

• Homogeneity of a Physical Equation

- An equation is said to be homogeneous if base units on L.H.S are identical to the base unit on the R.H.S (for eq.)
- For any equation to be classified as a correct Equation, it must satisfy the test of homogeneity.

$$\textcircled{1} \quad d = s \times t$$

$$\begin{array}{l} \downarrow \quad \downarrow \\ m = (ms^{-1}) \times (s) \\ m = m \quad [\text{homogeneous}] \end{array}$$

$$\textcircled{2} \quad v = f \lambda$$

$$\begin{array}{l} \downarrow \quad \downarrow \\ ms^{-1} = (s^{-1})(m) \\ ms^{-1} = ms^{-1} \end{array}$$

$$\textcircled{3} \quad P = \rho \cdot g \cdot h$$

$$\begin{array}{l} \downarrow \quad \downarrow \quad \downarrow \\ kgm^{-3} \cdot ms^{-2} \cdot (m) \\ kgm^{-3} \cdot ms^{-2} = kgm^{-1}s^{-2} \quad [\text{homogeneous}] \end{array}$$

$$\textcircled{4} \quad v = u + at$$

If an eq. contains more than one term on any one side, then to check for homogeneity, firstly break down the equation

- Compare v with u
$$\begin{array}{l} \downarrow \quad \downarrow \\ ms^{-1} \quad ms^{-1} \end{array}$$

- Compare v with at
$$\begin{array}{l} \downarrow \quad \downarrow \quad \downarrow \\ ms^{-1} \quad (ms^{-2})(s) \\ ms^{-1} \end{array}$$

Hence now the equation can be classified as a correct eq / homogeneous Eq

Q: In the following example suggest which equation is correct / homogeneous?

$$\textcircled{1} \quad E = mv$$

$$kgm^2s^{-2} = (kg)(ms^{-1})$$

not homogeneous

$E = \text{Energy}$
 $m = \text{mass}$
 $v = \text{velocity}$

$$\textcircled{2} \quad v = fg$$

$$ms^{-1} = (s^{-1})(ms^{-2})$$

$$= ms^{-3}$$

not homogeneous

$f = \text{freq}$
 $g = \text{acc. of free fall}$
 $\lambda = \text{wavelength}$

$$\textcircled{3} \quad E = \frac{1}{2} f v^3$$

$$kgm^2s^{-2} = (s^{-1})(ms^{-1})^3$$

not homogeneous

$$\textcircled{4} \quad v = \sqrt{g \lambda} \quad [\text{correct / homogeneous}]$$

$$ms^{-1} = \sqrt{(ms^{-2})(m)}$$

$$= \sqrt{m^2 s^{-2}}$$

$$ms^{-1} = ms^{-1}$$

Ques: Given that the equation shown below is homogeneous, use this information to find the base units of P and Q ?

$$3 \left(L + \frac{a^2}{P} \right) = Q T^2 \sin \theta$$

$L = \text{Length}$ $a = \text{radius}$ $T = \text{time}$

Simplify

$$3L + \frac{3a^2}{P} = Q T^2 \sin \theta$$

$3L$ compares with $Q T^2 \sin \theta$

$$\downarrow$$

$$m = Q s^2$$

$$Q = ms^{-2} \quad \text{Answer}$$

ignore (no units)

$\frac{3a^2}{P}$ compares with $Q T^2 \sin \theta$

$$\downarrow$$

$$\frac{m^2}{P} = (ms^{-2})(s^2)$$

ignore

Cross multiply P on the other side to get

$$P = m \quad \text{Answer}$$