

CHAPTER 1 : ALGEBRA

NOTES BY:
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SUBTOPIC 1 : PARTIAL FRACTIONS

- If the power of any variable in the denominator is equal or less than the power of any variable in numerator, the fraction is called improper fraction.

Examples :

$$\textcircled{i} \frac{x+1}{x-1} \quad \textcircled{ii} \frac{x^2+4}{(x+1)(x+4)} \quad \textcircled{iii} \frac{x^3+5}{x^2+4x+2}$$

(same powers) (same powers) (larger power)

- If the power of any variable in denominator is larger than any numerator variable's power then it is called proper fraction

Examples :

$$\textcircled{i} \frac{x+1}{x^2+5} \quad \textcircled{ii} \frac{x^2+6}{(x+1)(x+2)(x+3)}$$

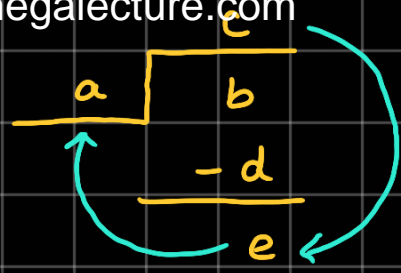
- We use long division method to convert an improper fraction to a proper fraction

Examples

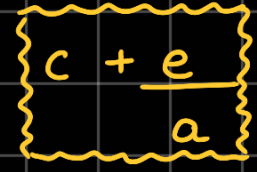
$$\textcircled{i} \frac{6x}{2x+1} \quad \textcircled{ii} \frac{3x^3-2}{2x-1} \quad \textcircled{iii} \frac{x^3+2x^2-3x+1}{x^2+x+3}$$

(i)

$$\begin{array}{r} 3 \\ 2x+1 \overline{) 6x} \\ \underline{-(6x+3)} \\ -3 \end{array} \quad \frac{6x}{2x} = 3$$



we will write the fraction in clockwise



$$3 - \frac{3}{2x+1} \quad \underline{\underline{\text{Ans}}}$$

(ii)

$$\begin{array}{r} \frac{3}{2}x^2 + \frac{3}{4}x + \frac{3}{8} \\ 2x-1 \overline{) 3x^3 + 0x^2 + 0x - 2} \\ \underline{-(3x^3 - \frac{3}{2}x^2)} \\ \frac{3}{2}x^2 + 0x - 2 \\ \underline{-(\frac{3}{2}x^2 - \frac{3}{4}x)} \\ \frac{3}{4}x - 2 \\ \underline{-(\frac{3}{4}x - \frac{3}{8})} \\ -\frac{13}{8} \end{array}$$

- $\frac{3x^3}{2x} = \frac{3}{2}x^2$
- $\frac{3x^2/2}{2x} = \frac{3x^2}{4x} = \frac{3}{4}x$
- $\frac{3x/4}{2x} = \frac{3x}{8x} = \frac{3}{8}$

$$-2 + \frac{3}{8} = -\frac{13}{8}$$

$$\frac{3x^2}{2} + \frac{3x}{4} + \frac{3}{8} - \frac{13}{8} \quad \underline{\underline{\text{Ans}}} \\ 2x-1$$

(iii)

$$\begin{array}{r} x+1 \\ x^2+x+3 \overline{) x^3+2x^2-3x+1} \\ \underline{-(x^3+x^2+3x)} \\ x^2-6x+1 \\ \underline{-(x^2+x+3)} \\ -7x-2 \end{array}$$

$$\frac{x^3}{x^2} = x$$

$$\frac{x^2}{x^2} = 1$$

$$x+1 - \frac{7x-2}{x^2+x+3} \quad \underline{\underline{\text{Ans}}}$$

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• CASE 1 : LINEAR AND DISTINCT FACTORS

$$\frac{x+1}{(x-3)(x+4)}$$

- we will always first check if the fraction is proper or improper. If it is improper then first convert the fraction to proper fraction.

→ In the case above its proper so we will directly solve

- The roots $(x-3)(x+4)$ are in the linear form " $mx+c$ " and the roots are distinct ($x-3 \neq x+4$) in this case

- we will apply same number of constants as number of factors

$$\frac{x+1}{(x-3)(x+4)} \rightarrow \frac{A}{(x-3)} + \frac{B}{(x+4)}$$

$$\frac{x^2+6}{(x+1)(x+2)(x-3)} \rightarrow \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-3}$$

- Now we will solve using the cover method

$$\frac{x+1}{(x-3)(x+4)} = \frac{A(x+4) + B(x-3)}{(x-3)(x+4)}$$

$$x+1 = A(x+4) + B(x-3)$$

To find A, make B zero

To find B, make A zero

by putting $x=3$ [youtube.com/c/MegaLecture](https://www.youtube.com/c/MegaLecture) by putting $x=-4$

$$(3)+1 = A[(3)+4] + B[(3)-3] \quad (4)+1 = A[(-4)+4] + B[(-4)-3]$$

$$4 = A(7) + B(0)$$

$$-3 = A(0) + B(-7)$$

$$4 = 7A$$

$$-3 = -7B$$

$$A = 4/7$$

$$B = 3/7$$

$$\frac{A}{(x-3)} + \frac{B}{(x+4)}$$

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$$\frac{4/7}{(x-3)} + \frac{3/7}{(x+4)} \quad \underline{\underline{\text{Ans}}}$$

• CASE 2 : LINEAR AND REPEATED FRACTIONS

$$\frac{4x}{(x-1)(x+1)^2}$$

• In this case, the factors will be repeated

$$\frac{4x}{(x-1)(x+1)(x+1)}$$

• We will apply same number of constants as number of factors and then solve

$$\frac{6x^2}{(x-1)(x+4)^3} \rightarrow \frac{6x^3}{(x-1)(x+4)(x+4)(x+4)} = \frac{A}{(x-1)} + \frac{B}{(x+4)} + \frac{C}{(x+4)^2} + \frac{D}{(x+4)^3}$$

$$\frac{4x}{(x+1)^2(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-1)}$$

Example :

$$\frac{2x+1}{(1+3x)(x-2)^2} = \frac{A}{(1+3x)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\frac{2x+1}{(1+3x)(x-2)^2} = \frac{A(x-2)(x-2)^2 + B(1+3x)(x-2)^2 + C(1+3x)(x-2)}{(1+3x)(x-2)(x-2)^2}$$

$$= \frac{\cancel{(x-2)} [A(x-2)^2 + B(1+3x)(x-2) + C(1+3x)]}{(1+3x)\cancel{(x-2)}(x-2)^2}$$

$$\frac{2x+1}{(1+3x)(x-2)^2} = \frac{A(x-2)^2 + B(1+3x)(x-2) + C(1+3x)}{(1+3x)(x-2)^2}$$

$$2x+1 = A(x-2)^2 + B(1+3x)(x-2) + C(1+3x)$$

To find A, make B and C zero by putting $x = -1/3$

$$2(-1/3) + 1 = A[(-1/3) - 2]^2 + B(1 + 3(-1/3))((-1/3) - 2) + C(1 + 3(-1/3))$$

$$\frac{1}{3} = A(-7/3)^2 + B(0) + C(0)$$

$$\frac{1}{3} = \frac{49}{9} A$$

$$A = 3/49$$

To find C, make A and B zero by putting $x = 2$

$$2(2) + 1 = A(2-2)^2 + B(1+3(2))(2-2) + C(1+3(2))$$

$$5 = A(0) + B(0) + C(7)$$

$$5 = 7C$$

$$C = 5/7$$

To find C, firstly we put the values of A and C in the equation and then put any value of x of our choice which has not been used above. (for e.g we can't use $x = \frac{-1}{3}$ or $x = 2$)

$$A = \frac{3}{49} \quad C = \frac{5}{7} \quad x = 0$$

$$2(0)+1 = A(0-2)^2 + B(1+3(0))(0-2) + C(1+3(0))$$

$$1 = A(4) + B(-2) + C(1)$$

$$1 = \left(\frac{3}{49} \times 4\right) + \left(\frac{5}{7} \times 1\right) + B(-2)$$

$$1 = \frac{47}{49} - 2B$$

$$\frac{2}{49} = -2B$$

$$B = -\frac{1}{49}$$

$$\frac{A}{(1+3x)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\frac{\frac{3}{49}}{1+3x} - \frac{\frac{1}{49}}{x-2} + \frac{\frac{5}{7}}{(x-2)^2} \quad \underline{\underline{\text{Ans}}}$$

• CASE 3 : IRREDUCIBLE QUADRATIC FACTORS

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$$a^2 - b^2 = (a+b)(a-b)$$

$$a^2 + b^2 = (a+ib)(a-ib)$$

- The expression $a^2 + b^2$ can only expand in complex numbers hence it is a irreducible quadratic factor

Example :

$$\frac{4x}{(x^2+9)(1+x)} = \frac{Ax+B}{x^2+9} + \frac{C}{1+x}$$

$$\frac{4x}{(x^2+9)(1+x)} = \frac{(Ax+B)(1+x) + C(x^2+9)}{(x^2+9)(1+x)}$$

$$4x = (Ax+B)(1+x) + C(x^2+9)$$

$$4x = Ax + Ax^2 + B + Bx + Cx^2 + 9C$$

$$4x = Ax^2 + Cx^2 + Bx + Ax + B + 9C$$

$$0x^2 + 4x + 0 = x^2(A+C) + x(B+A) + B+9C$$

$$0x^2 = x^2(A+C) \quad 4x = x(B+A) \quad 0 = B+9C$$

$$0 = A+C$$

$$4 = B+A$$

$$B = -9C$$

$$A = -C$$

$$A = 4 - B$$

$$-C = 4 - (-9C)$$

$$-C = 4 + 9C$$

$$-4 = 10C$$

$$C = -\frac{4}{10}$$

$$C = -\frac{2}{5}$$

$$A = -(-\frac{2}{5}) \quad B = -9(-\frac{2}{5})$$

$$A = \frac{2}{5}$$

$$B = \frac{18}{5}$$

* This part can directly be calculated using calculator as these steps are not required to show

$$\frac{Ax+B}{x^2+9} + \frac{C}{1+x} \rightarrow \frac{\frac{2}{5}x + \frac{18}{5}}{x^2+9} - \frac{2/5}{1+x}$$

ANS