned}
$$

(b) The distance from Earth to a star is $8.5 \times 10^{16} \mathrm{~m}$. Calculate the time for light to travel from the star to Earth in Gs. $\rightarrow 10^{9} \mathrm{~s}$

$$
v^{\downarrow}=3 \times 10^{8} \mathrm{~m}^{-1}
$$

$$
\left.\begin{aligned}
& s=v t \\
& \Rightarrow 8.5 \times 10^{16}=\left(3 \times 10^{8}\right) . t \\
& \Rightarrow t=2.833 \times 10^{8} \mathrm{~s}
\end{aligned} \right\rvert\, \begin{aligned}
t & =0.2833 \times 10^{9} \mathrm{~s} \\
& =0.2833 \mathrm{GS}
\end{aligned}
$$

time =
$\qquad$ 0.283 Gs [2]
(c) The following list contains scalar and vector quantities.

Underline all the scalar quantities.
acceleration force mass power temperature weight
(d) A boat is travelling in a flowing river. Fig. 1.1 shows the velocity vectors for the boat and the river water.


Fig. 1.1
The velocity of the boat in still water is $14.0 \mathrm{~ms}^{-1}$ to the east. The velocity of the water is $8.0 \mathrm{~ms}^{-1}$ from $60^{\circ}$ north of east.
(i) On Fig. 1.1, draw an arrow to show the direction of the resultant velocity of the boat. [1]
(ii) Determine the magnitude of the resultant velocity of the boat.

Scale:
$1 \mathrm{~cm}=2 \mathrm{~ms}^{-1}$

length of resultant velocity vector $=6.1 \mathrm{~cm}$
So:
magnitude of resultant velocity $=6.1 \times 2=12.2 \mathrm{~ms}^{-1}$ magnitude of velocity $=$ $\qquad$ $\mathrm{ms}^{-1}[2]$

## (Q1/23/M/J/15)

$$
10^{9} \mathrm{~m} \longleftarrow
$$

15 (a) The distance between the Sun and the Earth is $1.5 \times 10^{11} \mathrm{~m}$. State this distance in $\underline{\mathrm{Gm}}$.

$$
1.5 \times 10^{11} \mathrm{~m}=150 \times 10^{9} \mathrm{~m}=150 \mathrm{Gm}
$$

$$
\text { distance }=
$$

$\qquad$ Gm [1]
(b) The distance from the centre of the Earth to a satellite above the equator is 42.3 Mm . The radius of the Earth is $6380 \mathrm{~km} \rightarrow R=6.38 \times 10^{6} \mathrm{~m} \quad r=42.3 \times 10^{6} \mathrm{~m} \leftrightarrows$ A microwave signal is sent from a point on the Earth directly below the satellite.
$\Longrightarrow v=3 \times 10^{8} \mathrm{~ms}^{-1}$
Calculate the time taken for the microwave signal to travel to the satellite and back.

$\quad$ distance travelled $=$ speed $\times$ time
$\Rightarrow 2(r-R)=v t$
$\Rightarrow 2\left(42.3 \times 10^{6}-6.38 \times 10^{6}\right)=\left(3 \times 10^{8}\right) . t$
$\Rightarrow t=0.23947$
time $=\ldots \ldots . . . . . . . . . . . .23 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~$ s [2]
(c) The speed $v$ of a sound wave through a gas of density $\rho$ and pressure $P$ is given by

$$
v=\sqrt{\frac{C P}{\rho}}
$$

where $C$ is a constant.
Show that $C$ has no unit.

$$
\begin{aligned}
C & =\frac{v^{2} \rho}{P} \\
& =\frac{v^{2} \rho}{F / A} \\
& =\frac{v^{2} \rho}{m a / A} \\
& =\frac{v^{2} \rho A}{m a}
\end{aligned} \quad \text { Units of } C=\frac{\left(\mathrm{ms}^{-1}\right)^{2} \cdot \mathrm{~kg} \mathrm{~m}^{-3} \cdot \mathrm{~m}^{2}}{\mathrm{~kg} \cdot \mathrm{~ms}^{-2}} \quad \text { Has no units. }
$$

(d) Underline all the scalar quantities in the list below.

$$
\text { acceleration energy momentum power } \quad \text { weight }
$$

(e) A boat travels across a river in which the water is moving at a speed of $1.8 \mathrm{~ms}^{-1}$. The velocity vectors for the boat and the river water are shown to scale in Fig. 1.1.


Fig. 1.1 (shown to scale)
In still water the speed of the boat is $3.0 \mathrm{~m} \mathrm{~s}^{-1}$. The boat is directed at an angle of $60^{\circ}$ to the river bank.
(i) On Fig. 1.1, draw a vector triangle or a scale diagram to show the resultant velocity of the boat.
(ii) Determine the magnitude of the resultant velocity of the boat.

$$
\begin{aligned}
& \text { length of resultant velocity vector }=4.4 \mathrm{~cm} \\
& \text { So: } \\
& \begin{aligned}
\text { magnitude of resultant velocity } & =4.4 \times 0.6 \\
& =2.6 \mathrm{~ms}^{-1}
\end{aligned}
\end{aligned}
$$

$\qquad$ 2.6 $\mathrm{ms}^{-1}[2]$

## (Q1/23/M/J/16)

9 (a) A list of quantities that are either scalars or vectors is shown in Fig. 1.1.

| quantity | scalar | vector |
| :---: | :---: | :---: |
| distance | $\checkmark$ |  |
| energy | $\checkmark$ |  |
| momentum |  | $\checkmark$ |
| power | $\checkmark$ |  |
| time | $\checkmark$ |  |
| weight |  | $\checkmark$ |

Fig. 1.1
Complete Fig. 1.1 to indicate whether each quantity is a scalar or a vector.
One line has been completed as an example.
(b) A girl runs 120 m due north in 15 s . She then runs 80 m due east in 12 s .
(i) Sketch a vector diagram to show the path taken by the girl. Draw and label her resultant displacement R.

(NOT to scale)
(ii) Calculate, for the girl,

1. the average speed,

$$
\begin{aligned}
& \begin{array}{l}
\text { average } \\
\text { speed }
\end{array}=\frac{\text { total distance covered }}{\text { total time tallen }}=\frac{120+80}{15+12}=7.407 \\
& \text { average speed }= \\
& 7.4 \\
& \mathrm{~ms}^{-1} \text { [1] }
\end{aligned}
$$

2. the magnitude of the average velocity $v$ and its angle with respect to the direction of the initial path.
$\longrightarrow \theta$

## (Q1/23/M/J/17)

5 (a) Two forces, with magnitudes 5.0 N and 12 N , act from the same point on an object. Calculate the magnitude of the resultant force $R$ for the forces acting
(i) in opposite directions,
$12-5=7$
$R=$ $\qquad$ N [1]
(ii) at right angles to each other.
 $R=\ldots \quad 13$ N [1]
(b) An object X rests on a smooth horizontal surface. Two horizontal forces act on X as shown in Fig. 1.1.


Fig. 1.1 (not to scale)
A force of 55 N is applied to the right. A force of 18 N is applied at an angle of $115^{\circ}$ to the direction of the 55 N force.
(i) Use the resolution of forces or a scale diagram to show that the magnitude of the resultant force acting on X is 65 N .
Scale:
$1 \mathrm{~cm}=10 \mathrm{~N}$

length of resultant force vector $=6.5 \mathrm{~cm}$
So:
magnitude of resultant force $=6.5 \times 10$ $=65 \mathrm{~N}$
(ii) Determine the angle between the resultant force and the 55 N force.

$$
\rightarrow \theta=?
$$

When measured with a protractor, angle ' $\theta$ ' is found to be $15^{\circ}$

$$
\text { angle }=\ldots \ldots \ldots
$$

(c) A third force of 80 N is now applied to X in the opposite direction to the resultant force in (b). $\rightarrow m$
The mass of $X$ is 2.7 kg .

$$
\rightarrow a=?
$$

Calculate the magnitude of the acceleration of $X$.


$$
\begin{aligned}
& f_{\text {net }}=m a \\
\Rightarrow & 80-65=2.7 \times a \\
\Rightarrow & a=5.555
\end{aligned}
$$

$$
\text { acceleration }=\text {..................6............................. }{ }^{-2}[3]
$$

[Total: 9]

## (Q1/22/O/N/14)

18 (c) An object $B$ is on a horizontal surface. Two forces act on $B$ in this horizontal plane. $A$ vector diagram for these forces is shown to scale in Fig. 1.1.


Fig. 1.1

A force of 7.5 N towards north and a force of 2.5 N from $30^{\circ}$ north of east act on B.
The mass of $B$ is $750 \mathrm{~g} . \rightarrow m=0.750 \mathrm{~kg}$
(i) On Fig. 1.1, draw an arrow to show the approximate direction of the resultant of these two forces.

$$
\begin{equation*}
\rightarrow F=? \tag{1}
\end{equation*}
$$

(ii) 1. Show that the magnitude of the resultant force on $B$ is 6.6 N .

$$
\begin{aligned}
& \text { By cosine rule: } \\
& F^{2}=7.5^{2}+2.5^{2}-2(7.5)(2.5) \cos 60^{\circ} \\
& \Rightarrow F=6.6 \mathrm{~N}
\end{aligned}
$$

$$
r a=?
$$

2. Calculate the magnitude of the acceleration of $B$ produced by this resultant force.

$$
\begin{aligned}
& \text { Use the equation: } \\
& \text { F }=\text { ma }
\end{aligned}
$$

magnitude $=$ $\qquad$ 8.8 $\qquad$ $\mathrm{ms}^{-2}[2]$
$r \theta=$ ?
$r$ i.e. direction of resultant force (F)
(iii) Determine the angle between the direction of the acceleration and the direction of the 7.5 N force.

> * By Newton's $2^{\text {nd }}$ law of motion $(F=m a)$, acceleration AlwAYS has the same direction as the resultant force.

By sine rule:

$$
\left.\frac{F}{\sin 60^{\circ}}=\frac{2.5}{\sin \theta} \quad \text { (where } F=6.6 N\right)
$$

$$
\Rightarrow \theta=19^{\circ}
$$

## 9 Vector Resolution into Perpendicular Components

Let us consider a force of magnitude $F$ acting at point $O$ at angle $\theta$ to the horizontal as shown in Fig. 1.1.9. If we draw a normal from the head of $F$ onto horizontal axis, we get $F_{x}$ (the horizontal component of force $F$ ). Likewise, drawing a normal from the head of $F$ onto vertical axis gives $F_{y}$ (the vertical component of force $F$ ).


Figure 1.1.9
Note that the vector addition of forces $F_{x}$ and $F_{y}$ gives force $F$. This implies that force $F$ may be represented by two perpendicular forces $F_{x}$ and $F_{y}$ acting together (i.e. simultaneously). $F_{x}$ and $F_{y}$ are called the perpendicular components of force $F$. Now, in the right-angled triangle OAB:

$$
\begin{gathered}
F=\sqrt{F_{x}^{2}+F_{y}^{2}} \\
F_{x}=F \cos \theta \\
F_{y}=F \sin \theta \\
\frac{F_{y}}{F_{x}}=\tan \theta
\end{gathered}
$$

where $F, F_{x}$ and $F_{y}$ represent the side-lengths of the triangle OAB.

## Notes

- In a right-angled triangle, the side opposite to the right angle is the hypotenuse (hyp.), and the side opposite to angle $\theta$ is equal to 'hyp. $\times \sin \theta$. So, if:


Figure 1.1.10
then:

$$
F_{x}=F \sin \theta
$$

and:

$$
F_{y}=F \cos \theta
$$

- Any vector quantity (e.g. displacement, velocity, momentum etc.) can be resolved into its perpendicular components.


## Example

If:


$$
\begin{aligned}
& \text { mass of cube }(m)=0.5 \mathrm{Kg} \\
& \text { angle }(\theta)=30^{\circ}
\end{aligned}
$$

Figure 1.1.11
then determine the magnitudes of normal reaction $R$ and friction $f$ acting on the cube resting on the inclined plane.

## Solution

Resolving the weight $W$ of the cube into perpendicular components (one along the inclined plane, and the other, perpendicular to the plane) gives:


Now as the cube is stationary, so all the four forces acting on it ( $R, f, W \cos \theta$ and $W \sin \theta$ ) must be cancelling out each other. In other words, any force acting on the cube in one direction must be equal in magnitude to the force acting on it in the opposite direction; so:

$$
\begin{aligned}
& f=W \sin \theta \\
& =m g \sin \theta \\
& =(0.5)(9.81) \sin 30 \\
& =2.5 \mathrm{~N} \\
& \begin{aligned}
R & =W \cos \theta \\
R & =m g \cos \theta \\
& =(0.5)(9.81) \cos 30 \\
& =4.2 \mathrm{~N}
\end{aligned}
\end{aligned}
$$

## Note

Component of weight $W$ of a body down an inclined plane is always equal to ' $W \sin \theta$, where $\theta$ is the angle the plane makes with the horizontal.

## Now it's your turn

Do, in your workbook, question 20 (of chapter 1.1).

## (Q1/23/M/J/21)

20 (a) A property of a vector quantity, that is not a property of a scalar quantity, is direction. For example, velocity has direction but speed does not.
(i) State two other scalar quantities and two other vector quantities. scalar quantities $\qquad$ mass $\qquad$ and $\qquad$ time vector quantities: .............force and $\qquad$ acceleration $\qquad$
(ii) State two properties that are possessed by both scalar and vector physical quantities.

1. magnitude $\qquad$
2. unit $\qquad$
(b) A ship at sea is travelling with a velocity of $13 \mathrm{~ms}^{-1}$ in a direction $35^{\circ}$ east of north in still water, as shown in Fig. 1.1.


Fig. 1.1
(i) Determine the magnitudes of the components of the velocity of the ship in the north and the east directions.


(ii) The ship now experiences a tidal current. The water in the sea moves with a velocity of $2.7 \mathrm{~ms}^{-1}$ to the west.

$$
\rightarrow v_{x}=?
$$

Calculate the resultant velocity component of the ship in the east direction.

```
\(v_{x}=\) ship velocity in east direction - water velocity in west direction
    \(=7.5-2.7\)
    \(=4.8\)
                resultant east component of velocity \(=\)
```

$\qquad$

``` 4.8 8 \(\rightarrow\) velocity in north direction, \(v_{y}=11 \mathrm{~ms}^{-1}\)
```

(iii) Use your answers in (b)(i) and (b)(ii) to determine the magnitude of the resultant velocity of the ship.

$$
\longrightarrow v_{k}=4.8 \mathrm{~ms}^{-1}
$$

$$
\rightarrow v=?
$$

$$
\begin{gathered}
\text { Use the equation: } \\
v^{2}=v_{x}^{2}+v_{y}^{2}
\end{gathered}
$$

magnitude of resultant velocity $=$
12 $\qquad$ ms
(iv) Use your answers in (b)(i) and (b)(ii) to determine the angle between north and the resultant velocity of the ship.

```
                                    CO=?
```


angle =
24


[^0]:    Now it's your turn
    Do, in your workbook, question 19 (of chapter 1.1).

