Physics (A-level)

Circular motion (chap.7):

- One **radian** (rad) is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle

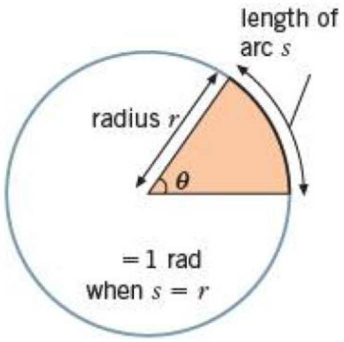


Figure 7.1 θ in radians = arc/radius

$$\theta = \frac{\text{length of arc}}{\text{radius of circle}} \left(= \frac{s}{r}\right)$$

- The **angular speed** is defined as the rate of change of angular displacement

$$\text{angular speed } \omega = \frac{\Delta\theta}{\Delta t}$$

- Figure 7.2, v → constant, in Δt object moves along the arc Δs and sweeps out at ΔØ:
    - Δs = rΔØ and dividing both sides by Δt:
        - Δs/Δt = rΔØ/Δt
    - v = rω

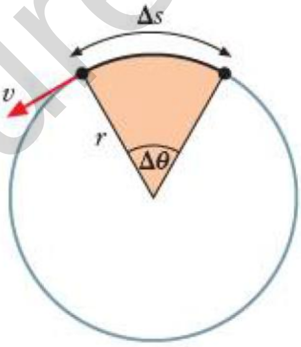


Figure 7.2 Angular velocity $\omega = v/r$

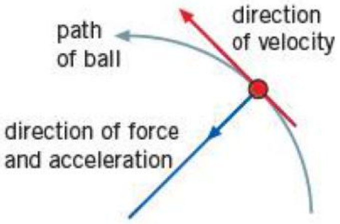


Figure 7.3 A ball swung in a circle on the end of a string

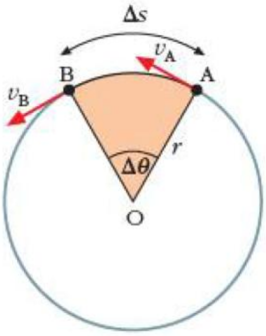


Figure 7.4 Diagram for proof of $a = v^2/r$

- Both the centripetal acceleration and force are towards the center (90° to that of the instantaneous velocity)
- Figure 7.4 & 7.5 shows the angle between two radii OA and OB & v_A and v_B (ΔØ)
    - Triangles OAB and CDE are similar
    - Consider angle ΔØ to be so small that arc AB approximated as a straight line
    - DE/CD = AB/OA
    - $\Delta v/v_A = \Delta s/r$
    - $\Delta v = \Delta s(v_A/r)$ and dividing both sides by Δt
    - $\Delta v/\Delta t = (\Delta s/\Delta t)(v_A/r)$
    - $A = v^2/r$

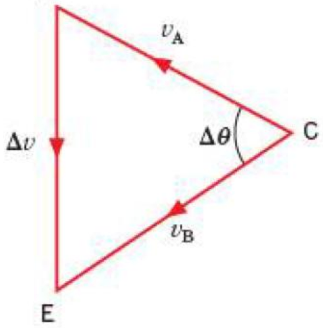


Figure 7.5 Vector diagram for proof of $a = v^2/r$

$$\text{centripetal acceleration} = \frac{v^2}{r} = r\omega^2$$

$$\text{centripetal force} = \frac{mv^2}{r} = mr\omega^2$$

Gravitational fields (chap.8):

- A **gravitational field** is a region of space where a mass experiences a force
- **Newton's law of gravitation** states that two point masses attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of their separation:

$$F = \frac{Gm_1m_2}{r^2}$$

$$F \propto m_1m_2/r^2$$

- G (gravitational constant) = 6.67 × 10^{-11} N m^2 kg^{-2}
- Newton's law specifies that the two masses are **point masses**, however the law still holds where the diameter/size of the masses is small compared to their separation
- Differences between gravitational fields and electric fields:
  - The electric field acts on charges, whereas the gravitational field acts on masses
  - The electric field can be attractive or repulsive, whereas gravitational field always attractive
- The gravitational field outside a spherical uniform mass is radial (all the lines of gravitational force appear to radiate from the centre of the sphere)
- Circular motion:
  - $F_{grav} = F_{circ}$
  - $GMm/r^2 = mv^2/r$
  - The period T of the planet in its orbit is the time required for the planet to travel a distance 2πr:
    - V = 2πr/T
  - $GMm/r^2 = m(4\pi^2r^2/T^2)/r$
  - $T^2 = (4\pi^2/GM)r^3$

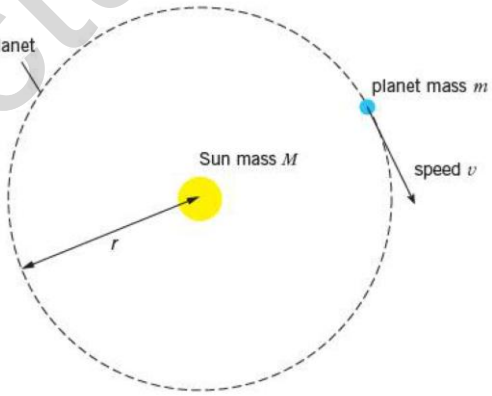


Figure 8.2 Circular orbit of a planet about the Sun

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

  - The right hand-side of the equation shows the constants (π and G), where M is the same (mass of the sun in the e.g.) when we are considering the relation between T and r
- **Kepler's third law of planetary motion** states that for planet or satellites describing circular orbits about the same central body, the square of the period is proportional to the cube of the radius of the orbit ($T^2 \propto r^3$)
- **Geostationary orbit** refers to communication satellites (called geostationary satellites) that are in equatorial orbits with exactly the same period of rotation as the Earth (24 hours), and move in the same direction as the Earth (west to east) so that they are always above the same point on the Equator
- The **gravitational field strength** at a point is defined as the force per unit mass acting on a small mass placed at that point
- Newton's second law: F = ma. Thus the gravitational field strength is given by g = F/m

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

- For small distances above the Earth's surface, g is approximately constant and is called the acceleration of free fall
- **Gravitational potential** at a point in a gravitational field is defined as the work done per unit mass in bringing a unit mass from infinity to the point

$$\phi = -\frac{GM}{r}$$

  - The gravitational potential is negative due to the **always** attractive gravitational force, hence there is work done by the test mass, decreasing its potential
- **g.p.e:** the work done in bringing an object from infinity to the point
- For a body of mass m, then the gravitational potential energy of the body will be *m times* as large as for the body of the unit mass

$$\text{Gravitational potential energy} = m\phi = -\frac{GMm}{r}$$

Example questions:

A satellite is orbiting the Earth. For an astronaut in the satellite, his sensation of weight is caused by the contact force from his surroundings. The astronaut reports that he is 'weightless', despite being in the Earth's gravitational field. Suggest what is meant by the astronaut reporting that he is 'weightless'.

- gravitational force provides the centripetal force
- gravitational force is 'equal' to the centripetal force
  - (accept Gm1m2 / x2 = mxω2 or $F_C = F_G$)
- 'weight'/sensation of weight/contact force/reaction force is difference between FG and FC which is zero

Explain why the centripetal force acting on both stars has the same magnitude.

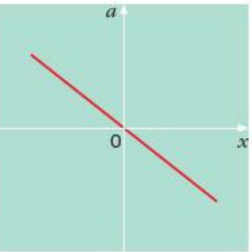
- gravitational force provides/is the centripetal force
- same gravitational force (by Newton III)

Oscillations (chap.13):

- The time taken for one complete oscillation or vibration is referred to as the **period T** of the oscillation
- The number of oscillations or vibrations per unit time is the **frequency f**
- Frequency f = 1/T
- The distance from the equilibrium position is known as the **displacement** (vector quantity)
- The **amplitude** (scalar quantity) is the maximum displacement
- **Simple harmonic motion** is defined as the motion of a particle about a fixed point such that its acceleration is proportional to its displacement from the fixed point, and is directed towards the point

$$a = -\omega^2 x$$

- A sinusoidal displacement-time graph is a characteristic of s.h.m.
- **Harmonic** oscillators move in s.h.m.



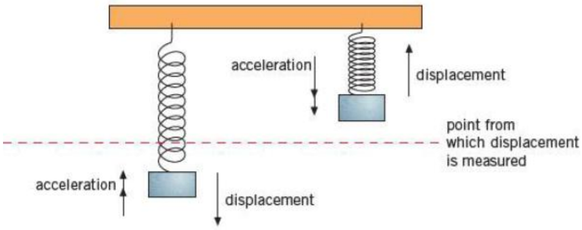


Figure 13.3 Directions of displacement and acceleration are always opposite.

- ω is known as the **angular frequency** of the oscillation
  - $\omega = 2\pi f$
- Newton's second law states that the force acting on the body is proportional to the acceleration of the body; hence the **restoring force** is proportional to the displacement and acting towards the fixed point
- Solution of equation for s.h.m.:

$$x = x_0 \sin \omega t$$

or

$$x = x_0 \cos \omega t$$

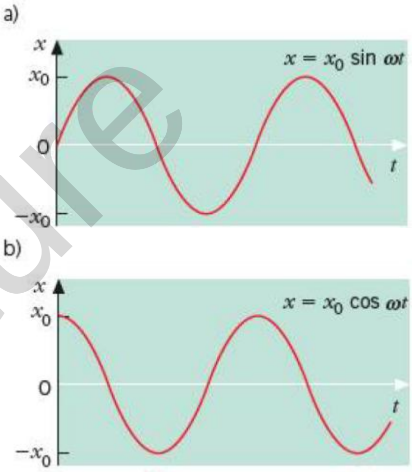


Figure 13.5 Displacement–time curves for the two solutions to the s.h.m. equation

  - X_0 → amplitude of oscillation
  - V → the gradient of displacement-time graph

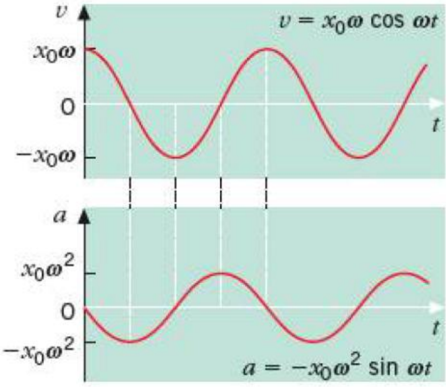
$$v = x_0\omega \cos \omega t \text{ when } x = x_0 \sin \omega t$$

  - For the case where x is zero at time t = 0, displacement and velocity are given by:
    - $x = x_0 \sin \omega t$
    - $v = x_0\omega \cos \omega t$
  - Applying $\sin^2\emptyset + \cos^2\emptyset = 1$:

$$x^2/x_0^2 + v^2/x_0^2\omega^2 = 1$$ leading to:

$$v^2 = x_0^2\omega^2 - x^2\omega^2$$ hence:

$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$



  - a → the gradient of velocity-time graph

$$a = -x_0\omega^2 \sin \omega t \text{ when } x = x_0 \sin \omega t$$

$$a = -\omega^2 x$$

- The K.E. of the particle oscillating with s.h.m. is ½mv^2:

$$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

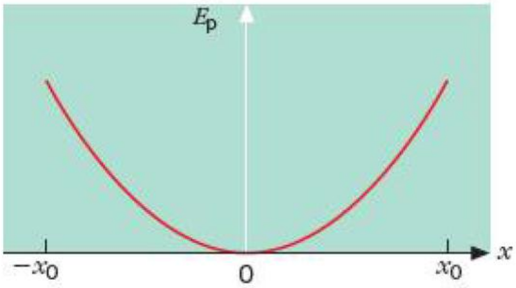


Figure 13.11 Variation of potential energy in s.h.m.

- The restoring force is F = ma:

$$F_{res} = -m\omega^2 x$$

- The potential energy:

$$E_p = \frac{1}{2}m\omega^2 x^2$$

- The total energy E_{tot} of the oscillating particle:

$$E_{tot} = E_k + E_p$$
$$= \frac{1}{2}m\omega^2 (x_0^2 - x^2) + \frac{1}{2}m\omega^2 x^2$$

$$E_{tot} = \frac{1}{2}m\omega^2 x_0^2$$

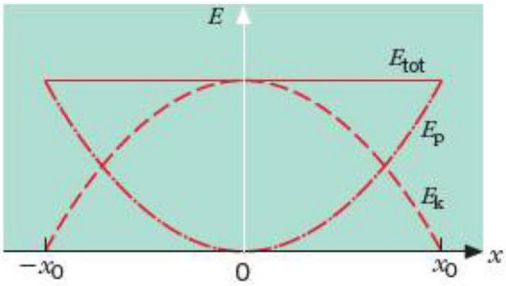


Figure 13.12 Energy variations in s.h.m.

- A particle is said to be undergoing **free oscillations** when the only external force acting on it is the restoring force (vibrating at its natural frequency):
    - No force to dissipate energy, hence constant amplitude and total energy remains constant, so s.h.m. are free oscillations
- In real situations, however, resistive forces cause the oscillator's energy to be dissipated, eventually converted into thermal energy. The oscillations are said to be **damped**
    - Light damping: the amplitude decreases gradually with time (T of the oscillation is slightly greater than the corresponding free oscillation)
    - Heavy damping: the oscillations will die away more quickly
    - Critically damped: the displacement decreases to zero in the shortest time, without any oscillation
    - Overdamping: the displacement decreases to zero in a longer time than for critical damping

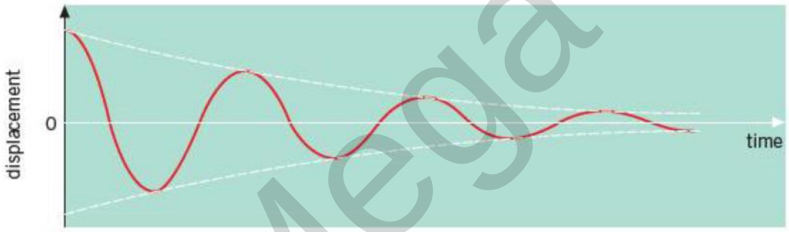


Figure 13.13 Lightly damped oscillations

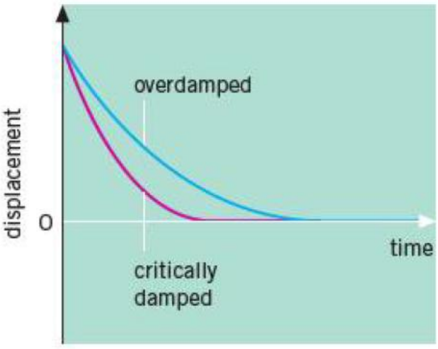


Figure 13.14 Critical damping and overdamping

- When a vibrating body undergoes free (undamped) oscillations, it vibrates at its **natural frequency**
- Periodic forces will make the object vibrate at the frequency of the applied force (forced vibrations)
- During forced oscillations, at first the amplitude is small, but increases with increasing frequency, reaches a maximum amplitude, then decreases (shown in a **resonance curve**)
    - **Resonance** occurs when a natural frequency of vibration of an object is equal to the driving frequency, giving a maximum amplitude of vibration
    - The frequency at which resonance occurs is called the **resonant frequency**

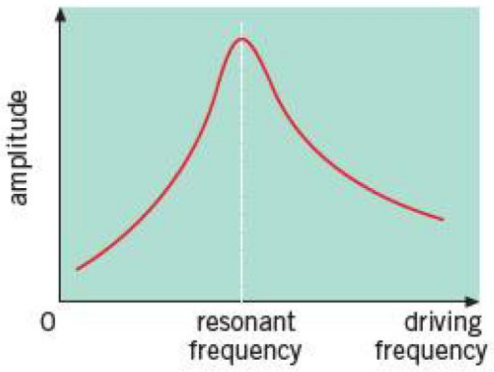


Figure 13.17 Resonance curve

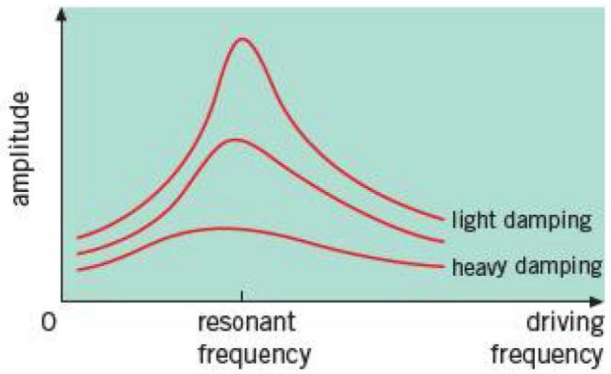


Figure 13.18 Effect of damping on the resonance curve

- As the degree of damping increases:
    - The amplitude of oscillation at all frequencies reduced
    - The frequency at maximum amplitude gradually shifts towards lower frequencies
    - The peak becomes flatter

Temperature (Chapter 11):

- Thermal energy is transferred from a region of higher temperature to a region of lower temperature
- **Thermal equilibrium** is a condition when two or more objects in contact have the same temperature so that there is no net flow of energy between them
- °C has two fixed point – the melting point of pure ice and the boiling point of pure water – divides the range between them into 100 equal intervals (changes when pressure changes or has impurities)
- Thermodynamic scale (Kelvin scale) is said to be an absolute scale, as it is not defined in terms of a property of any particular substance, based on the idea that the K.E. increases with increases in temperature; it has two fixed points:
    - Absolute zero (0 K) – (minimum internal energy of all substances, 0 K.E. and minimum electrical potential energy)
    - The triple point of water, the temperature at which ice, water and water vapour can co-exist, which is defined as 273.16 K (0.01 °C)

$$\theta/°\text{C} = T/\text{K} - 273.15 \text{ or } T/\text{K} = \theta/°\text{C} + 273.15$$

Feature	Resistance thermometer (thermistor)	Thermocouple thermometer
Robustness	Very robust	Robust
Range	Narrow range	Can be very wide
Size	Larger, has greater thermal capacity hence slower acting	Smaller, has smaller thermal capacity hence quicker acting and can measure temp. at a point
Sensitivity	High sensitivity over a narrow range	Can be sensitive according to the metals chosen
linearity	Fairly linear over a narrow range	Non-linear so requires calibration
Remote operation	Long conducting wires allow the operator to be at a distance from the thermometer	Long conducting wires allow the operator to be at a distance from the thermometer

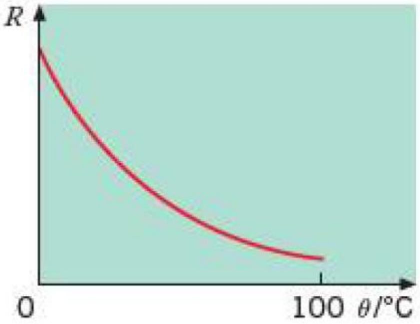


Figure 11.8 Resistance *R* of a thermistor over a small range of temperatures

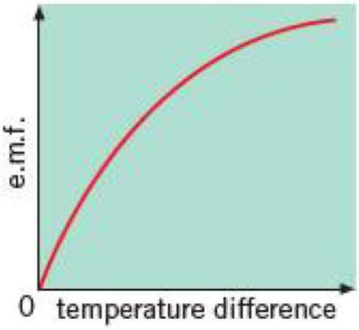


Figure 11.10 Graph of thermocouple e.m.f. against temperature

Thermal properties of materials (Chapter 12):

- A solid has fixed volume and shape (particles are close together, tightly bonded to their neighbors, and vibrating about fixed points)
  - During transition between solid and liquid, the energy supplied does not increase the K.E, hence the temperature of the solid, instead it is used to overcome the intermolecular forces between the atoms or molecules – increasing the electrical potential energy of the molecules, this increase is the latent heat of fusion of solid
- A liquid has fixed volume, no fixed shape and similar density as to solid
  - During transition between liquid and gas, the intermolecular forces in the liquid must be overcome, the latent heat of vaporization
- The graph shows that:
  - The electrical potential energy of two atoms very close together is large and negative
  - As the separation increases, their potential energy also increases
  - When atoms are completely separated, their potential energy is maximum and has a value of zero

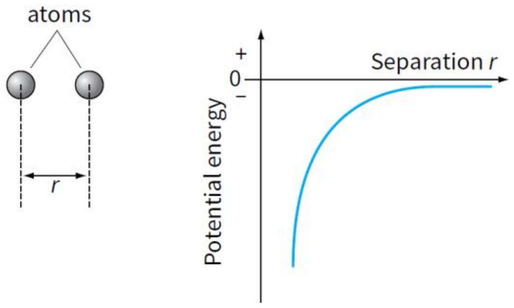


Figure 21.5 The electrical potential energy of atoms is negative and increases as they get further apart.

- A gas has no fixed shape or volume (widely separated and free to move around within their container)
- Latent heat of vaporization > latent heat of fusion, due to the greater energy required to completely separate the molecules than to break the rigid bonds in the solid (melting involves breaking of fewer bonds per molecule); energy is required to push back the atmosphere as liquid turns to vapour, vol. of vapour > vol. of liquid
- During evaporation, the most energetic molecules are most likely escape the surfaces of the liquid and hence reducing the average K.E. thus its temperature (cooling effect)
- **The internal energy** of a system is the sum of the random distribution of kinetic and potential energies of its atoms or molecules
  - Can be increased by heating and/or compression
- **First law of thermodynamics**: The increase in internal energy of a body is equal to the thermal energy transferred to it by heating plus the mechanical work done to it
  - $\Delta U = q + w$

- **Specific heat capacity:** The energy required per unit mass of a substance to raise its temperature by 1 K (or 1 °C). Unit: J kg^{-1} K^{-1}

$$\Delta Q = mc\Delta\theta$$

  - Assuming no energy losses to the surroundings:

$$M \times c \times (T_f - T_i) = I \times V \times t$$

  - Assuming energy losses (e.g. in 300s):
    - First reading:

$$\text{thermal energy supplied} = \text{thermal energy gained by liquid} + \text{energy losses to surroundings}$$

$$V_1 \times I_1 \times 300 = M_1 \times c \times (T_o - T_i) + h$$

    - Second reading:

$$V_2 \times I_2 \times 300 = M_2 \times c \times (T_o - T_i) + h$$

    - Subtracting:

$$(V_2 \times I_2 \times 300) - (V_1 \times I_1 \times 300) = (M_2 - M_1) \times c \times (T_o - T_i)$$

Hence, thermal energy losses have been eliminated and c can be det

- **Specific latent heat of fusion:** The energy required per unit mass of a substance to change it from solid to liquid without a change in temperature. Unit: J kg−1

$$\Delta Q = mL_f$$

- **Specific latent heat of vaporization:** The energy required per unit mass of a substance to change it from liquid to gas without a change in temperature. Unit: J kg−1

$$\Delta Q = mL_v$$

  - Assuming no energy losses to the surroundings:

$$(M - m) \times L = V \times I \times 300$$

  - Assuming energy losses (e.g. in 300s):
    - First reading:

$$\text{thermal energy supplied by heater} = \text{energy used to vaporise water} + \text{energy losses to surroundings}$$

$$V_1 \times I_1 \times 300 = M_1 \times L + h$$

    - Second reading:

$$V_2 \times I_2 \times 300 = M_2 \times L + h$$

    - Subtracting:

$$(V_2 I_2) \times 300 - (V_1 I_1) \times 300 = (M_2 - M_1) \times L$$

- Exchanges of heat energy examples:

1 A mass of 0.30 kg of water at 95 °C is mixed with 0.50 kg of water at 20 °C. Calculate the final temperature of the water, given that the specific heat capacity of water is 4200 J kg^{-1} K^{-1}.
Hint: always start by writing out a word equation containing all the gains and losses of heat energy.
heat energy lost by hot water = heat energy gained by cold water

$$(m \times c \times \Delta\theta_1) = (M \times c \times \Delta\theta_2)$$

$$0.30 \times 4200 \times (95 - \theta) = 0.50 \times 4200 \times (\theta - 20)$$

where θ is the final temperature of the water.

$$1260 \times (95 - \theta) = 2100 \times (\theta - 20)$$

$$119700 - 1260\theta = 2100\theta - 42000$$

$$161700 = 3360\theta$$

$$\theta = \mathbf{48\,°C}$$

2 A mass of 12 g of ice at 0 °C is placed in a drink of mass 210 g at 25 °C. Calculate the final temperature of the drink, given that the specific latent heat of fusion of ice is 334 kJ kg^{-1} and that the specific heat capacity of water and the drink is 4.2 kJ kg^{-1} K^{-1}.
energy lost by drink = energy gained by melting ice + energy gained by ice water

$$(m \times c \times \Delta\theta_1) = (M \times L_f) + (M \times c \times \Delta\theta_2)$$

$$M \times c \times \Delta\theta_2 = \frac{12}{1000} \times 4.2 \times 1000 \times (\theta - 0)$$

$$= 50.4\theta$$

$$\frac{210}{1000} \times 4.2 \times 1000 \times (25 - \theta) = \frac{12}{1000} \times 334 \times 1000 + 50.4\theta$$

where θ is the final temperature of the drink. Simplifying,

$$22050 - 882\theta = 4008 + 50.4\theta$$

$$18\,042 = 932.4\theta$$

$$\theta = \mathbf{19\,°C}$$

Electric fields (Chapter 17):

- For any point outside a spherical conductor, the charge on the sphere may be considered to act as a point charge at the centre of the sphere
- **Coulomb's law**: Any two point charges exert an electrical force on each other that is proportional to the product of their charges and inversely proportional to the square of the distance between them

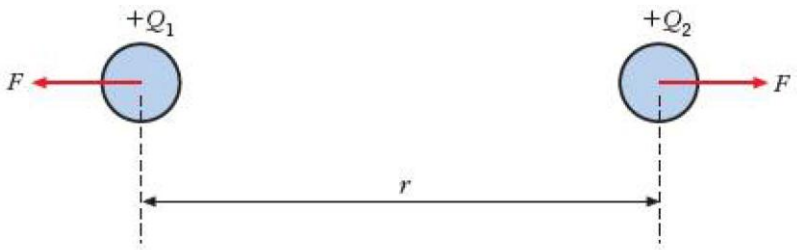


Figure 17.2 Force between charged spheres

- Figure 17.2 shows for point charges Q1 and Q2 a distance r apart. Coulomb's law gives the force F as:

$$F \propto Q_1Q_2/r^2$$

or

$$F = \frac{kQ_1Q_2}{r^2}$$

where k is a constant of proportionality, the value of which depends on the insulating medium around the charges and the system of units employed. In SI units, F is measured in newtons, Q in coulombs and r in metres. Then the constant k is given as

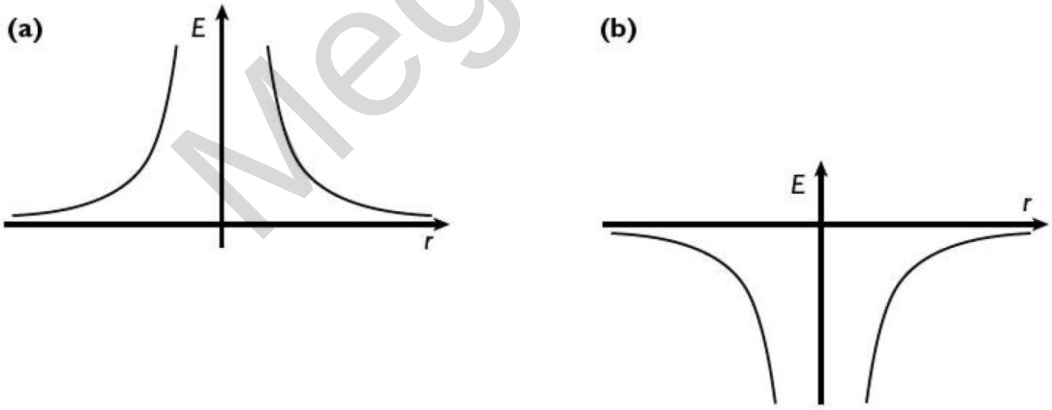
$$k = \frac{1}{4\pi\varepsilon_0}$$

and so

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$$

- When the charges are in a vacuum (or air), the quantity ε_0 is called the **permittivity of free space**
    - ➢ $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$
- **Electric field strength**: The force per unit positive charge at a point. Unit: Vm^{-1} or NC^{-1}
- The electric field E at the location of q is given by $E = F/q$, hence the electric field due to the isolated point charge is:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$



The electric field near (a) a positive point charge and (b) a negative point charge

- **Electric potential:** The energy per unit charge due to a charged body's position in an electric field. Unit: V (volt)
- Similar to electric field strength, electric potential is defined as the potential energy per unit positive charge (e.g. at point A, having potential energy E_{PA}):
    - ➢ $V_A = E_{PA}/Q$

- The electric field strength is equal to the negative of the potential gradient at that point:

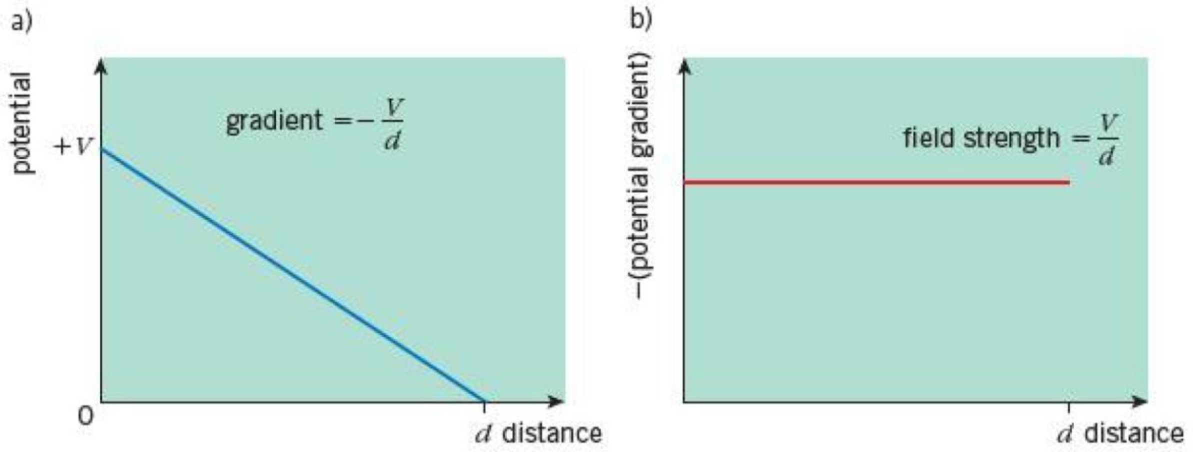


Figure 17.3 Graphs of the potential and the (negative of the) potential gradient for a uniform electric field.

- The potential *V* is given by:

$$V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

- Electric fields and gravitational fields:
    - force ∝ 1 / r 2
    - potential ∝ 1 / r
    - gravitation force (always) attractive
    - electric force attractive or repulsive

Ideal gases (Chapter 10):

- **Boyle's law:** The pressure exerted by a fixed mass of gas is inversely proportional to its volume, provided the temperature of the gas remains constant
    - P1V1 = P2V2
- **Charles's law:** The volume occupied by a gas at constant pressure is directly proportional to its thermodynamic (absolute) temperature

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

- **Gay-Lussac's law:** $P_1/T_1 = P_2/T_2$
- Combining the three gas laws into a single relation:

$$pV \propto T$$

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

- Another series of experiments provide (using the number *n* of moles of the gas):

$pV \propto nT$ **or** $pV = nRT$

- The previous equation can be expressed in the form: $pV = NkT$
  - N is the number of molecules in the gas
  - k is the **Boltzmann constant** ($1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$)
- The molar gas constant R and the Boltzmann constant k are connected through N_A:

$k = R/N_A$

- **Avogadro constant**:
  - Amount of substance containing N_A particles/molecules/atoms
  - Amount of substance which contains the same number of particles/molecules/atoms as there are atoms in 12g of carbon-12
- R, **molar gas constant** or **universal gas constant** (same value for all gases), having the value of $8.3\,\mathrm{J\,K^{-1}\,mol^{-1}}$
- An **ideal gas** is one which obeys the equation of state $pV = nRT$ or $pV = NkT$
- **Mole:** The amount of matter which contains the same number of atoms/nuclei as there are in 12 g of carbon-12
- Tiny pollen grains suspended in water shows the jerky, erratic, random motion (**Brownian motion**), due to the bombardment from all sides of the water molecules. Brownian motion can be reproduced by observing the motion of tiny soot particles in smoke, these particles move in a jerky motion too, proving the idea of rapid, random motion as required by the molecular model
- The **kinetic theory of gases** is a theory which links these microscopic properties of particles (atoms or molecules) to the macroscopic properties of a gas
- The assumptions of the kinetic theory of an ideal gas are:
  - Time of collisions negligible compared to time between collisions
  - No intermolecular forces except during collisions
  - Random motion of molecules
  - Large number of molecules
  - Total volume of molecules negligible compared to volume of containing vessel / Average separation large compared with the size of the molecules

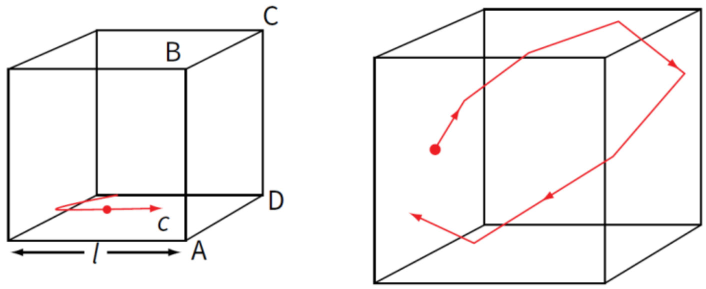


Figure 22.10 A single molecule of a gas, moving in a box.

- **Molecule in a box:**

- Consider a collision in which a single molecule with mass *m* moving with speed *c* parallel to one side of the box
- Collision striking side ABCD of the cube; elastically rebounded to the opposite direction (velocity is $-c$; momentum changes from mc to $-mc$), so momentum from the single collision is:

$$\begin{aligned}\text{change in momentum} &= -mc-(+mc)\\ &= -mc-mc=-2mc\end{aligned}$$

- Between consecutive collisions with side ABCD, the molecule travels a distance of 2l at speed c. Hence:

$$\text{time between collisions with side ABCD} = \frac{2l}{c}$$

- Using Newton's second law:

$$\text{force} = \frac{\text{change in momentum}}{\text{time taken}} = \frac{2mc}{2l/c} = \frac{mc^2}{l}$$

- The area of side ABCD is l^2, hence pressure is:

$$\text{pressure} = \frac{\text{force}}{\text{area}} = \frac{mc^2/l}{l^2} = \frac{mc^2}{l^3}$$

- For the large number *N* of molecules, we write the average value of c^2 as $<c^2>$, and multiply by *N* to find the total pressure:

$$\text{pressure } p = \frac{Nm<c^2>}{l^3}$$

- Considering three dimensions, divide by 3 to find the pressure exerted:

$$\text{pressure } p = \frac{1}{3}\frac{Nm<c^2>}{l^3}$$

- l^3 is equal to the volume *V* of the cube, hence:

$$p = \frac{1}{3}\frac{Nm}{V}<c^2> \qquad \text{or} \qquad pV = \frac{1}{3}Nm<c^2>$$

- *Nm/V* is equal to the density *ρ* of the gas, and we can write:

$$p = \frac{1}{3}\rho <c^2>$$

- So pressure of gas depends only on its density and the mean square speed of its molecules

- The mean **translational kinetic energy** of an atom (or molecule) of an ideal gas is proportional to the thermodynamic temperature:

Since the average kinetic energy of a molecule is

$$\langle E_k \rangle = \tfrac{1}{2} m \langle c^2 \rangle$$

there seems to be a link between our kinetic theory equation for pV and energy. We can find this relation by re-writing the pV equation as

$$pV = \frac{2}{3} N \left(\frac{1}{2} m \langle c^2 \rangle \right) = \frac{2}{3} N \langle E_k \rangle$$

$$pV = \frac{2}{3} N \langle E_k \rangle = NkT$$

and

$$\langle E_k \rangle = \frac{3}{2} kT$$

The average speed of the molecules:

$$\langle E_k \rangle = \frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} kT$$

$$\langle c^2 \rangle = 3kT/m$$

and

$$\sqrt{\langle c^2 \rangle} = \sqrt{3kT/m}$$

- The quantity $\sqrt{\langle c^2 \rangle}$ is called the **root-mean-square (r.m.s.)** speed (**c_{rms}**) of the molecules

Alternating current (Chapter 24):

- The magnitude of the power dissipated in a resistor is given by the expression:
  I^2R or VI or V^2/R

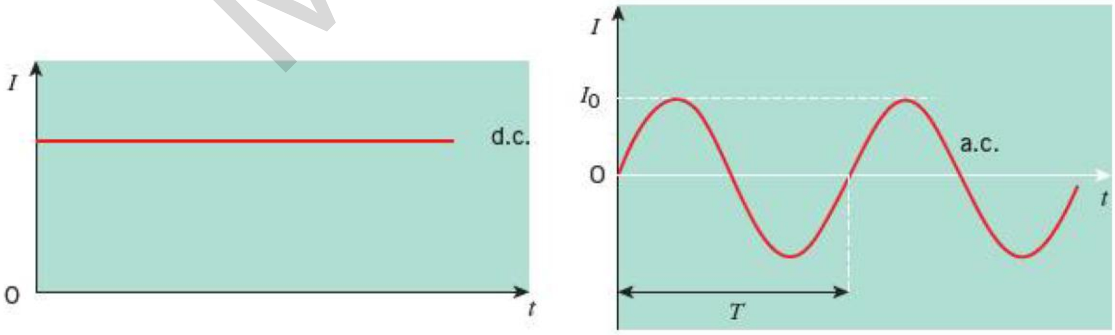


Figure 24.1 Direct and alternating currents

- Representing current and voltage:

$$I = I_0 \sin \omega t$$
$$V = V_0 \sin \omega t$$

- The **peak value** of the current or voltage is I_0 or V_0
- The time taken for one complete cycle of the a.c. is the period (T) of the current $T = 2\pi/\omega$
- The frequency is the number of complete cycles per unit time $f = \omega/2\pi$

The power generated in a resistance R is given by the usual formula

$$P = I^2R$$

but here the current I must be written as

$$I = I_0 \sin \omega t$$

Thus

$$P = I_0^2 R \sin^2 \omega t$$

- Since I_0 and R are constants, the average value of *P* will depend on the average value of $\sin^2 \omega t$, which is 1/2, hence the average power <*P*> delivered to the resistor is:

$$\langle P \rangle = \tfrac{1}{2} I_0^2 R = \tfrac{1}{2} V_0^2 / R$$

    - This is half the maximum power
- Average value of the square of the current or voltage:

$$\langle I^2 \rangle = \tfrac{1}{2} I_0^2 \text{ and } \langle V^2 \rangle = \tfrac{1}{2} V_0^2$$

$$I_{rms} = \sqrt{\langle I^2 \rangle} = I_0/\sqrt{2} = 0.707 I_0$$
$$V_{rms} = \sqrt{\langle V^2 \rangle} = V_0/\sqrt{2} = 0.707 V_0$$

- The square root of $\langle I^2 \rangle$ is called the root-mean-square, or r.m.s.
- The r.m.s value of the alternating current or voltage is that value of the direct current or voltage that would produce thermal energy at the same rate in a resistor

Example

A 1.5 kW heater is connected to the domestic supply, which is quoted as 240 V. Calculate the peak current in the heater, and its resistance.

The r.m.s version of the power/current/voltage equation is $I_{rms} V_{rms}$ = mean power.

This gives $I_{rms} = 1.5 \times 10^3 / 240 = 6.3$ A.

The peak current $I_0 = \sqrt{2}\, I_{rms} =$ **8.8 A**

The resistance $R = V_{rms}/I_{rms} = 240/6.3 =$ **38 Ω**

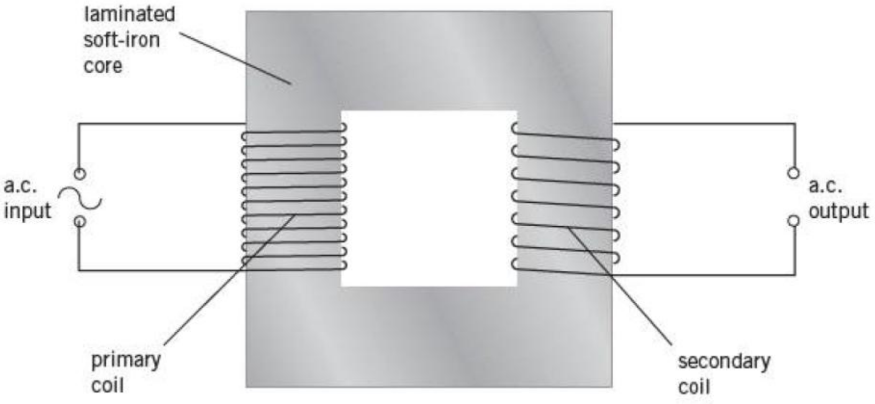


Figure 24.2 Simple transformer

- The alternating current in the primary coil gives rise to an in phase alternating magnetic flux, which threads through the secondary coil, in turn causes an induced e.m.f in the secondary coil – Faraday's law of electromagnetic induction
- For an ideal transformer (100% efficient):

$$\text{input power} = \text{output power}$$

$$V_pI_p = V_sI_s$$

and

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

- When $V_s > V_p$, $N_s > N_p$: step-up transformer
- When $V_s < V_p$, $N_s < N_p$: step-down transformer
- Sources of power losses in a transformer:
  - Loss of magnetic flux between the primary and secondary coils
  - Heating of the core due to eddy currents
  - Heating of the core due to repeated magnetisation and demagnetisation
- Soft iron core reduces heating due to repeated magnetisation and demagnetisation – not eddy currents – while laminating it reduces energy losses as thermal energy due to eddy currents
- Long-distance electrical transmission are prone to power losses due to heating of the cables (the I^2R effect); reduced if the power is transmitted at high voltage – done by using a.c. rather than d.c. for transformers to be usable
- **Rectification:** The process of converting alternating current (a.c.) into direct current (d.c.)

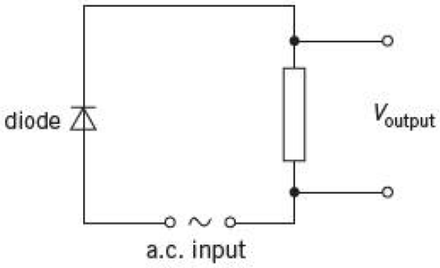


Figure 24.4 Single-diode circuit for half-wave rectification

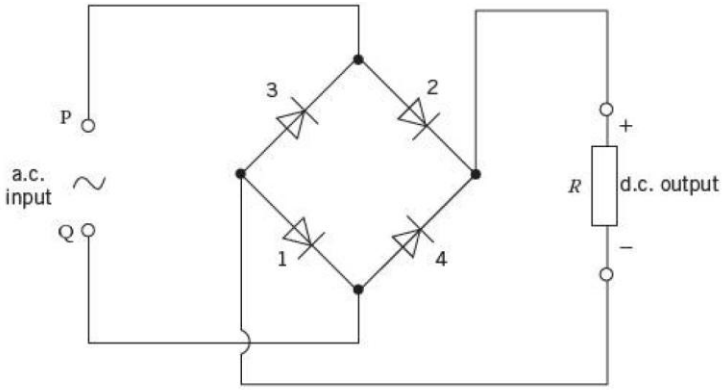


Figure 24.7 Four-diode (bridge) circuit for full-wave rectification

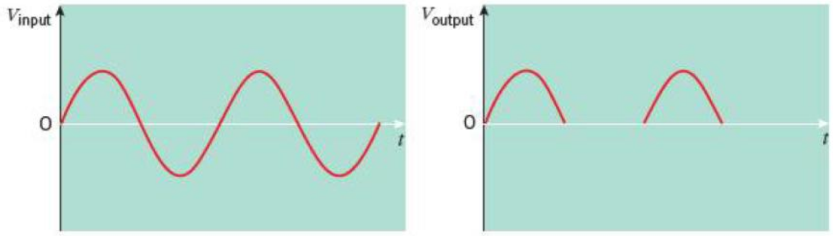


Figure 24.5 Half-wave rectification

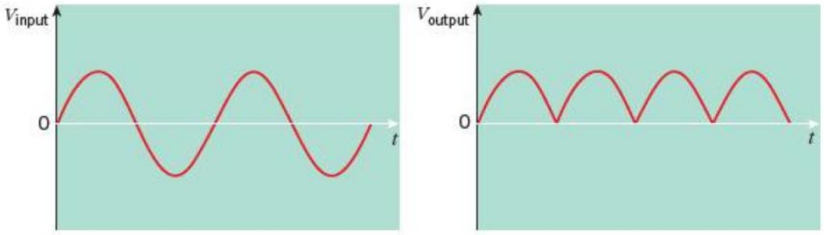


Figure 24.6 Full-wave rectification

- Full-wave rectification involves four diodes, and is referred to as a **bridge rectifier** circuit
- The inputs terminals are P and Q. If P is positive during the first half-cycle, diodes 1 and 2 will conduct; in the next half-cycle, Q is positive so diodes 3 and 4 will conduct. Thus the resistor will always have its upper terminal positive and its lower terminal negative. However, the output is not good a good approximation to the steady voltage, hence a capacitor across the output terminals is used:

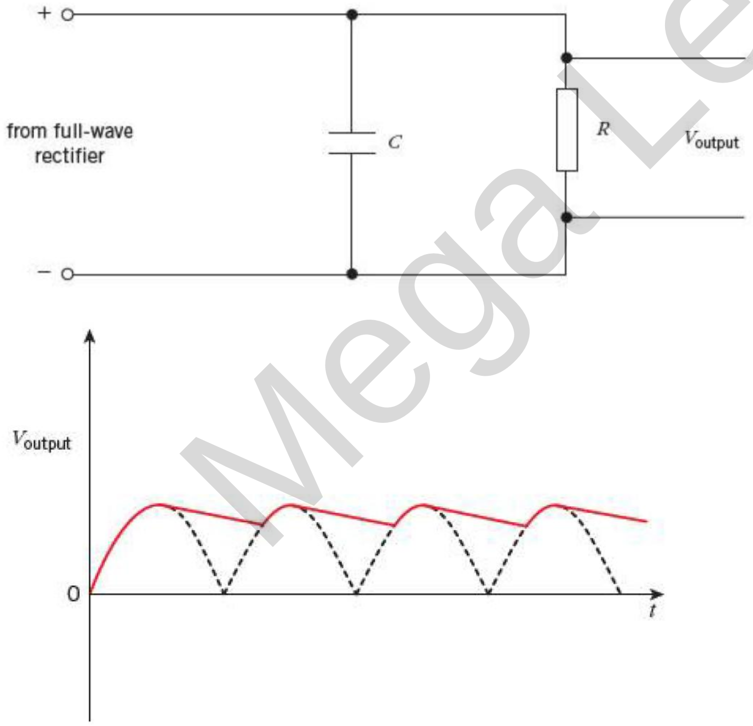


Figure 24.8 Smoothing by capacitor

- The capacitor charges up on the rising part of the half-cycle, then discharges through the resistor as the output voltage falls; is done to reduce the fluctuations in the unidirectional output, where the process is called **smoothing**
- The important factor is the time constant of the resistor-capacitor circuit. If the product of the capacitance *C* and the load *R* is much larger than the half-period of the original supply,

the ripple on the direct current or voltage will be small. Reducing the time constant will increase the ripple:

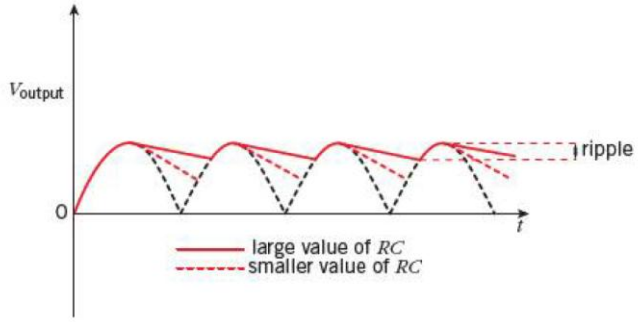


Figure 24.9 Magnitude of the ripple

Physics (A-level)

Capacitance (Chapter 18):

- Every capacitor has two leads, each connected to a metal plate, where in between there is an insulating material called the **dielectric**; to store energy, these two plates must be given equal and opposite electric charges
- Uses of capacitors in electrical circuits, other than for the smoothing of direct current:
    - Storing energy
    - Blocking d.c.
    - In oscillator circuits
    - In tuning circuits
    - In timing circuits
- Explain why the capacitor stores energy but not charge:
    - Charges on plates are equal and opposite , hence there is no resultant charge; energy is stored because there is a charge separation
- The **capacitance** of a capacitor is the charge stored on one plate per unit of potential difference between the plates. Given by the equation:

$$C = \frac{Q}{V}$$

- - *Q* is the magnitude of the charge on each of the capacitor's plates
    - *V* is the potential difference across it
- **Farad**: The unit of capacitance (abbreviated F). 1 F = 1 C V^{-1}
- During charging of a capacitor, power supply is used to push electrons from one plate to another; power supply does work on the electrons, increasing their potential energy, which is recovered during discharging
- The area under a graph of p.d. against charge is equal to work done

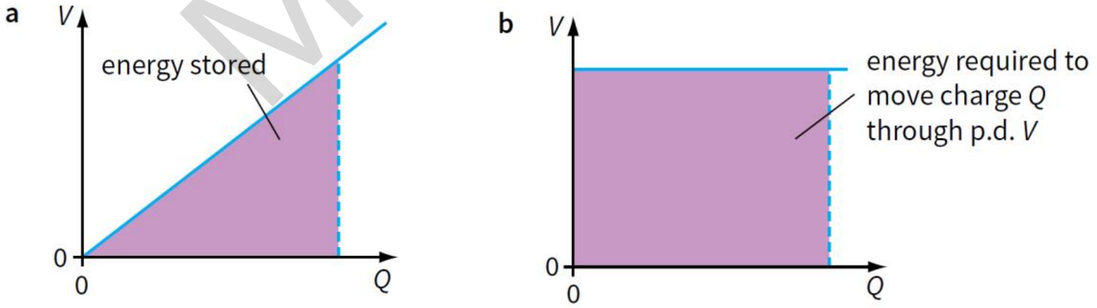


Figure 24.7 The area under a graph of voltage against charge gives a quantity of energy. The area in **a** shows the energy stored in a capacitor; the area in **b** shows the energy required to drive a charge through a resistor.

-

- Hence for a capacitor, energy stored is given by:

$$W=\tfrac{1}{2}QV \quad \& \quad W=\tfrac{1}{2}CV^2 \quad \& \quad W=\tfrac{1}{2}\frac{Q^2}{C}$$

- Capacitors in parallel (same p.d. across each capacitor; total charges equal to the sum of charges):
  - The total charge is given by:

$$Q=Q_1+Q_2=C_1V+C_2V$$

  - Since V is the common factor:

$$Q=(C_1+C_2)V$$

  - Comparing this with $Q = C_{\text{total}}V$:

$$C_{\text{total}}=C_1+C_2+C_3+\cdots$$

- Capacitors in series (p.d. is divided among capacitors; each capacitor stores same charges):
  - $V=V_1+V_2$
  - $\dfrac{Q}{C_{\text{total}}}=\dfrac{Q}{C_1}+\dfrac{Q}{C_2}$
  - $\dfrac{1}{C_{\text{total}}}=\dfrac{1}{C_1}+\dfrac{1}{C_2}$

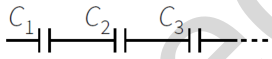
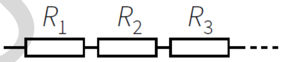
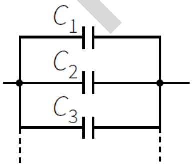
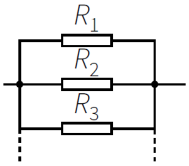
	Capacitors	**Resistors**
In series		
	store same charge	have same current
	$\frac{1}{C_{\text{total}}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots$	$R_{\text{total}}=R_1+R_2+R_3+\cdots$
In parallel		
	have same p.d.	have same p.d.
	$C_{\text{total}}=C_1+C_2+C_3+\cdots$	$\frac{1}{R_{\text{total}}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots$

Table 24.3 Capacitors and resistors compared.

Magnetic fields and electromagnetism (Chapter 26 TB):

- **Magnetic fields**: A force field in which a magnet, a wire carrying current, or a moving charge experiences a force
- Magnetic fields are produced by current-carrying conductors (moving charges (e.g. free electrons)) or by permanent magnets (movement of the electrons within the atoms)
- An electromagnet makes use of the magnetic field created by an electric current; which a coil is used to concentrate the magnetic field (field lines are closer together) – also called **solenoid** – creating north pole and south pole; strength can be increased by addition of ferrous core, due to its easily magnetised property

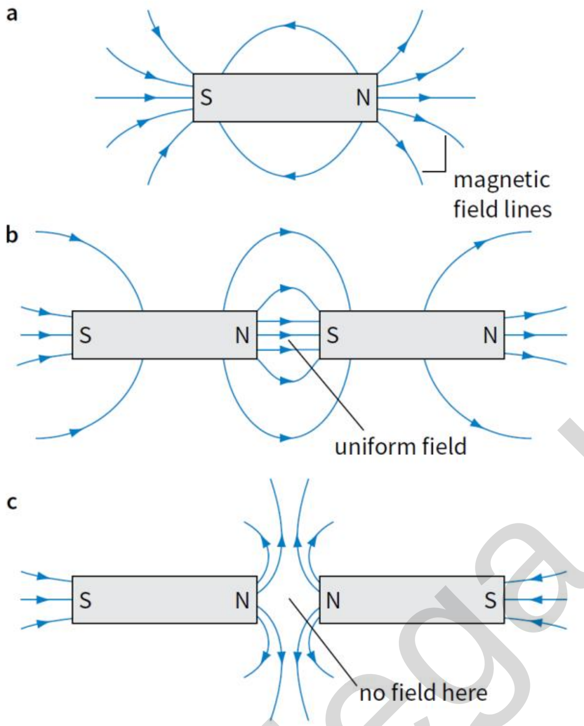


Figure 26.2 Magnetic field patterns: **a** for a bar magnet; **b** for two attracting bar magnets; **c** for two repelling bar magnets.

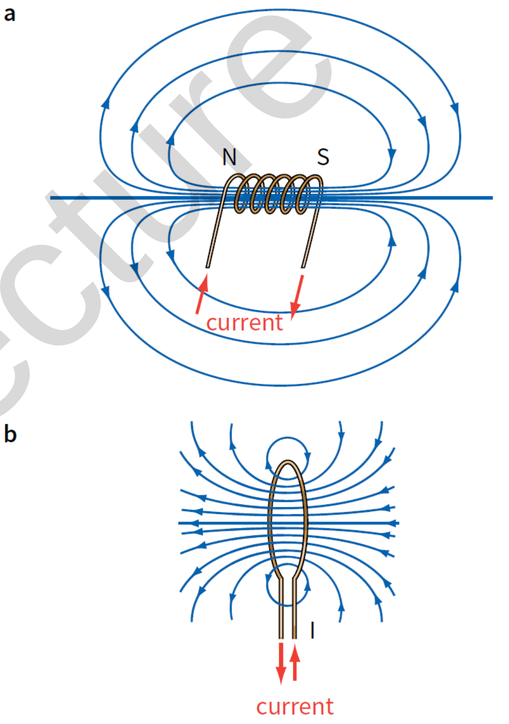
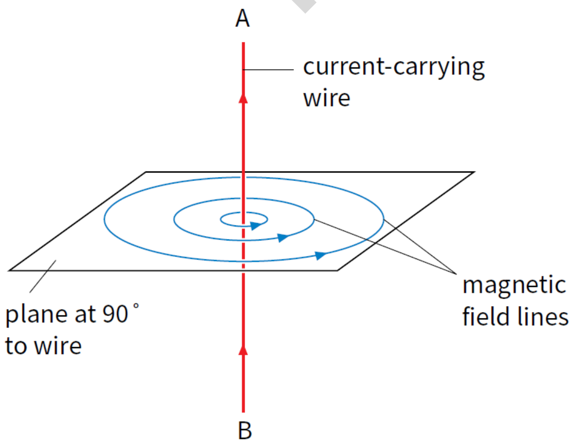


Figure 26.3 Magnetic field patterns for **a** a solenoid, and **b** a flat circular coil.



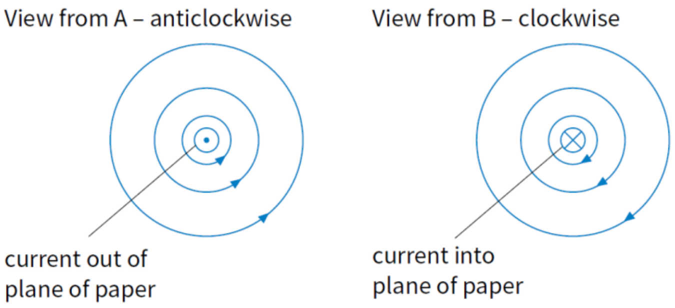


Figure 26.4 The magnetic field pattern around a current-carrying wire. The diagram also shows the convention used to indicate the direction of current.

- The **right-hand grip rule** gives the direction of magnetic field lines in an electromagnet

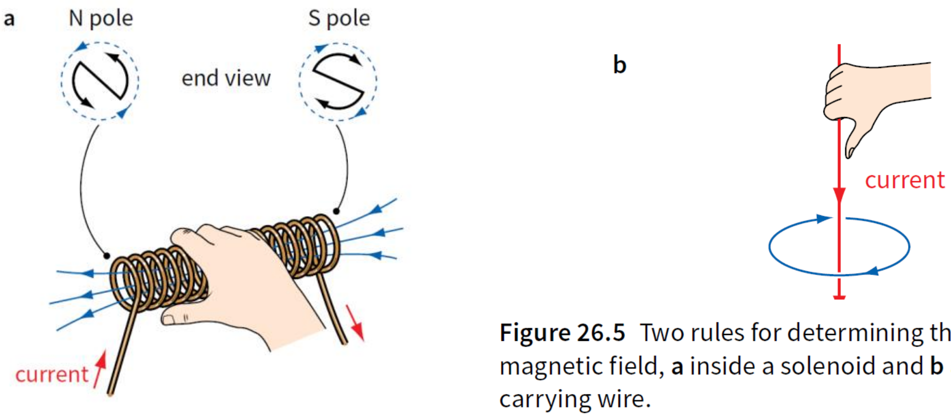


Figure 26.5 Two rules for determining the direction of a magnetic field, **a** inside a solenoid and **b** around a current-carrying wire.

- A current-carrying wire is surrounded by a magnetic field, which will interact with an external magnetic field, giving rise to a force on the conductor, similar to the fields of two interacting magnets:

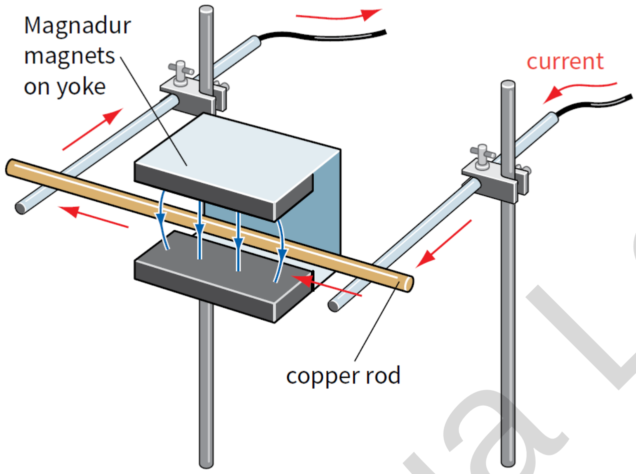


Figure 26.8 The copper rod is free to roll along the two horizontal aluminium 'rails'.

- The magnetic force created by the **motor effect** (e.g. a current in a coil's magnetic field interacts with a second magnetic field produced by a permanent magnet):

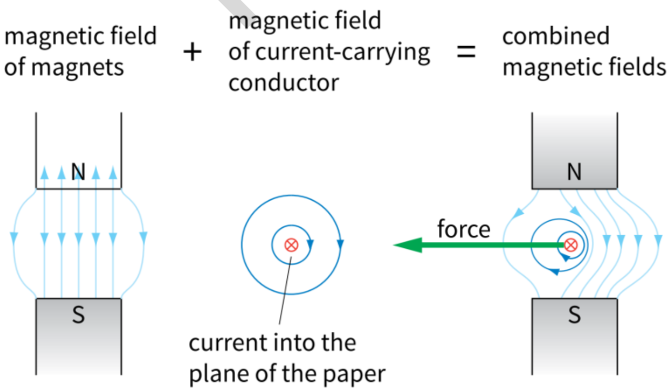


Figure 26.10 In the field of a permanent magnet, a current-carrying conductor experiences a force in accordance with Fleming's left-hand rule. The fields due to the permanent magnet and the current (left and centre) combine as shown on the right.

- The strength of a magnetic field is known as its **magnetic flux density**, B, representing the number of magnetic field lines passing through a region per unit area
- Gravitational field strength *g* at a point is defined as the force per unit mass:

$$g = \frac{F}{m}$$

- Electric field strength *E* is defined as the force per unit positive charge:

$$E = \frac{F}{Q}$$

- The **magnetic flux density** at a point in space is the force experienced per unit length by a long straight conductor carrying unit current and placed at **right angles** to the field at that point

$$B = \frac{F}{IL}$$

- **Tesla:** the magnetic flux density is 1T when a wire carrying a current of 1A placed at **right angles** to the magnetic field experiences a force of 1 N per metre of its length
- The force on the conductor is given by:

$$F = BIL$$

- Measuring *B* with a **current balance**:

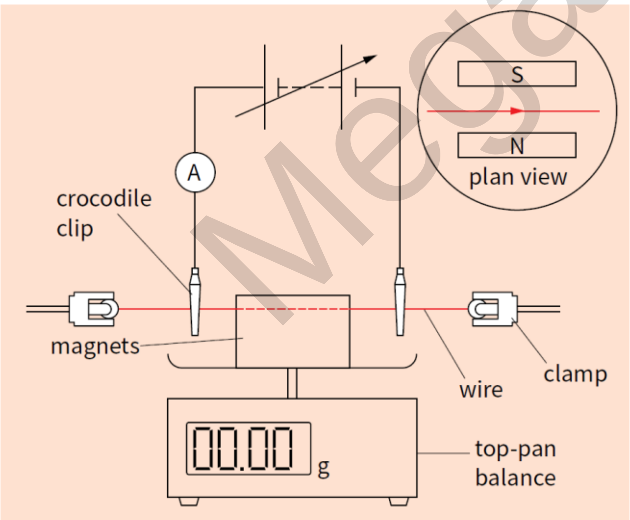


Figure 26.14 An arrangement to determine magnetic flux density in the laboratory.

- ➢ The magnetic field is roughly uniform
- ➢ The length *L* of the current-carrying wire measured using ruler
- ➢ When there is no current in the wire, the magnet arrangement is placed on the top an and the balance is zeroed
- ➢ When the current, *I*, flows in the wire, ammeter shows the value

- The wire will experience an upwards force and, according to Newton's third law of motion, there is an equal and opposite force on the magnets, hence pushed downwards causing readings on the balance
- F, I and L are now known, the magnetic flux density B between the magnets is given by:

$$B = \frac{F}{IL}$$

- Currents crossing fields:

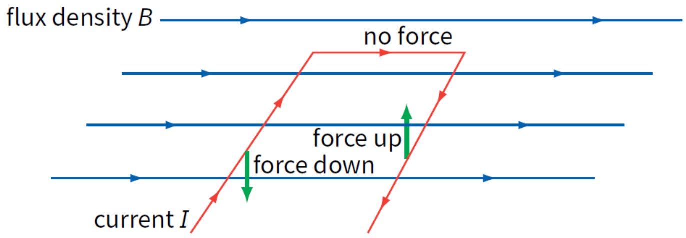


Figure 26.18 The force on a current-carrying conductor crossing a magnetic field.

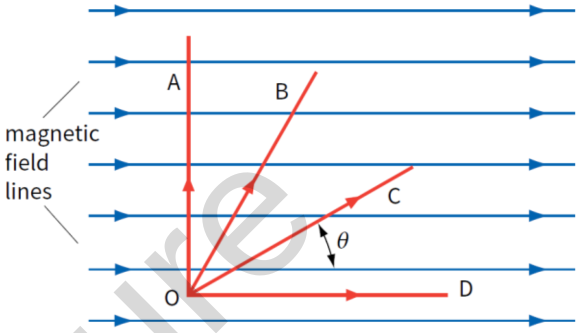


Figure 26.20 The force on a current-carrying conductor depends on the angle it makes with the magnetic field lines.

$$F = (B\sin\theta)IL$$

or simply:

$$F = BIL\sin\theta$$

- Figure 26.22a, their magnetic fields circle around, and in the space between the wires there is an extra-strong field, creating repulsive forces on the two wires
- Figure 26.22b, in the space between the two wires, the magnetic fields cancel out, hence the wires are pushed together; the two forces are equal and opposite to one another (Newton's third law of motion)

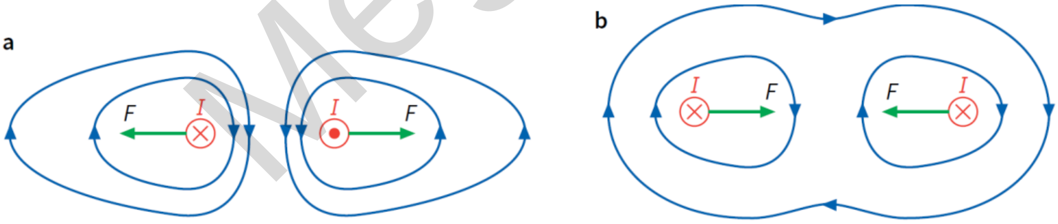


Figure 26.22 The forces between current-carrying wires.

Base units	Derived units	because
m, kg, s	newton N = kg m s^{-2}	$F = ma$
	joule J = kg m^2 s^{-2}	$W = Fd$
	watt W = kg m^2 s^{-3}	$P = \frac{W}{t}$
m, kg, s, A	coulomb C = A s	$Q = It$
	volt V = kg m^2 A^{-1} s^{-3}	$V = \frac{W}{Q}$
	tesla T = kg A^{-1} s^{-2}	$B = \frac{F}{IL}$

Table 26.1 How derived units relate to base units in the SI system.

The force between two 1 kg masses 1 m apart = 6.7×10^{-11} N

The force between two charges of 1 C placed 1 m apart = 9.0×10^{9} N

The force per meter on two wires carrying a current of 1 A placed 1 m apart = 2.0×10^{-7} N

- Electric force is strongest and gravity is the weakest, however over larger distances and with objects of large mass, the gravitational field becomes the most significant

Charged particles (Chapter 27 TB):

- The direction of conventional electric current is the direction of flow of positive charge; when electrons are moving, the conventional current is regarded as flowing in the opposite direction

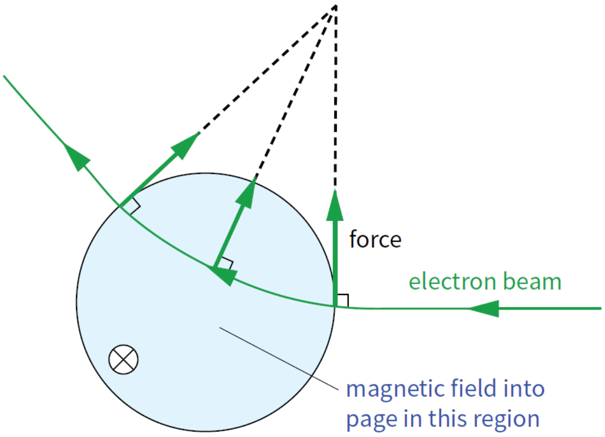


Figure 27.3 A beam of electrons is deflected as it crosses a magnetic field. The magnetic field into the plane of the paper is represented by the cross in the circle.

- The size of force on a moving charge in a uniform magnetic field depends on:
    - The magnetic flux density *B* (strength of the magnetic field)
    - The charge *Q* on the particle
    - The speed *v* of the particle
    - Hence if motion at right angles to the magnetic field:

$$F = BQv$$

- Hence if motion at an angle of θ to the magnetic field:

$$F = BQv \sin\theta$$

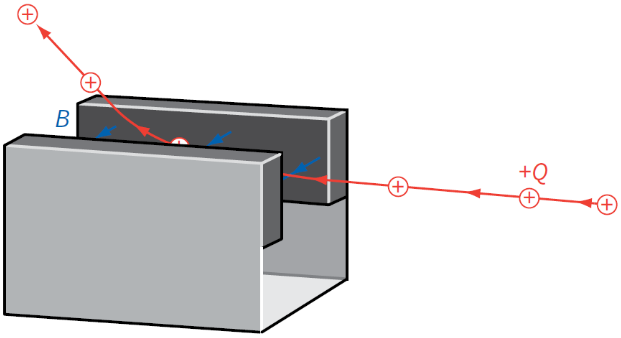


Figure 27.6 The path of a charged particle is curved in a magnetic field.

- The two equations $F = BIL$ and $F = BQv$ are consistent with one another:

Since current I is the rate of flow of charge, we can write:

$$I = \frac{Q}{t}$$

Substituting in $F = BIL$ gives:

$$F = \frac{BQL}{t}$$

Now, $\frac{L}{t}$ is the speed v of the moving particle, so we can write:

$$F = BQv$$

- For an electron, with a charge of -*e*, the magnitude of the force on it is:

$$F = Bev \quad (e = 1.60 \times 10^{-19}\,\mathrm{C})$$

- The force on a moving charge is sometimes called 'the *Bev* force'; it is this force acting on all the electrons in a wire which gives rise to 'the *BIL* force'
- When a charged particle moves at right angles to a uniform magnetic field, magnetic force *F* is always perpendicular to its velocity, hence *F* acts as a **centripetal force** (force directed towards the centre of the circle):

$$\text{centripetal force} = \frac{mv^2}{r}$$

The centripetal force is provided by the magnetic force Bev.
Therefore:

$$Bev = \frac{mv^2}{r}$$

Cancelling and rearranging to find r gives:

$$r = \frac{mv}{Be}$$

  - The equation rewritten in terms of momentum *p* of the particle:

$$p = Ber$$

- The equation $r = \frac{mv}{Be}$ shows that:
  - Faster moving particles move in bigger circles ($r \propto v$)
  - Particles with bigger masses move in bigger circles, due to more inertia ($r \propto m$)
  - Stronger field makes particles move in tighter circles ($r \propto 1/B$)
- **The charge-to-mass ratio of an electron** involves finding the charge-to-mass ratio $\frac{e}{m_e}$ known as the specific charge on the electron
  - Using the equation for an electron travelling in a circle in a magnetic field:

$$\frac{e}{m_e} = \frac{v}{Br}$$

- *B* and *r* are measurable, but *v* is not; hence cathode-anode voltage (V_{ca}) is used, which p.d. causes each electron to accelerate; if an individual electron has charge –*e* then an amount of work $e \times V_{ca}$ is done one each electron, which is its K.E. as it leaves the anode:

$$eV_{ca} = \tfrac{1}{2}m_e v^2$$

- Where m_e is the electron mass; *v* is the speed of the electron
- Eliminating *v* from the two equations gives:

$$\frac{e}{m_e} = \frac{2V_{ca}}{r^2B^2}$$

- Velocity selector:

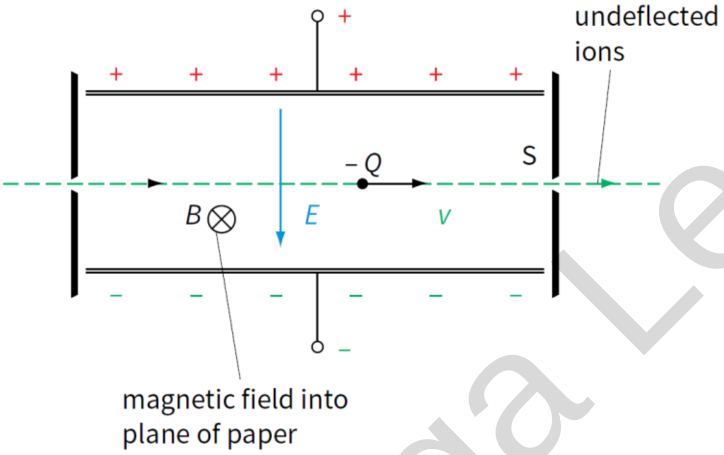


Figure 27.11 A velocity selector – only particles with the correct combination of charge, mass and velocity will emerge through the slit S.

- **Hall effect**: the production of voltage across a conductor when a current flows through the conductor at right angles to a magnetic field
  - A small current flows through the probe across the ends; when magnetic field is applied, electrons are pushed sideways by the magnetic force, hence accumulating along one side of the probe – **Hall effect** – and lack of electrons on the other side, resulting to an electric field between the two sides
  - The charge is detected by a small voltage across the probe: **Hall voltage**
  - The greater the magnetic flux density, the greater the Hall voltage

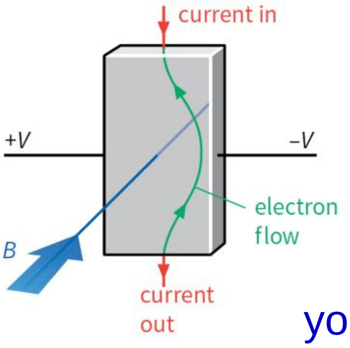


Figure 27.12 Electrons are deflected as they move through the Hall probe.

- The electric field strength *E* is related to the Hall voltage V_H by:

$$E = \frac{V_H}{d}$$

  - *d* is the width of the slice

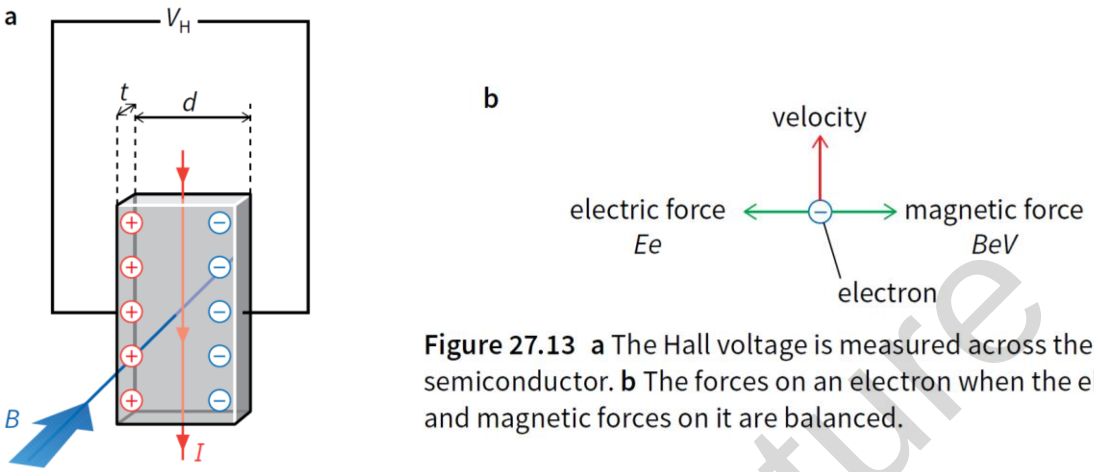


Figure 27.13 **a** The Hall voltage is measured across the slice of semiconductor. **b** The forces on an electron when the electric and magnetic forces on it are balanced.

- Equating electric and magnetic forces:

$$eE = Bev$$

Substituting for *E* we have:

$$\frac{eV_H}{d} = Bev$$

  - By substituting *v* from the equation *I* = *nAve*, where *A* is the cross-sectional area of the conductor and *n* is the number density of the conducting particles:

$$\frac{eV_H}{d} = \frac{BeI}{nAe}$$

Making V_H the subject of the equation (and cancelling *e*) gives:

$$V_H = \frac{BId}{nAe}$$

But the area of the side face of the conductor $A = d \times t$, where *t* is the thickness of the slice. Substituting and cancelling gives:

$$V_H = \frac{BI}{nte}$$

- Where a more general equation can be given by:

$$V_H = \frac{BI}{ntq}$$

- Velocity selection on charged particles:
  - Where an electric force is given by:

$F_E = qE$

  - If the velocity if the particles before entry into the field is *v*, then they will follow a parabolic path:

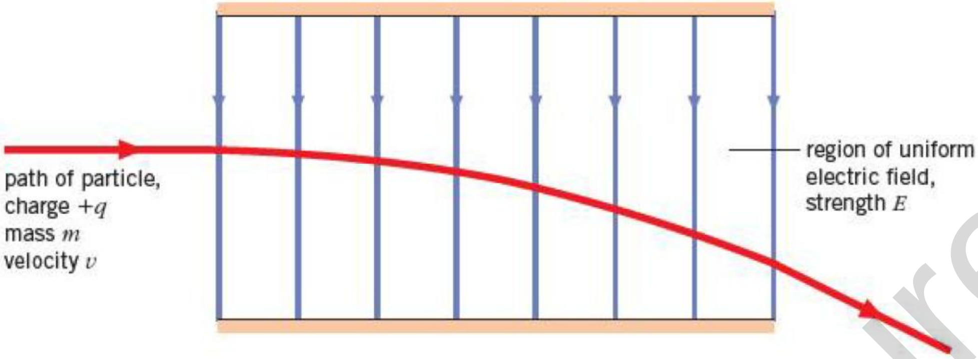


Figure 22.21

  - During equilibrium between electric force and magnetic force, the particle will pass through the fields not deflected, where forces are given by:

$Bqv = qE$ and $v = E/B$

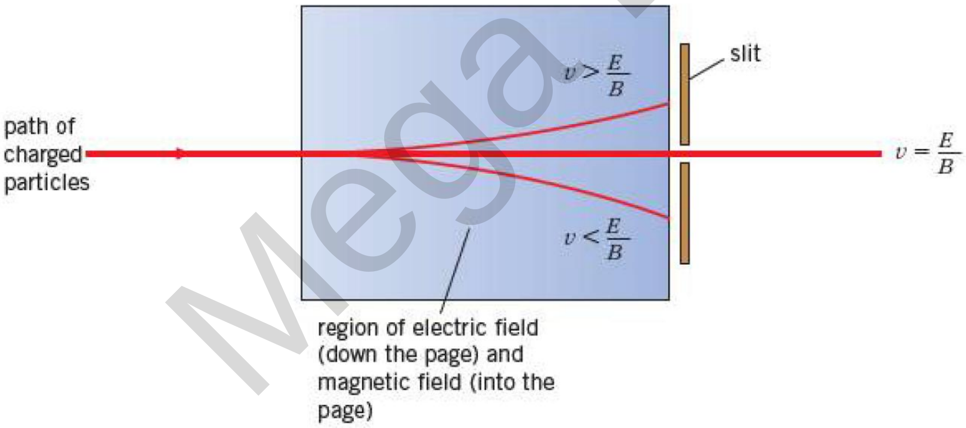


Figure 22.22 Velocity selector

It is required to select charged ions which have a speed of $4.2 \times 10^6\,\mathrm{m\,s^{-1}}$. The electric field strength in the velocity selector is $3.2 \times 10^4\,\mathrm{V\,m^{-1}}$. Calculate the magnetic flux density required.

$v = E/B$

$B = 3.2 \times 10^4 / 4.2 \times 10^6$

$= \mathbf{7.6 \times 10^{-3}\,T}$

  - This is due to the increase in magnetic force *Bqv* when there is an increase in speed, hence they will be deflected

Physics (A-level)

Electromagnetic induction (Chapter 23):

- For a straight wire, the induced current or e.m.f. depends on:
  - The magnitude of the magnetic flux density
  - The length of the wire in the field
  - The speed of movement of the wire
- For a coil of wire, the induced current or e.m.f. depends on:
  - The magnitude of the magnetic flux density
  - The cross-sectional area of the coil
  - The number of turns of wire
  - The rate at which the coil turns in the field
- In electromagnetic induction, Fleming's right-hand (generator) rule is used:

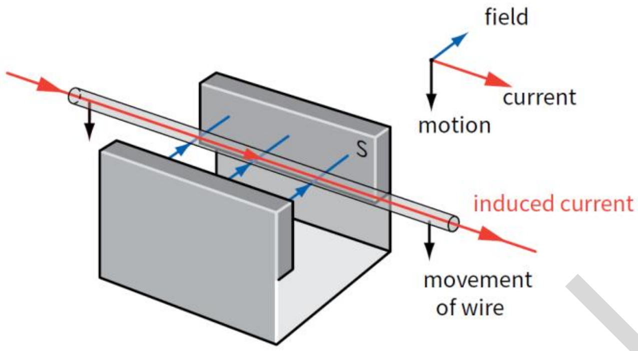


Figure 28.9 Deducing the direction of the induced current using Fleming's right-hand rule.

  - By pushing the wire through the magnetic field, work is done and transformed into electrical energy (e.m.f.)
- We can picture the **magnetic flux density** as the number of magnetic field lines passing through a region **per unit area**; we can picture **magnetic flux** as the total number of magnetic field lines passing through an area *A*; therefore the **magnetic flux** Φ is defined as the product of magnetic flux density and the cross-sectional area *A* (normal to the field):

$$\Phi = BA$$

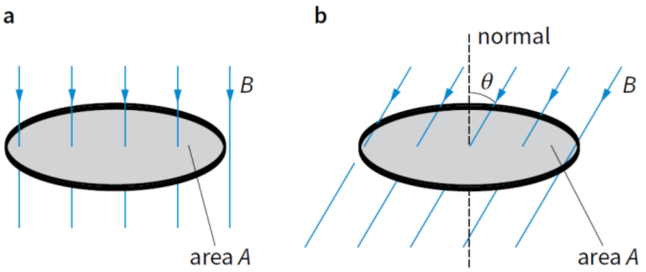


Figure 28.12 **a** The magnetic flux is equal to *BA* when the field is normal to the area. **b** The magnetic flux becomes $BA\cos\theta$ when the field is at an angle θ to the normal of the area.

- For an coil with *N* turns, the **magnetic flux linkage** is defined as the product of the magnetic flux and the number of turns:

$$\text{magnetic flux linkage} = N\Phi \qquad \text{or} \qquad \text{magnetic flux linkage} = BAN\cos\theta$$

- The unit for magnetix flux or flux linkage is the **Weber** (Wb)
- **One Weber** (1 Wb) is the flux that passes through an area of 1m^2 when the magnetic flux density is 1 T. ($1\text{Wb} = 1\ \text{Tm}^2$)
- e.m.f. is induced whenever there is a **change** in magnetic flux linking the circuit; this can be done by:
  - Changing the magnetic flux density *B*
  - Changing the area *A* of the circuit
  - Changing the angle θ
- Electromagnetic induction occurs whenever a conductor cuts across lines of magnetic flux
- **Faraday's law of electromagnetic induction** (to determine the magnitude of the induced e.m.f. in a circuit): the magnitude of the induced e.m.f. is proportional to the rate of change of magnetic flux linkage

$$E = \frac{\Delta(N\Phi)}{\Delta t}$$

- **Lenz's law**: any induced current or e.m.f. will be established in a direction so as to produce effects which oppose the change that is producing it

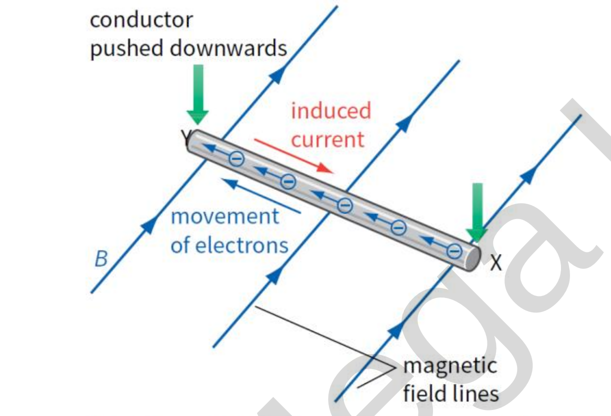


Figure 28.20 Showing the direction of the induced current.

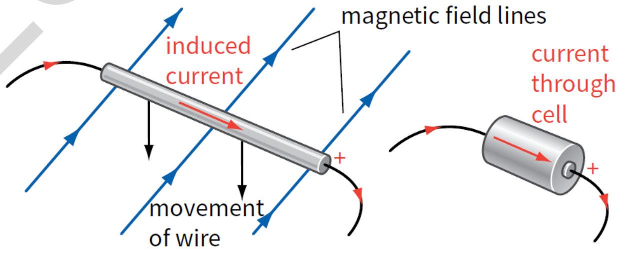


Figure 28.21 A moving conductor in a magnetic field is a source of e.m.f., equivalent to a cell.

- Conservation of energy in electromagnetic induction:

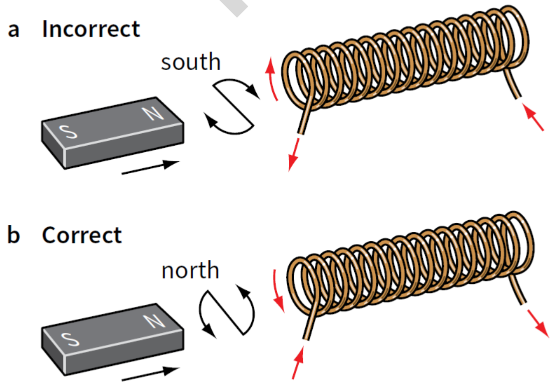


Figure 28.22 Moving a magnet towards a coil: the direction of the induced current is as shown in **b**, not **a**.

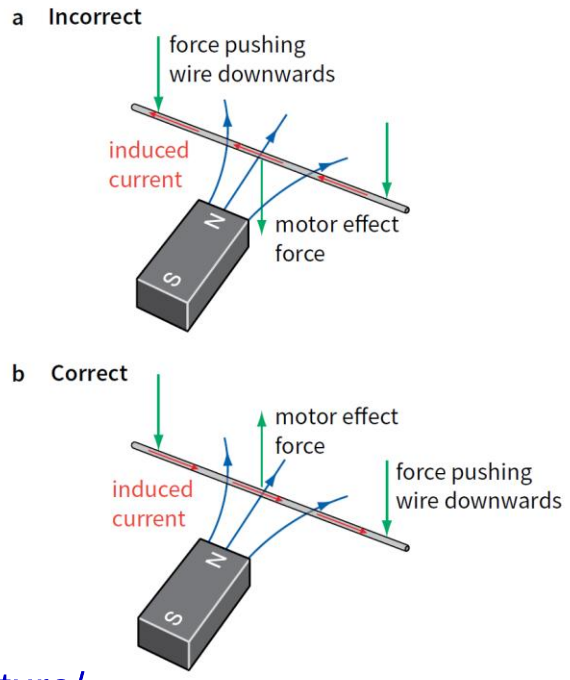


Figure 28.23 Moving a conductor through a magnetic field: the direction of the induced current is as shown in **b**, not **a**.

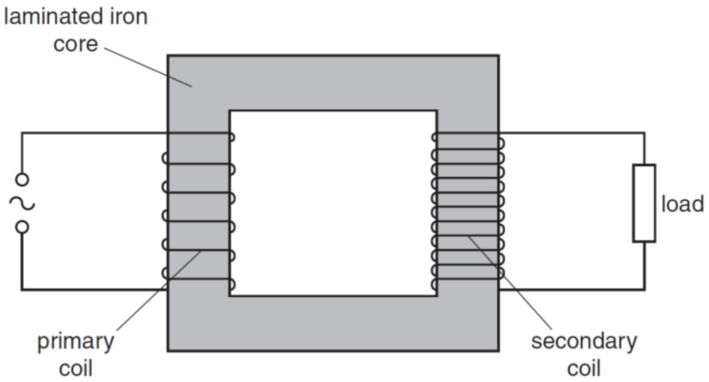


Fig. 6.1

- The coils are wounded on a core made of iron to concentrate the magnetic flux, and reduce magnetic flux losses
- The core is laminated to reduce energy or heat losses caused by the eddy currents
- Thermal energy is generated in the core due to the change in magnetic flux in the core, inducing current in it, which in turn gives rise to a heating effect, due to the induction of different e.m.f in different parts of the core, forming eddy currents in the core
- An e.m.f is induced in the secondary coil, a current in the primary coil gives rise to this induced e.m.f explanation: alternating current gives rise to changing magnetic flux in the core, which links to the secondary coil; by Faraday's law, changing of magnetic flux induces e.m.f in the secondary coil
- Using Faraday's law, the difference in phase of the p.d. across the load and e.m.f can be explained: the magnetic flux is in phase with the e.m.f or current in the primary coil; e.m.f or p.d. across the secondary coil is proportional to the rate of change of magnetic flux; hence e.m.f of the supply is not in phase with p.d. across secondary
- Alternating current is used in transmitting electrical energy as voltage can be easily and efficiently changed
- High voltages are used in transmitting electrical energy as for the same power transmission, high voltages means low currents, hence less energy will be lost in the transmission of the cables

Quantum physics (Chapter 25):

- Phenomena such as interference and diffraction provide evidence for a wave nature
- The photoelectric effect provides evidence for a particulate nature of electromagnetic radiation – all consists of photons
- **Photon**: packet of discrete amount of energy of electromagnetic radiation; energy (*E*, in joules (J)) = Planck constant (value: 6.63×10^{-34}) × frequency
  - Photon's energy is directly proportional to the frequency of the electromagnetic wave, hence high-frequency radiation means high-frequency photons

$E = hf$, and substituting the wave equation c = $f\lambda$: $E = \frac{hc}{\lambda}$

- **Gamma radiation** (γ-radiation): photons of electromagnetic radiation emitted from the nuclei
- **One electronvolt** (1 eV) is the energy transferred when an electron travels through a potential difference of one volt, therefore 1eV = 1.60×10^{-19} J

- To convert from eV to J, multiply by 1.60×10^{-19}.
- To convert from J to eV, divide by 1.60×10^{-19}.

- When an electron is accelerated through a p.d., V, its K.E. increases given by:

$$eV = \tfrac{1}{2}mv^2$$

  - Rearranging the equation gives the electron's speed:

$$v = \sqrt{\frac{2eV}{m}}$$

  - Equation applies to all charged particle: photons (+ e) and ions
- The **threshold frequency** is defined as the minimum frequency for electrons to be emitted from the surface of a metal
- The **work function energy**, Φ, of a metal is the minimum amount of energy required by an electron to escape its surface

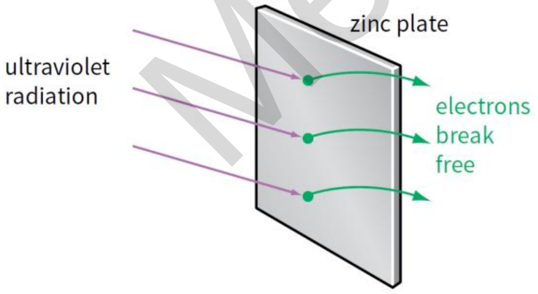


Figure 30.8 The photoelectric effect. When a photon of ultraviolet radiation strikes the metal plate, its energy may be sufficient to release an electron.

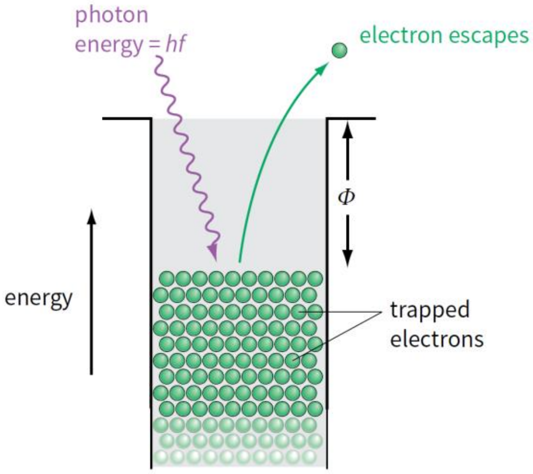


Figure 30.9 A single photon may interact with a single electron to release it.

- Some of the energy from the photons is needed for the electrons to escape from the energy well; the rest is the electron's K.e.
- Electrons from the surface of the metal is removed
- A single photon can only interact (exchange energy) with a single electron

- A surface electron is removed **instantaneously** from the metal surface when the energy of the incident photon is greater than, or equal to, the work function Φ of the metal
- Energy of photon = work function energy + maximum K.e. of the electron

$$hf = \Phi + \text{k.e.}_{\text{max}} \quad \text{or} \quad hf = \Phi + \tfrac{1}{2}mv_{\text{max}}{}^2$$

  - If the photon is absorbed by an electron that is lower in the energy well, the electron will have less K.e. than K.e.$_{\text{max}}$
- If the incident radiation has a frequency equal to the threshold frequency, f_0, then the K.e. of the electrons is zero:

$$hf_0 = \Phi$$

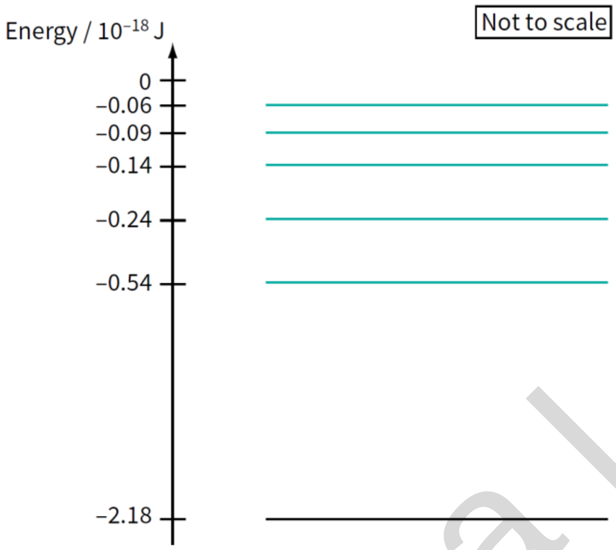


Figure 30.15 Some of the energy levels of the hydrogen atom.

- An electron in a hydrogen atom can only have one of these values of energy, no in betweens
- The energy levels have negative values due to the attractive forces within the atom by the atomic nucleus, hence external energy needs to be supplied to emit an electron from the atom; zero energy results in 'free' electrons
- Atoms of different elements have different line spectra as they have different spacings between their energy levels

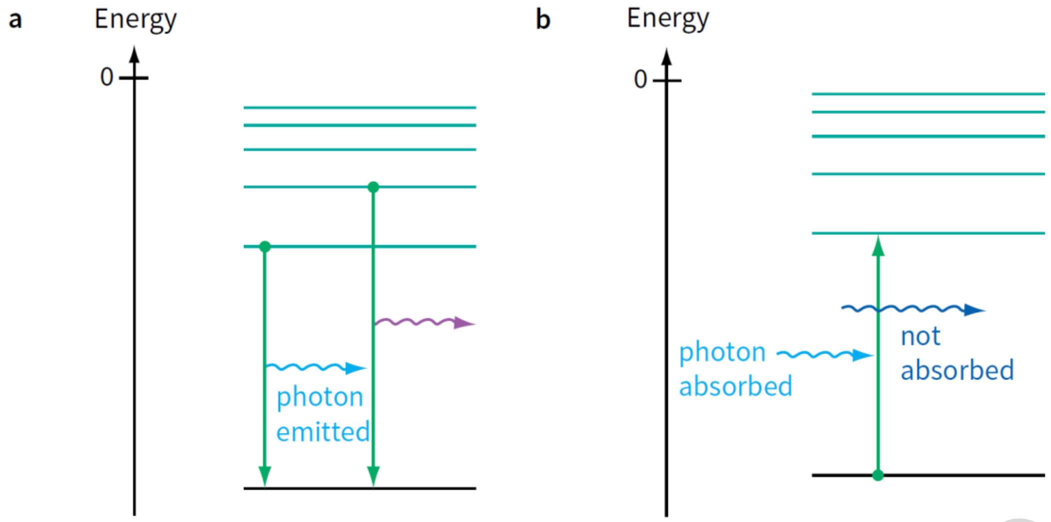


Figure 30.16 **a** When an electron drops to a lower energy level, it emits a single photon. **b** A photon must have just the right energy if it is to be absorbed by an electron.

- When an electron changes its energy from one level E_1 to another E_2, it either emits or absorbs a single photon; the energy equals to the difference in energies between the two levels:

$$\text{photon energy} = \Delta E \,, \quad hf = E_1 - E_2 \quad \text{or} \quad \frac{hc}{\lambda} = E_1 - E_2$$

- E.g. figure 30.15, falling from second level to the lowest energy level (ground state):

$$\text{photon energy} = \Delta E$$

$$hf = [(-0.54) - (-2.18)] \times 10^{-18}\,\text{J}$$

$$hf = 1.64 \times 10^{-18}\,\text{J}$$

- Wavelength and frequency can then be calculated:

The frequency is:

$$f = \frac{E}{h} = \frac{1.64 \times 10^{-18}}{6.63 \times 10^{-34}}$$

$$f = 2.47 \times 10^{15}\,\text{Hz}$$

The wavelength is:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}}{2.47 \times 10^{15}}$$

$$\lambda = 1.21 \times 10^{-7}\,\text{m} = 121\,\text{nm}$$

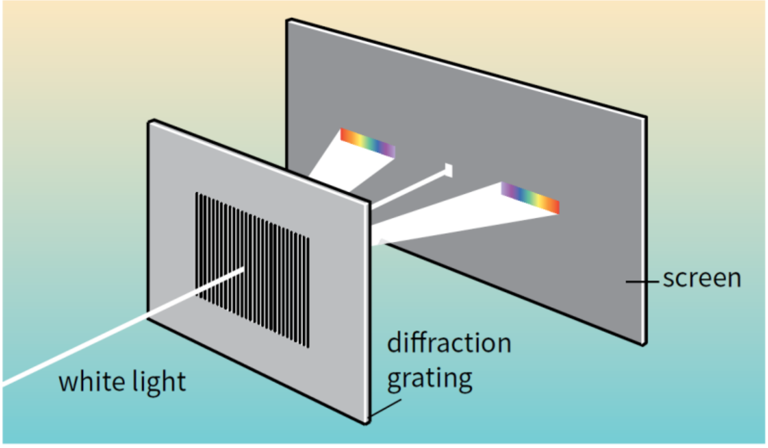


Figure 30.11 White light is split up into a continuous spectrum when it passes through a diffraction grating.

- Line spectrum can be seen
- The line spectra that show the composition of light emitted by hot gases are called **emission line spectra**
- The line spectra observable when white light passed through cool gases are called **absorption line spectra**

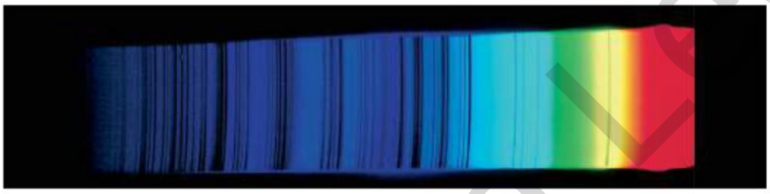


Figure 30.14 The Sun's spectrum shows dark lines. These dark lines arise when light of specific wavelengths coming from the Sun's hot interior is absorbed by its cooler atmosphere.

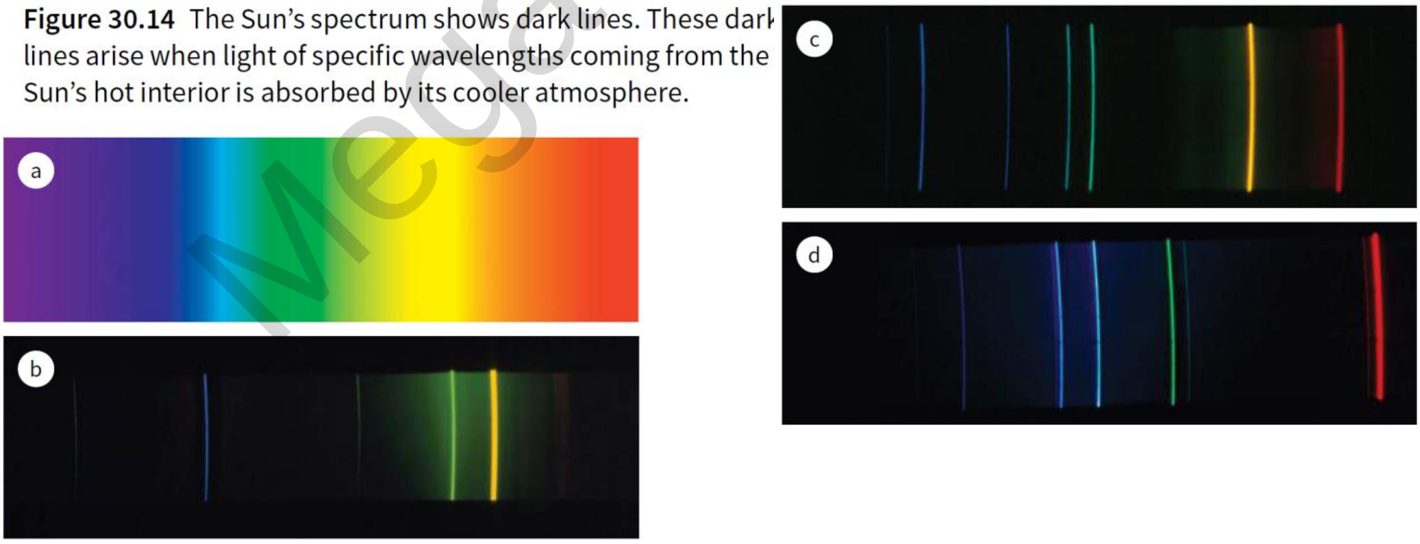


Figure 30.12 Spectra of **a** white light, and of light from **b** mercury, **c** helium and **d** cadmium vapour.

- Wave-particle duality of light:
  - Light interacts with matter (e.g. electrons) as a particle – the photon (photoelectric effect)

  - Light travels through space as a wave (diffraction and interference of light using slits)
- De Broglie wavelength given by:

$$\lambda = \frac{h}{p} \quad \text{or} \quad \lambda = \frac{h}{mv}$$

  - *h* is the Planck constant
  - *p* is the momentum
- Investigating wave nature of electrons through electron diffraction:

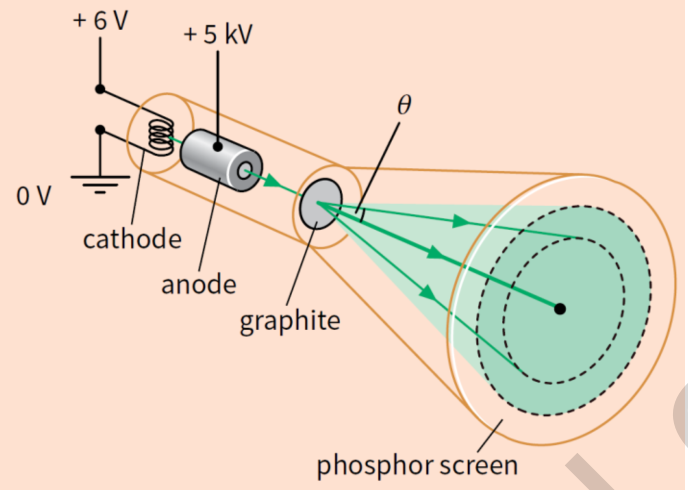


Figure 30.25 Electrons are accelerated from the cathode to the anode; they form a beam which is diffracted as it passes through the graphite film.

  - Electron beam in a vacuum
  - Incident on thin metal target / carbon film (graphite)
  - Fluorescent screen (phosphor screen)

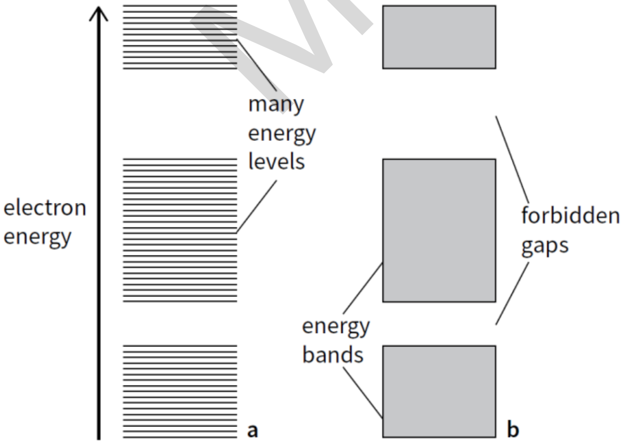


Figure 30.19 **a** In a solid, the electron energy levels are very close together. **b** The energy levels form bands with forbidden gaps between them.

- **Forbidden gap**: a range of energy values which an electron in a solid cannot have
- **Conduction band**: a range of electron energies in a solid, which are free to move throughout the material

- **Valence band**: a range of electron energies in a solid, which electrons are bound to individual atoms
- In isolated atoms (gases), electron energy levels have discrete values, while in a solid, there are energy bands; this is because in a solid electrons in neighbouring atoms are very close together and influence / interact with each other; this changes their electron energy levels; many atoms in lattice cause spread of energy levels into a band

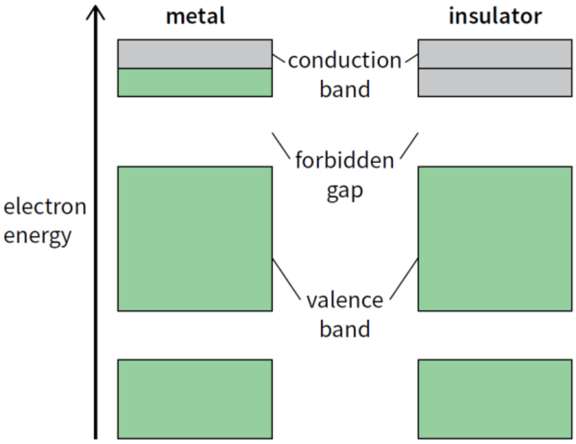


Figure 30.20 How the energy bands are filled in a metal and in an insulator.

  - Conduction band is partially filled in metals, hence it can conduct electricity (free electrons)
- An intrinsic semiconductor (pure material) conducts electric current, but only very slightly:

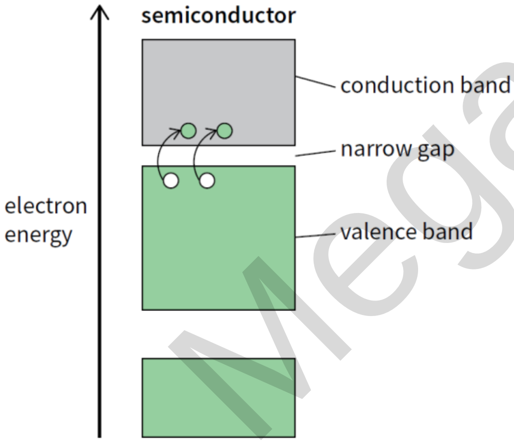


Figure 30.21 Electron energy bands in a semiconductor.

  - It is similar to an insulator (full valence electrons and empty conduction band), difference is its small gap between the bands
  - With heating, more electrons will gain the energy to jump up into the conduction band; leaving holes in the valence band; these electrons and holes are charge carriers; increase in current, hence reduced resistance
  - Unlike metal when heated, as its resistance will increase due to the decrease in free electron density, as its atoms vibrate more, the electrons collide more frequently with the vibrating atoms
- In LDR (consisting of intrinsic semiconductor) at constant temperature, it will be dependent on light intensity, as photons of light give energy to electrons in the valence band;

electrons move up into the conduction band; leaving holes in the valence band; these electrons and holes are charge carriers; increase in current, hence reduced resistance

- **De Broglie wavelength**: the wavelength associated with a moving particle dependent on its momentum
- Observations of photoelectric effect has proven that either electrons are emitted immediately or they are not emitted at all with exposure of light on a metal surface, suggest why this observation does not support a wave theory of light:
    - For a wave, electron can 'collect' energy continuously, electron will be emitted at all frequencies of electromagnetic radiation after a sufficiently long delay
- Explain how the line spectrum of hydrogen provides evidence for the existence of discrete electron energy levels in atoms:
    - Each line represents photon of specific energy
    - Photon emitted as a result of energy change of an electron
    - There are specific energy changes, hence discrete levels

(b) Some electron energy levels in atomic hydrogen are illustrated in Fig. 7.1.

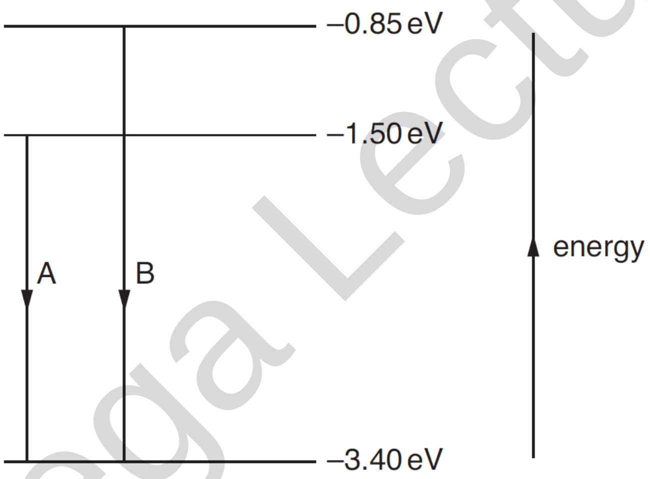


Fig. 7.1

Two possible electron transitions A and B giving rise to an emission spectrum are shown.
These electron transitions cause light of wavelengths 654 nm and 488 nm to be emitted.

(i) On Fig. 7.1, draw an arrow to show a third possible transition. [1]

(ii) Calculate the wavelength of the emitted light for the transition in **(i)**.

(b) **(i)** arrow from –0.85 eV level to –1.5 eV level B1 [1]

(ii) $\Delta E = hc/\lambda$ C1

$= (1.5 - 0.85) \times 1.6 \times 10^{-19}$ C1

$= 1.04 \times 10^{-19}$ J

$\lambda = (6.63 \times 10^{-34} \times 3.0 \times 10^{8})/(1.04 \times 10^{-19})$

$= 1.9 \times 10^{-6}$ m A1 [3]

(c) The light in a beam has a continuous spectrum of wavelengths from 400 nm to 700 nm. The light is incident on some cool hydrogen gas, as illustrated in Fig. 7.2.

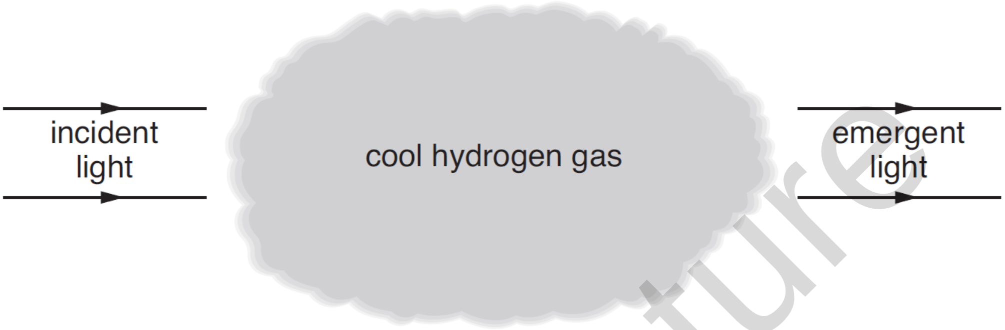


Fig. 7.2

Using the values of wavelength in **(b)**, state and explain the appearance of the spectrum of the emergent light.

....

(c) spectrum appears as continuous spectrum crossed by dark lines B1
two dark lines B1
electrons in gas absorb photons with energies equal to the excitation energies M1
light photons re-emitted in all directions A1 [4]

- Light of a particular wavelength is incident on a metal surface and gives rise to a photoelectric current and the wavelength is reduced, and the intensity is kept constant, hence state and explain the effect on the photoelectric current:
  - ➢ Each photon has more energy
  - ➢ Fewer photons per unit time
  - ➢ Fewer electrons per unit time, hence less current

Nuclear physics (Chapter 26):

- In α decay, the nucleon number decreases by 4; proton number decreases by 2
- In β^- decay, the nucleon number is unchanged; the proton number increases by 1
- In β^+ decay, the nucleon number is unchanged; the proton number decreases by 1
- In γ emission, no change in nucleon and proton number

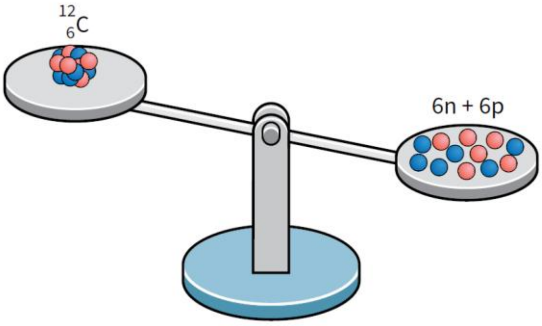


Figure 31.3 The mass of a nucleus is less than the total mass of its component protons and neutrons.

- Association between energy and mass:

$$E = mc^2$$

- Where c is 3.00×10^8 m s^{-1}
- The mass of a system increases when energy is supplied to it; when energy is released from a system, mass decreases:

$$\Delta E = \Delta mc^2$$

Particle	Rest mass / 10^{-27} kg
^1_1p	1.672 623
^1_0n	1.674 929
$^{12}_{6}C$ nucleus	19.926 483

Table 31.1 Rest masses of some particles. It is worth noting that the mass of the neutron is slightly greater than that of the proton (roughly 0.1% greater).

- **Mass defect** of a nucleus is equal to the difference between the total mass of the individual, separate nucleons and the mass of the nucleus
  - E.g. particles in Fig. 31.3:

$$\text{mass before} = (6 \times 1.672\,623 + 6 \times 1.674\,929) \times 10^{-27}\,\text{kg}$$
$$= 20.085\,312 \times 10^{-27}\,\text{kg}$$

$$\text{mass after} = 19.926\,483 \times 10^{-27}\,\text{kg}$$

$$\text{mass difference } \Delta m = (20.085\,312 - 19.926\,483) \times 10^{-27}\,\text{kg}$$
$$= 0.158\,829 \times 10^{-27}\,\text{kg}$$

- Loss in mass implies that energy is released:

$$E = mc^2$$
$$= 0.158\,829 \times 10^{-27} \times (3.00 \times 10^8)^2$$
$$\approx 1.43 \times 10^{-11}\,\text{J}$$

- Atomic and nuclear masses often given in the **atomic mass unit** (u)
  - 1 u = 1.6605 × 10^{-27}

Nuclide	Symbol	Mass / u
proton	$^{1}_{1}\text{p}$	1.007 825
neutron	$^{1}_{0}\text{n}$	1.008 665
helium-4	$^{4}_{2}\text{He}$	4.002 602
carbon-12	$^{12}_{6}\text{C}$	12.000 000
potassium-40	$^{40}_{19}\text{K}$	39.963 998
uranium-235	$^{235}_{92}\text{U}$	235.043 930

Table 31.3 Masses of some nuclides in atomic mass units. Some have been measured to several more decimal places than are shown here.

- **Mass excess** = mass (in u) – nucleon number
  - So the mass excess for U-235 is 235.043 930 – 235 = 0.043 930 u
- E.g. decay of a nucleus of uranium-238:

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

  - Δ*m* is equivalent to the energy released as K.E. of the products, where using accurate values:

$$\text{mass of } {}^{238}_{92}\text{U nucleus} = 3.952\,83 \times 10^{-25}\,\text{kg}$$

$$\text{total mass of } {}^{234}_{90}\text{Th nucleus and } \alpha\text{-particle } ({}^{4}_{2}\text{He})$$
$$= 3.952\,76 \times 10^{-25}\,\text{kg}$$

$$\text{change in mass } \Delta m = (3.952\,76 - 3.952\,83) \times 10^{-25}\,\text{kg}$$
$$\approx -7.0 \times 10^{-30}\,\text{kg}$$

  - Hence energy released in the decay:

$$\text{energy released} \approx 7.0 \times 10^{-30} \times (3.0 \times 10^{8})^{2}$$
$$\approx 6.3 \times 10^{-13}\,\text{J}$$

- **Binding energy**: the minimum energy needed to pull a nucleus apart into its separate nucleons
- To find the binding energy per nucleon for a nuclide:
  - Find mass defect for the nucleus.
  - Use Einstein's mass–energy equation, find the binding energy of the nucleus (mass defect × c^2)
  - Divide the binding energy of the nucleus by the number of nucleons to calculate the binding energy per nucleon
- The greater the value of the binding energy per nucleon, the more tightly bound the nucleons that make up the nucleus

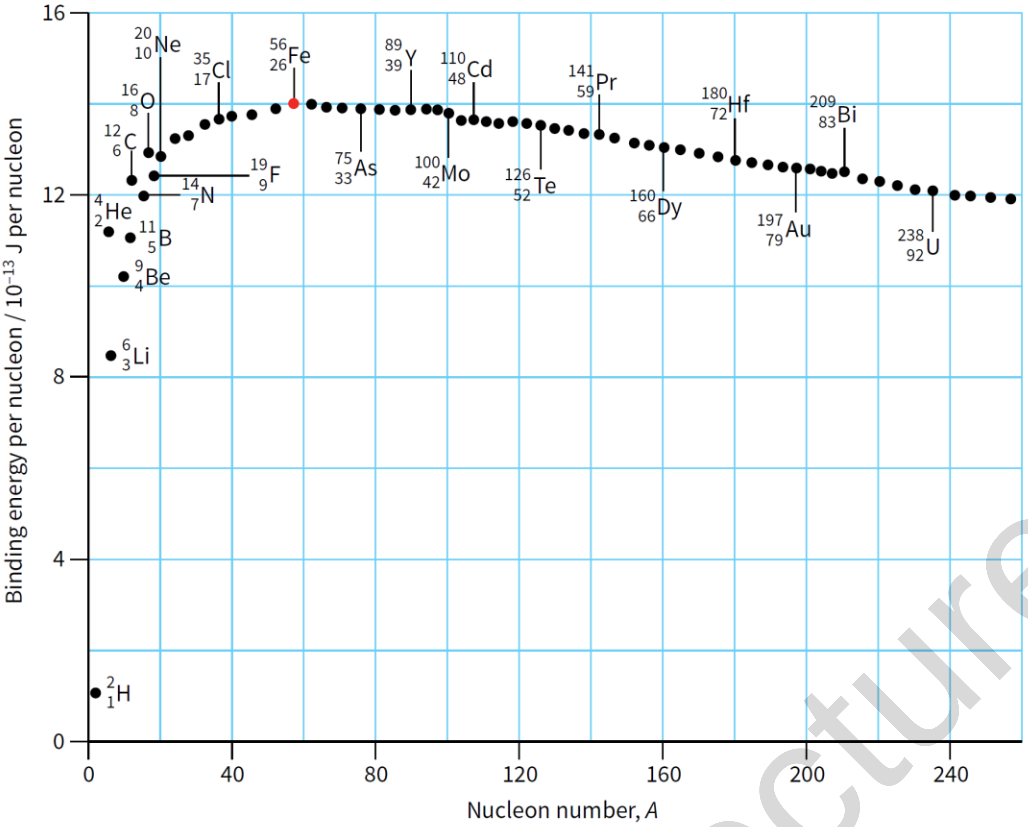


Figure 31.4 This graph shows the binding energy per nucleon for a number of nuclei. The nucleus becomes more stable as binding energy per nucleon increases.

Calculate the binding energy per nucleon for the nuclide $^{56}_{26}$Fe.

mass of neutron = 1.675×10^{-27} kg

mass of proton = 1.673×10^{-27} kg

mass of $^{56}_{26}$Fe nucleus = 9.288×10^{-26} kg

Step 1 Determine the mass defect.

number of neutrons = 56 − 26 = 30

mass defect

$= (30 \times 1.675 \times 10^{-27} + 26 \times 1.673 \times 10^{-27}) - 9.288 \times 10^{-26}$

$= 8.680 \times 10^{-28}$ kg

Step 2 Determine the binding energy of the nucleus.

binding energy = Δmc^2

$= 8.680 \times 10^{-28} \times (3.00 \times 10^8)^2$

$= 7.812 \times 10^{-11}$ J

Step 3 Determine the binding energy per nucleon.

$$\text{binding energy per nucleon} = \frac{7.812 \times 10^{-11}}{56}$$

$\approx 14 \times 10^{-13}$ J

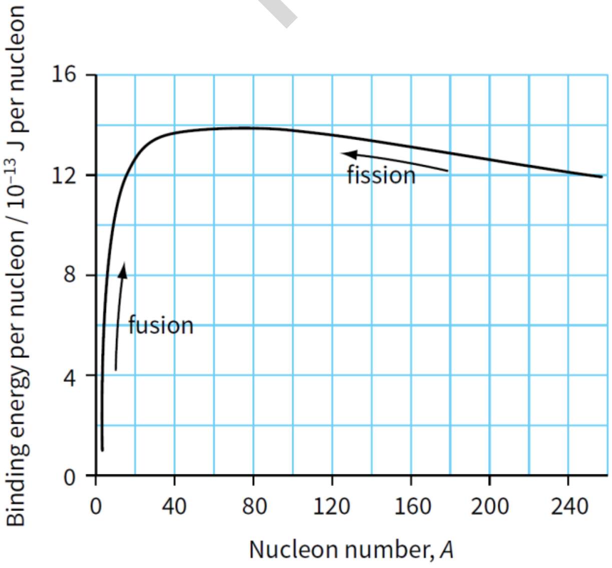


Figure 31.6 Both fusion and fission are processes that tend to increase the binding energy per nucleon of the particles involved.

- **Nuclear fission**: heavy / large nucleus splits into two nuclei of approximately equal masses
    - Binding energy of nucleus = $B_E \times A$
    - Binding energy of parent nucleus is less than the sum of the two binding energies fragments
- **Nuclear fusion**: process by which two very light nuclei join together to form a heavier nucleus
    - Binding energy of nucleus = $B_E \times A$
    - Binding energy of parent nuclei is less than the final binding energy nucleus of the product
- Graph of count rate against time, where the fluctuations on either side are caused by the randomness of the decay:

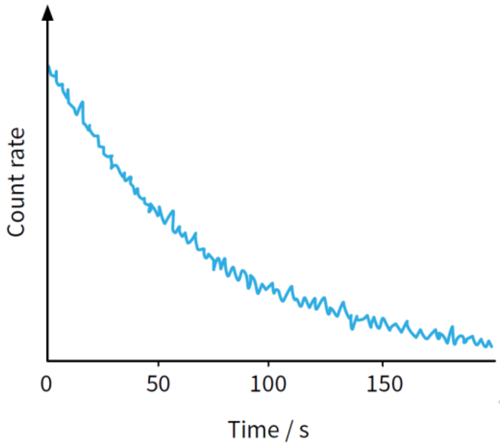


Figure 31.8 Count rate showing randomness of decay.

- Nuclear decay is **spontaneous** because:
    - The decay of a particular nucleus is not affected by the presence of other nuclei
    - The decay of nuclei cannot be affected by chemical reactions or external factors such as temperature and pressure
- Nuclear decay is **random** because:
    - Impossible to predict when a particular nucleus in a sample is going to decay
    - Each nucleus in the sample has the same chance of decaying per unit time
- Difference between the actual half-life value and calculated without any faulty equipments can be caused by:
    - Random nature of decay
    - Background radiation
    - Daughter product is also radioactive
- Difference between the activity of a sample might not be equal to the measured count rate due to:
    - Background count rate / radiation
    - Multiple possible counts for each decay
    - Radiation emitted in all directions
    - Dead-time of counter
    - Daughter product might also be unstable / emits radiation
    - Self-absorption of radiation in a sample or absorption in air

- **Decay constant** (λ): the probability that an individual nucleus will decay per unit time interval; unit: h^{-1} or s^{-1} or day^{-1} or $year^{-1}$, etc.
- The **activity** *A* of a radioactive sample is the rate at which nuclei decay or disintegrate; unit: Bq

$$A = \lambda N$$

  - N is the number of undecayed nuclei present in the sample
- Activity can be thought of the number of α- or β- particles emitted from the source per unit time:

$$A = \frac{\Delta N}{\Delta t}$$

  - N is the number of emissions (decays) in a small time interval of Δt

A radioactive source emits β-particles. It has an activity of 2.8×10^7 Bq. Estimate the number of β-particles emitted in a time interval of 2.0 minutes. State one assumption made.

Step 1 Write down the given quantities in SI units.

$A = 2.8 \times 10^7\,\text{Bq} \qquad \Delta t = 120\,\text{s}$

Step 2 Determine the number of β-particles emitted.

$A = \frac{\Delta N}{\Delta t} \qquad \Delta N = A\Delta t$

$\Delta N = 2.8 \times 10^7 \times 120 = 3.36 \times 10^9 \approx 3.4 \times 10^9$

We have assumed that the activity remains constant over a period of 2.0 minutes.

A sample consists of 1000 undecayed nuclei of a nuclide whose decay constant is $0.20\,\text{s}^{-1}$. Determine the initial activity of the sample. Estimate the activity of the sample after 1.0 s.

Step 1 Since activity $A = -\lambda N$, we have:

$A = 0.20 \times 1000 = 200\,\text{s}^{-1} = 200\,\text{Bq}$

Step 2 After 1.0 s, we might expect 800 nuclei to remain undecayed.
The activity of the sample would then be:

$A = 0.2 \times 800 = 160\,\text{s}^{-1} = 160\,\text{Bq}$

(In fact, it would be slightly higher than this. Since the rate of decay decreases with time all the time, less than 200 nuclei would decay during the first second.)

- **Count rate**: the number of particles (beta or alpha) or gamma-ray photons detected per unit time by a Geiger-Muller tube, where it is always a fraction of the sample's activity
- Radioactive decay follows an exponential decay pattern:

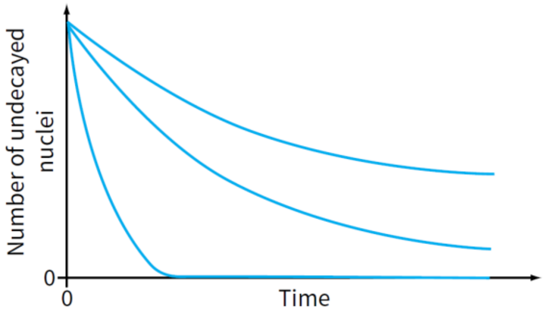


Figure 31.9 Some radioactive materials decay faster than others.

- The **half-life** $t_{1/2}$ of a radioisotope is the mean time taken for half of the active nuclei in a sample to decay

- Activity is proportional to the number of undecayed nuclei ($A \propto N$); hence in a time equal to one half-life, the activity of the sample will also halved

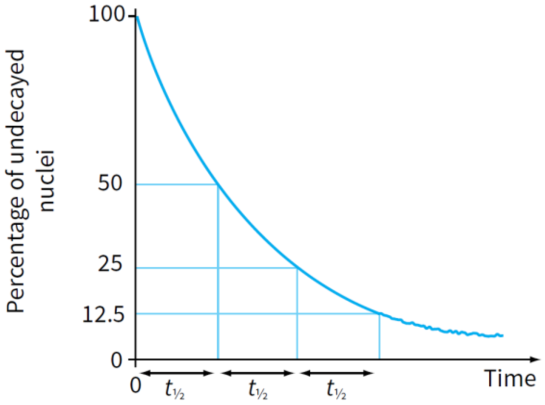


Figure 31.10 All radioactive decay graphs have the same characteristic shape.

- The activity A of a sample is proportional to the number of undecayed nuclei given by:

$$A = A_0 e^{(-\lambda t)}$$

- Starting with N_0 undecayed nuclei, then the number N that remain undecayed after time t is given by:

$$N = N_0 e^{(-\lambda t)}$$

- Count rate is a fraction of activity, hence decreases exponentially with time too, given by:

$$R = R_0 e^{(-\lambda t)}$$

- In a time of one half-life $t_{1/2}$, the number of undecayed nuclei is halved giving:

$$N = N_0 e^{(-\lambda t)}$$

becomes:

$$\frac{N}{N_0} = e^{(-\lambda t_{1/2})} = \tfrac{1}{2}$$

Therefore:

$$e^{(\lambda t_{1/2})} = 2$$

$$\lambda t_{1/2} = \ln 2 \approx 0.693$$

   - The half-life and decay constant are inversely proportional to each other:

$$\lambda = \frac{0.693}{t_{1/2}}$$

Electronic sensors (Chapter 19 & 20):

Figure 25.2 Block diagram of an electronic sensor.

- An electronic sensor consists of a sensing device and a circuit that provides an output that can be registered as a voltage

- The sensing device (transducer – changes energy from one form into another) changes its property when there is a change in physical quantity, such as temperature (thermistor) and light intensity (light-dependent resistor); changes in resistance causes the processor to produce an output voltage that drives the output device (switching it on)
- A light-dependent resistor (LDR) is made of a high-resistance semiconductor; if light of high enough frequency falls on it, the resistance will be reduced

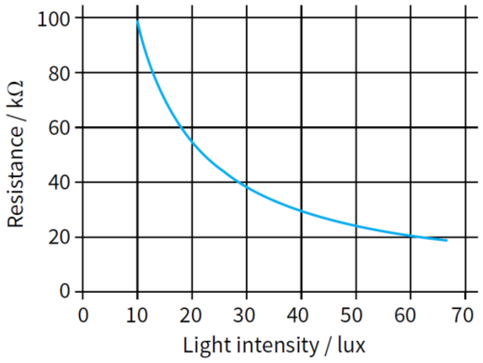


Figure 25.4 Resistance plotted against light intensity for an LDR.

  - ➢ LDR is placed in series with a fixed resistor to generate the change in voltage, forming a potential divider

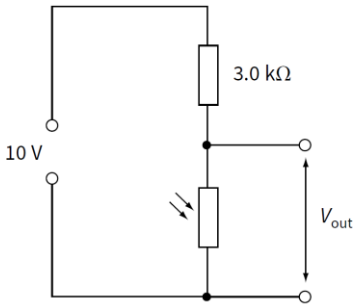


Figure 25.5 An LDR used in a sensor.

- The thermistors (negative temperature coefficient thermistors) where as temperature rises, the resistance of the thermistor falls; used the same way as the LDR

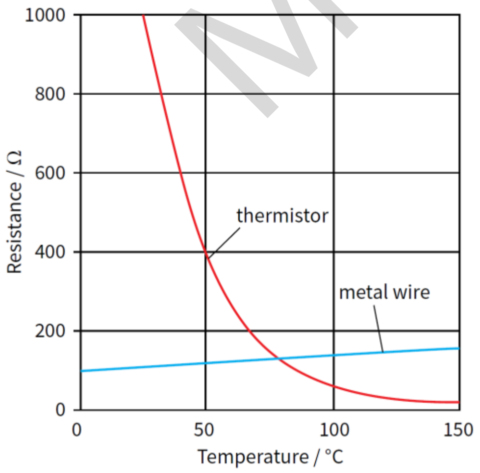


Figure 25.6 Variation of resistance with temperature.

- The metal-wire strain gauge uses the change in resistance of a metal wire as its length and cross-sectional area change; when stretched, wire becomes narrower and longer – increase resistance; when compressed, wire becomes shorter and wider, decreases resistance

- A metal-wire strain gauge consists of a thin wire placed between thin sheets of plastic; zigzags up and down its plastic base so that the length of wire used is longer than the actual strain gauge

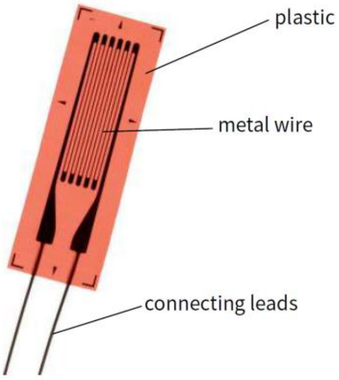


Figure 25.7 A metal-wire strain gauge.

- The wire of length *L*, cross-sectional area *A* and resistivity *ρ* has a resistance *R* given by:

$$R = \frac{\rho L}{A}$$

- If the wire increases in length by a small amount δL and the cross-sectional area *A* is unchanged, then the resistance of the wire increase by δR:

$$R + \delta R = \frac{\rho(L + \delta L)}{A}$$

- Subtracting the two equations give:

$$\delta R = \frac{\rho \delta L}{A}$$

- Expression divided by $R = \frac{\rho L}{A}$:

$$\frac{\delta R}{R} = \frac{\delta L}{L}$$

- Hence the change in resistance is directly proportional to the increase in length (extension), $\delta R \propto \delta L$

- Some crystals such as quartz crystals produce an electric field when a force is applied causing changes in the shape of the crystal, known as the **piezo-electric effect**
  - A piezo-electric crystal consists of positive and negative ions in a regular arrangement, hence when it is stressed, a small voltage is produced between the faces of the crystal, hence it acts as a transducer
  - In microphones, the crystal is made into a thin sheet with metal connections on opposite sides; when a sound wave hits one side of the sheet, the compressions and rarefactions cause the pressure to increase and decrease, hence the crystal

changes shape in response to these pressure changes and a small voltage is induced across the connections

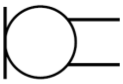


Figure 25.3 The symbol for a microphone.

- Acoustic guitars and other instruments often use a piezo-electric transducer to produce an electrical output; the microphone is stuck to the body of the guitar and the electrical output can be amplified and played back through loudspeakers

Electronics (Chapter 21):

- The goal of an amplifier is to produce a constant amplification or **gain** (all frequencies), hence **operational amplifier (op-amp)** has a very high gain and then provide an external circuit which reduces the gain but ensures that the overall gain is the same for signals of a greater range of frequencies

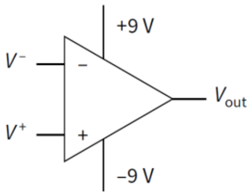


Figure 25.15 An operational amplifier and its symbol.

- The op-amp has two inputs:
  - Inverting input, marked (-)
  - Non-inverting input, marked (+)
- The function of op-amp is to use the potential difference between the two inputs (potential at inverting input (V^-) and potential at non-inverting input (V^+) to produce as large an output voltage V_{out} as possible
- The open-loop voltage gain G_0 is given by:

$$G_0 = \frac{\text{output voltage}}{\text{input voltage}}$$

  - Hence for the op-amp in Fig 25.15, the open-loop voltage gain is given by:

$$G_0 = \frac{V_{out}}{(V^+ - V^-)}$$

  - It is called an open-loop because there is no loop of resistors or other components linking the output back to the input – it is just the operational amplifier alone
- Unlike a transformer, an op-amp's output power is much greater than its input power; achieved by having two power supplies (e.g. +9 V and -9 V connections), and a zero volt line, or earth (all voltages are measured relative to this potential); one power supply will be between the +9 V and 0 V line and the other between -9 V connection and the 0 V line

- The actual voltage used for the power supplies can vary in different circuits (providing the power for the op-amp); the positive and negative supply voltages are equal in magnitude and may be written as $+V_s$ and $-V_s$, and are often left out for clarity
- The largest voltage an op-amp can produce is a value close to the supply voltage, e.g. between -9 V and +9 V, and when it reaches one of this values, it is saturated
- The main properties of an idea op-amp:
    - Infinite open-loop gain
        - Signals of a wide range of frequencies have equal gain before the feedback is applied
    - Infinite input resistance / impedance
        - No current is drawn from the supply, there are no ‘lost volts’ and the input voltage to the op-amp is large as possible; the resistance for an alternating voltage is known as impedance and no current passes into the input terminals
    - Zero output resistance / impedance
        - No ‘lost volts’ when current is supplied by the op-amp
    - Infinite bandwidth
        - The bandwidth of an op-amp is the range of frequencies that are amplified by the same amount; an ideal op-amp will amplify signals of all frequencies, and therefore has an infinite bandwidth
    - Infinite slew rate
        - An ideal op-amp changes its output instantaneously as the input is changed; an infinite slew rate means there is no time delay
- The op-amp as a comparator (to compare two potentials / voltages output depending upon which is greater):

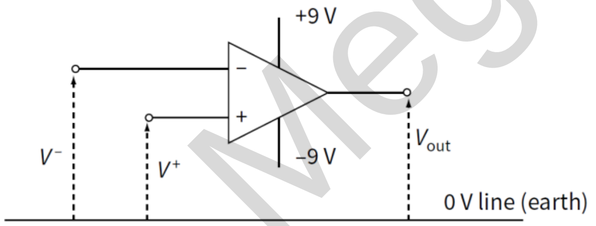


Figure 25.16 An op-amp used as a comparator.

- The output voltage is given by:

$$V_{out} = G_0 \times (V^+ - V^-)$$

- Suppose that $G_0 = 10^5$ and $V^+ = 0.15$ V and $V^- = 0.10$ V

$$V_{out} = 10^5 \times (0.15 - 0.10) = 5000\,\text{V}$$

- The op-amp is therefore saturated and V_{out} will be close to one of the power supply voltages, in this case +9 V:
    - If V^+ is slighly greater in magnitude than V^-, then V_{out} will have a magnitude equal to the positive power supply voltage

  - If V^+ is slighly smaller in magnitude than V^-, then V_{out} will have a magnitude equal to the negative power supply voltage
- The op-amp compares V^+ and V^- and tells which one is larger

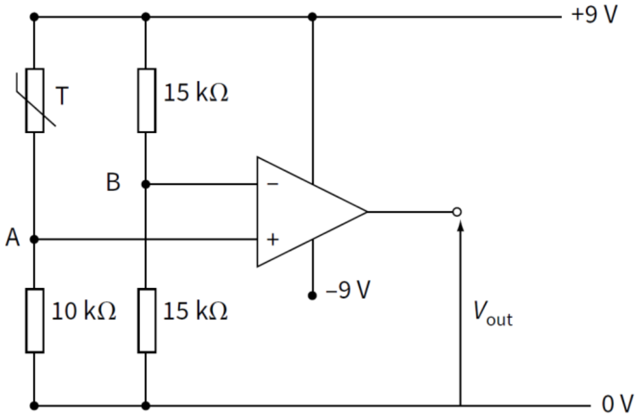


Figure 25.17 An op-amp used as a comparator to monitor temperature.

- A comparator circuit can be used to compare two temperatures of two light levels, where Fig 25.17 shows a circuit used to give a warning when the temperature sensed by thermistor T becomes smaller than a set-value
- The positive power supply to the op-amp is also used to supply voltage and current to the thermistor T and a 10 kΩ connected as a potential divider; another potential divider circuit connected to the inverting input of the op-amp

For the circuit shown in Figure 25.17, the resistance of the thermistor T is 8 kΩ at a temperature of 15 °C. What are V^- and V^+, the potentials at the inverting and non-inverting inputs? And what happens when the temperature falls so that the resistance of T rises above 10 kΩ?

Step 1 V^- and V^+ can be found by using the potential divider formula to find the potentials at points A and B. The potential at A is the p.d. across the 10 kΩ resistor. So:

$$\text{potential at A} = 9 \times \frac{10}{18} = 5.0\,\text{V}$$

The potential at B is easier to find, as the two 15 kΩ resistors share the 9 V equally.

$$\text{potential at B} = \frac{9}{2} = 4.5\,\text{V}$$

The op-amp acts as a comparator and, since V^+ is larger than V^-, the output will be the highest voltage that the op-amp can produce, in this case +9 V.

Step 2 The thermistor T is a negative temperature coefficient thermistor and so its resistance rises sharply and eventually becomes larger than 10 kΩ. Suppose it becomes 12 kΩ. Then:

$$\text{potential at A} = 9 \times \frac{10}{22} = 4.1\,\text{V}$$

Now V^+ is smaller than V^- and the op-amp output voltage is the lowest it can provide, near the negative supply voltage, in this case −9 V.

This switch from +9 V to −9 V is quite sudden because of the large open-loop voltage gain. The value of the temperature when the output voltage switches from +9 V to −9 V can be altered by adjusting the resistance of the resistor in series with the thermistor.

- Effects of negative feedback on the gain of an amplifier incorporating an op-amp:
  - Reduced gain
  - Increased stability

  - Greater bandwidth / less distortion
- Electrical feedback:

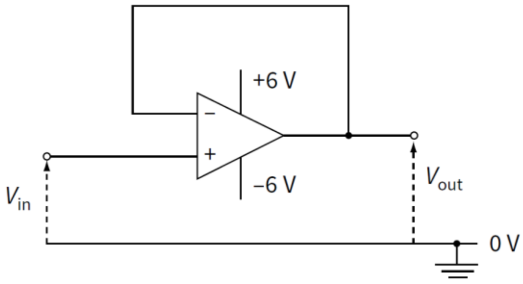


Figure 25.19 An op-amp with the output connected to the inverting input.

  - The potential V^- at the inverting input is always the same as the V_{out}, as they are connected by feedback loop
  - Assume that the open-loop voltage gain is infinite; the op-amp is not saturated
  - As V_{out} is fed back to V^-, the value of V^- increases and this reduces the difference between V^- and V^+, hence the difference becomes zero again and $V_{in} = V_{out}$

We know that $V_{out} = G_0 \times (V^+ - V^-)$, where G_0 is the open-loop voltage gain. Since $V_{out} = V^-$ and $V_{in} = V^+$ we have:

$$V_{out} = G_0(V_{in} - V_{out})$$

$$V_{out}(1 + G_0) = G_0 V_{in}$$

The closed-loop gain G is given by:

$$G = \frac{V_{out}}{V_{in}} = \frac{G_0}{(1 + G_0)}$$

  - Because G_0 is very high (~ 10^5), there is little difference between G_0 and (G_0 + 1), so the closed-loop gain is very close to 1, since the input voltage was +0.1 V the output voltage also +0.1 V, as long as the output voltage is smaller than the supply voltage (e.g. as long as V_{out} is between -6 V and +6 V)
- The inverting amplifier:
  - Uses negative feedback
  - The non-inverting input is connected to the 0 V line
  - Part of the output voltage is connected to the inverting input
  - The input voltage is connected to the inverting input

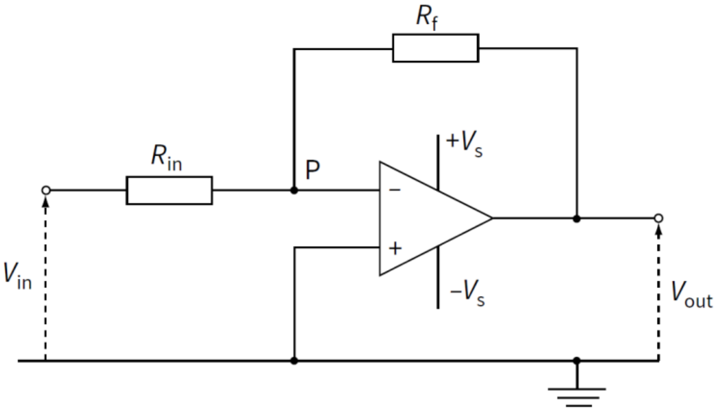


Figure 25.20 An inverting amplifier.

- Virtual earth approximation:
    - Op-amp has very large open-loop voltage gain
    - Op-amp saturates if $V^+ \neq V^-$
    - V^+ is at earth potential so V^- or P must be at earth
- If the current in input resistor R_{in} is I_{in} and the current in the feedback resistor R_f is I_f, then because point P is at 0 V:

$$I_{in} = \frac{V_{in}}{R_{in}} \quad \text{and} \quad I_f = \frac{V_{out}}{R_f}$$

- The input resistance of the op-amp is very high, so virtually no current enters or leaves the inverting input, hence $I_{in} = I_f$
- The output voltage has an opposite sign to that of the input voltage:

$$I_f = -I_{in} \quad \text{and} \quad \frac{V_{out}}{R_f} = -\frac{V_{in}}{R_{in}}$$

- Hence the gain of the inverting amplifier given by:

$$G = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

- As long as the op-amp is not saturated, the p.d. between V^+ and V^- is almost zero, hence $V^+ = V^-$

- The non-inverting amplifier:
    - The input voltage is applied to the non-inverting input
    - Part of the output voltage is fed back to the inverting input

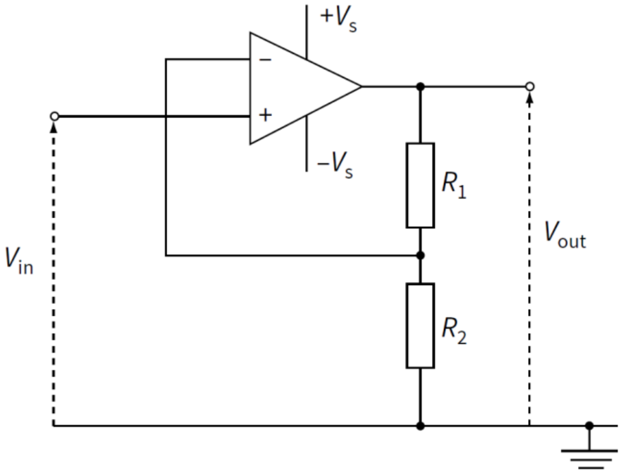


Figure 25.21 A non-inverting amplifier.

- As long as the op-amp is not saturated, the p.d. between V^+ and V^- is almost zero, hence $V^+ = V^-$
- Since the non-inverting input is connected to the input voltage, $V^+ = V^- = V_{in}$
- The two resistors (R_1 and R_2) forms a potential divider
- The total voltage across R_1 and R_2 is V_{out} and the voltage across R_2 is V_{in}
- The current in the two resistors can be written as:

$$\frac{V_{out}}{(R_1+R_2)} = \frac{V_{in}}{R_2}$$

- The gain is calculated from:

$$G = \frac{V_{out}}{V_{in}} = \frac{(R_1+R_2)}{R_2} = 1 + \left(\frac{R_1}{R_2}\right)$$

- Hence for a non-inverting amplifier the gain is given by:

$$G = 1 + \left(\frac{R_1}{R_2}\right)$$

- Input and output voltage has the same sign

- An output device may be required to monitor the output of an op-amp circuit:

❖ The relay:

- A typical op-amp can provide a maximum output current of 25 mA; maximum output voltage of 15 V
- To switch on larger currents and voltages op-amp is connected to relay

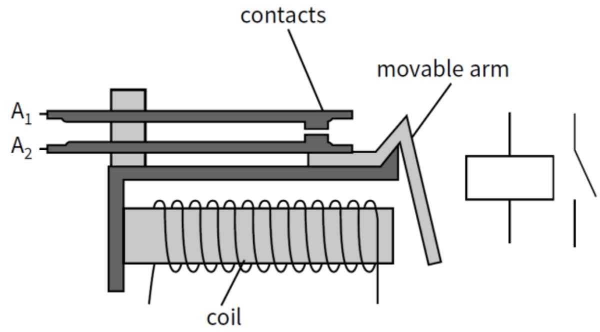


Figure 25.22 A relay and its circuit symbol.

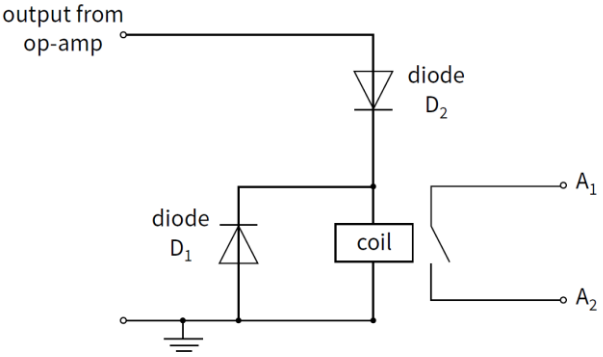


Figure 25.23 The output of an op-amp connected to a relay.

- Relay is just an electromagnetic switch operated by a small current in the coil
- There are two separate circuits: one to the coil and one involving the contacts A_1 and A_2; when a small current passes through the coil of the relay in Fig 25.22, the iron core attracts the movable arm and the contacts connected to contacts A_1 and A_2 close, completing the circuit
- A reverse-biased diode, D_1, is placed across the relay coil; when the op-amp switches off, the induced voltage in the coil causes the bottom of the coil to be more positive than the top of the coil, hence diode D_1 is able to pass current round the coil without damaging the op-amp
- Diode D_2 ensures that current can only flow from op-amp when the op-amp output is positive, hence the relay contacts are closed only when the output is positive

❖ The light-emitting diode (LED):

- LED only requires a current of 20 mA to produce a light output
- LED starts to conduct when the voltage across it is greater than about 2 V
- The value of the resistance of the series resistor *R* can be calculated, e.g. a current is 20 mA and max. voltage output from op-amp is 12 V, then there will be just 2 V across the LED and 12 – 2 = 10 V across the series resistor *R*; the series resistor required is:

$$R = \frac{10}{0.02} = 500\,\Omega$$

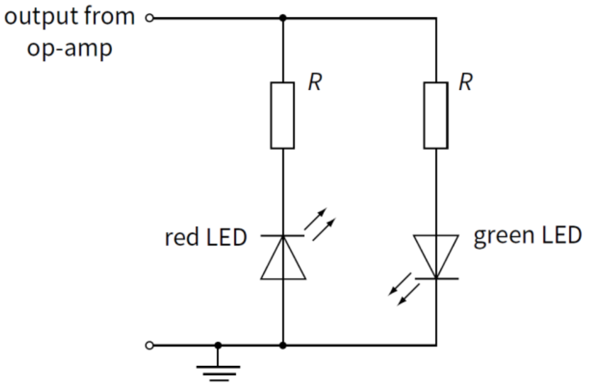


Figure 25.24 LEDs connected to the output of an op-amp.

- When the output of the op-amp is positive relative to earth, the green LED lights
- When the output of the op-amp is negative relative to earth, the red LED lights

❖ The calibrated meter:

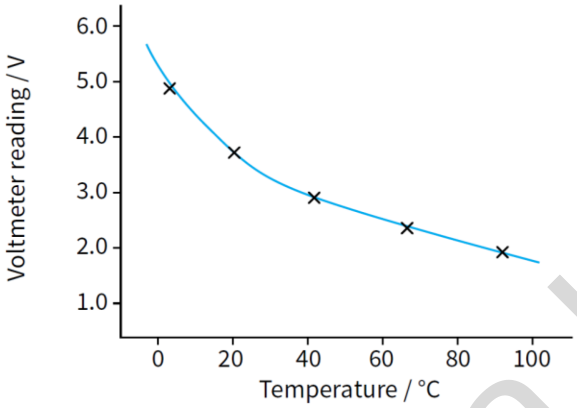


Figure 25.25 A calibration curve relates the output voltage of an op-amp to the variable it is being used to measure.

- Output voltage of op-amp is unlikely to be proportional to the physical quantity being measured, e.g. temperature
- With a digital voltmeter, calibration is needed by placing the temperature sensor and a thermometer in a water bath at a number of different temperatures; the calibration curve is used to change any voltmeter reading into a value for the physical quantity
- With an analogue voltmeter, calibration done in the same way, but result will be cause a change in the linearity of the scale

Communication (Chapter 16):

- To allow several radio stations to broadcast, each signal has a different **carrier wave** frequency which is altered and modulated
- **Modulation** is the variation of either **amplitude** or **frequency** of the carrier wave
- The **modulated carrier wave** is the actual wave transmitted:
  - High frequency wave
  - The amplitude or the frequency is varied in synchrony with the displacement of the information signal
  - The variation represents the information signal
- The signal is present during modulation of the modulated wave
- **Amplitude modulation** (AM):
  - Amplitude of the carrier wave varies in synchrony with the displacement of the information signal
  - The frequency of the modulated carrier wave is constant
- **Frequency modulation** (FM):
  - Frequency of carrier wave varies in synchrony with the displacement of the information signal
  - The amplitude of the modulated carrier wave is constant

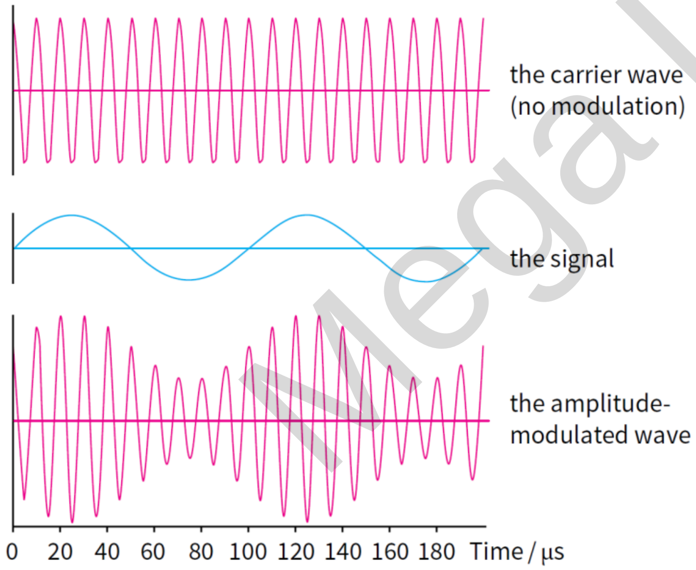


Figure 20.3 Amplitude modulation.

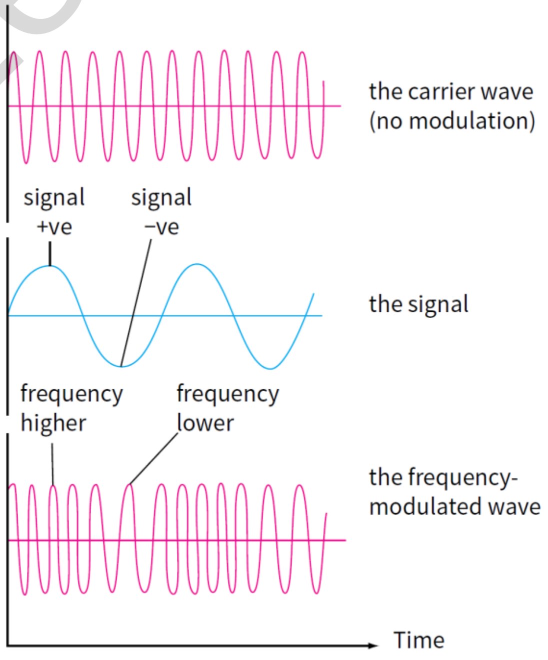


Figure 20.4 Frequency modulation.

A carrier wave of frequency 300 kHz and amplitude 5.0 V is frequency modulated by a sinusoidal signal of frequency 6 kHz and amplitude 2.0 V. The frequency deviation of the carrier wave is 30 kHz V^{-1}. Describe the modulated carrier wave produced.

Step 1 Consider the amplitude of the modulated signal. The amplitude of the carrier wave is unchanged at 5.0 V during frequency modulation. The signal alters the frequency of the carrier wave, not its amplitude.

Step 2 Now consider how the signal will modify the carrier frequency. The frequency shift produced by the signal is $\pm 2 \times 30 = \pm 60$ kHz, so the carrier wave varies in frequency between 240 and 360 kHz. This variation in frequency occurs 6000 times every second as the signal varies at this frequency.

- A carrier wave (only has 1 frequency, f_c) which is **amplitude modulated** by a single audio frequency, f_m, is equivalent to the carrier wave frequency together with two sideband frequencies (($f_c - f_m$) and ($f_c + f_m$))

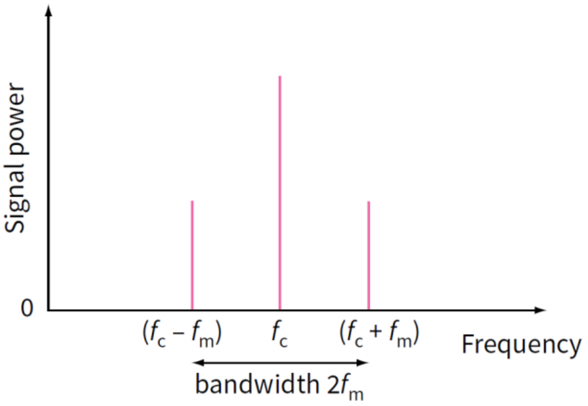


Figure 20.5 The frequency spectrum of a carrier wave modulated in amplitude by a signal of one frequency.

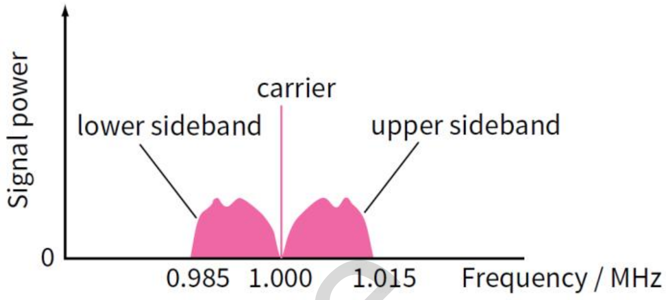


Figure 20.6 The frequency spectrum for an amplitude-modulated wave.

  - When music is transmitted, the carrier wave is modulated by a range of frequencies which change with time, resulting to a band of frequencies (upper and lower sidebands), stretching above and below the carrier frequency by the value of the highest modulating frequency
  - Figure 20.6 shows the frequency spectrum for a carrier wave of frequency 1 MHz modulated with frequencies between 0 and f_m = 15 kHz = 0.015 MHz; the highest frequency present in the spectrum is ($f_c + f_m$) = 1.015 MHz and the lowest frequency is ($f_c - f_m$) = 0.985 MHz
  - **Bandwidth** of a signal is the range of frequencies occupied by the amplitude-modulated waveform, the difference between the highest-frequency and lowest-frequency signal component:
    - E.g. fig 20.6, bandwidth = 1.015 – 0.985 = 0.030 MHz
    - E.g. fig 20.5, bandwidth = ($f_c + f_m$) – ($f_c - f_m$) = $2f_m$
  - Whereas in FM carrier wave has more than two sideband frequencies for each signal frequency, hence requires a greater bandwidth for each radio station
- Modulated carrier waves are used, rather than the direct transmission of electromagnetic waves having audio frequencies due to:
  - Shorter aerial required
  - Longer transmission range / lower transmitter power / less attenuation
  - Allows more than one station in a region
  - Less distortion
- Advantages of FM:
  - Electrical interference affects AM more than FM
  - Higher bandwidth can be used, due to greater range of frequencies available, hence better quality of sound
- Advantages of AM:

  - The actual receiver and transmitter used for AM are less complicated and cheaper than for FM transmission
  - The bandwidth needed for each AM transmission is less than FM transmission, hence more stations are available in any given frequency range
  - AM transmissions use lower frequencies than FM, hence can be diffracted, so can cover a larger area than FM transmissions, for the same power output
- **Digital signal**: signal consists of a series of 1s and 0s
- **Analogue signal**: signal that is continuously variable, having a continuum of possible values

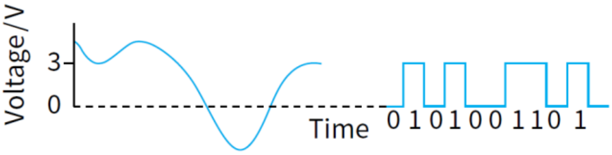


Figure 20.8 Analogue and digital signals.

- **Noise**: unwanted power on signal that is random
  - Amplification of a signal amplifies the noise at the same time
  - Regeneration will remove the noise from a digital signal
- Advantages of data transmission in digital form compared to analogue:
  - Noise can be eliminated, signal can be regenerated
  - Extra bits can be added to check for errors
  - Multiplexing possible
  - Digital circuits are more reliable / cheaper
  - Data can be encrypted for security
- The digital transmission of speech or music involves analogue-to-digital conversion (ADC) before transmission and digital-to-analogue conversion (DAC) after reception

Decimal number	Binary number	Decimal number	Binary number
0	0	6	110
1	1	7	111
2	10	8	1000
3	11	9	1001
4	100	10	1010
5	101	11	1011

Table 20.2 Binary and decimal numbers.

  - Each digit in the binary number is known as a **bit**
- The function of the ADC:
  - Samples the analogue signal at regular intervals and converts the analogue number to a digital number
- The effect of the sampling rate and the number of bits in each sample on the reproduction of an input signal:

E.g. a recording is made of some music. For this recording, the music is sampled at a rate of 44.1 kHz and each sample consists of a 16-bit word

- Suggest the effect on the quality of the recording of
    - Sampling at a higher frequency rather than a lower frequency:
        - Higher frequencies can be reproduced
    - Using a long word length rather than a shorter word length:
        - Smaller changes in loudness / amplitude can be detected
- The recording lasts for a total time of 5 minutes 40 seconds. Calculate the number of bits generated during the recording:
    - Bit rate = $44.1 \times 10^3 \times 16 = 7.06 \times 10^5\ \text{s}^{-1}$
    - Number = $7.06 \times 10^6 \times 340 = 2.4 \times 10^8$

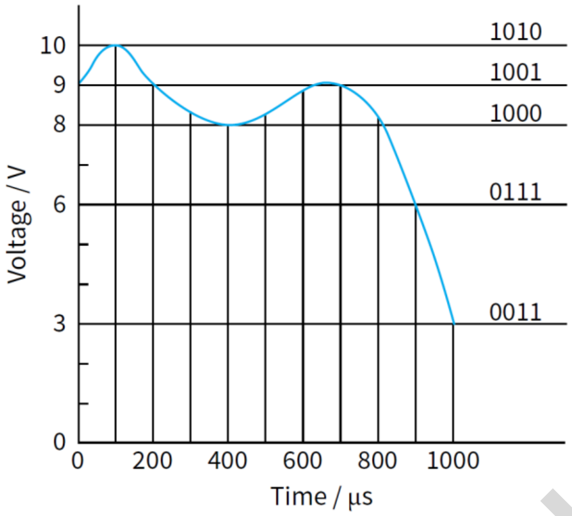


Figure 20.10 Analogue-to-digital conversion.

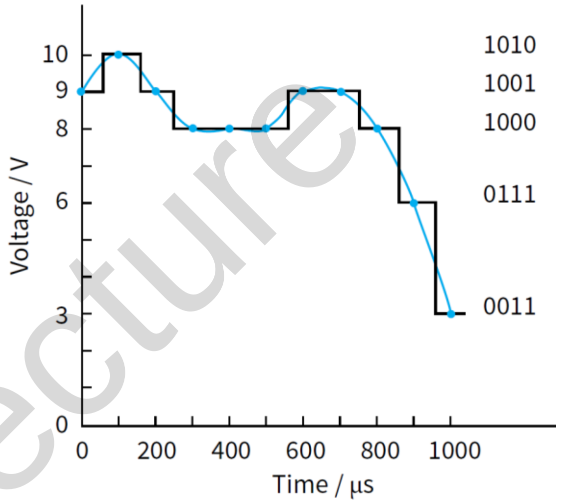


Figure 20.11 Digital-to-analogue conversion.

- There are differences in the digital signal in ADC and DAC; level of detail in the transmitted signal can be increased by:
    - Increase the number of bits in each number (similar to having an extra significant figure) to reduce **step height**, resulting to smaller changes in input signal can be seen/reproduced
    - Increase in sampling rate – the number of samples made per second, e.g. in fig. 20.10, sample is taken every 100μs, so the **step width/depth** is reduced
- **Signal attenuation**: loss of signal power
    - The decrease in signal power from the transmitted value P_1 to that received P_2 can be very high, hence the ratio of P_2 and P_1 is measured using logarithmic scale
    - The logarithm to base 10 of the ratio gives the number of bels (B); when multiplied by 10 gives the number of decibels (dB)

$$\text{number of B} = \lg\left(\frac{P_2}{P_1}\right)$$

$$\text{number of dB} = 10\lg\left(\frac{P_2}{P_1}\right)$$

For example, suppose P_2 is 1000 times greater than P_1:

$$\text{number of dB} = 10\lg\left(\frac{1000}{1}\right) = 30$$

Positive number due to increase in power (amplified signal); negative number due to attenuation

A signal of power 18.0 mW passes along one cable, where the attenuation is 20 dB. It then passes along another cable, where the attenuation is 30 dB. What is the power at the end of the two cables?

Step 1 Apply the decibel equation to each cable in turn.

In the first cable, if the input is P_1 and the output P_2, then:

$$20 = 10\lg\left(\frac{P_1}{P_2}\right)$$

Hint: *Notice that both sides of the equation produce a positive number since* $P_1 > P_2$.

In the second cable, the input is P_2, the output of the first channel. If the output is P_3, then:

$$30 = 10\lg\left(\frac{P_2}{P_3}\right)$$

Step 2 Add the two equations; this gives:

$$50 = 10\left[\lg\left(\frac{P_1}{P_2}\right) + \lg\left(\frac{P_2}{P_3}\right)\right]$$

Applying the 'log of a product rule' gives:

$$50 = 10\lg\left(\frac{P_1}{P_2} \times \frac{P_2}{P_3}\right) = 10\lg\left(\frac{P_1}{P_3}\right)$$

This shows that the total attenuation of the two cables is 50 dB, equal to the sum of the attenuations of the consecutive channels. Hence you can add attenuations to find the total attenuation (but be careful if a signal is being both amplified and attenuated).

Step 3 We have $P_1 = 18$ mW and we need to find P_3. Substituting gives:

$$50 = 10\lg\left(\frac{18}{P_3}\right)$$

so:

$$\lg\left(\frac{18}{P_3}\right) = \frac{50}{10} = 5$$

Taking inverse logs, or pressing the inverse lg button on your calculator, gives:

$$\left(\frac{18}{P_3}\right) = 10^5$$

$P_3 = 1.8 \times 10^{-4}$ mW

You could apply the decibel equation to each cable in turn and use the output of the first cable as the input to the second cable. You should find that the result is the same.

- Attenuation per unit length, units: dB km^{-1}, given by:

$$\text{attenuation per unit length (dB km}^{-1}\text{)} = \frac{\text{attenuation (dB)}}{\text{length of cable (km)}}$$

- When a signal travels along a cable, the level of noise is important, hence the signal-to-noise ratio, in dB, given by:

$$\text{signal-to-noise ratio} = 10\lg\left(\frac{\text{signal power}}{\text{noise power}}\right)$$

- In analogue signal, at regular intervals along a cable, repeaters amplify the signal and noise – multiplying both signal and noise by the same amount keeps the signal-to-noise ratio the same
- Regeneration of a digital signal at the same time as amplification removes most of the noise, ensuring that the signal-to-noise ratio remains high
- Regenerator amplifier do not amplify the noise that has been picked up on digital signals as for digital, only the 1 and 0 / 'high' and 'low' are necessary, variation between 'highs' and 'lows' caused by noise not required

The input signal to a cable has power 1.2×10^{-3} W. The signal attenuation per unit length in the cable is 14 dB km^{-1} and the average noise level along the cable is constant at 1.0×10^{-10} W. An acceptable signal-to-noise ratio is at least 30 dB.

Calculate the minimum acceptable power for the signal and the maximum length of the cable that can be used without a repeater.

Step 1 The signal-to-noise ratio must be at least 30 dB. Hence, using:

$$\text{signal-to-noise ratio} = 10 \lg\left(\frac{\text{signal power}}{\text{noise power}}\right)$$

we have:

$$30 = 10 \lg\left(\frac{P}{1 \times 10^{-10}}\right)$$

where P is the minimum acceptable power. Solving for P gives:

$= 1.0 \times 10^{-7}$ W

Step 2 A repeater is needed to regenerate the signal when the signal-to-noise ratio falls to 30 dB, i.e. its power is 10^3 times the noise level, and this is 1.0×10^{-7} W. We can calculate the attenuation needed to reduce the signal to this level:

$$\text{attenuation} = 10 \lg\left(\frac{1.2 \times 10^{-3}}{1.0 \times 10^{-7}}\right)$$

$$= 41 \text{ dB}$$

Hence the length of cable is $\frac{41}{14} = 2.9$ km.

If the cable is 10 km in length, the total attenuation is:

$14 \text{ dB km}^{-1} \times 10 \text{ km} = 140 \text{ dB}$.

The signal of power 1.2×10^{-3} W is attenuated to a power P where:

$$140 = 10 \lg\left(\frac{0.0012}{P}\right)$$

$P = 12 \times 10^{-17}$ W

You can see that the power in the signal is much smaller than the minimum acceptable power – it is even smaller than the noise level. The signal-to-noise ratio is now $10 \lg(12 \times 10^{-17} / 1.0 \times 10^{-8}) = -79$ dB, smaller than the acceptable +30 dB. A repeater is needed well before the end of the 10 km of cable.

- Different channels of communication:
- ❖ Wire-pairs:
  - ➢ Application: linking a land telephone to the local exchange
  - ➢ The potential difference between the two wires is the signal
  - ➢ Each wire acts as an aerial, picks up wanted electromagnetic waves and distorts the signal
  - ➢ However when two wires are close together, each wire picks up an equal amount of electrical interference, hence no additional potential difference between the two wires and so having the wires close together reduces the interference

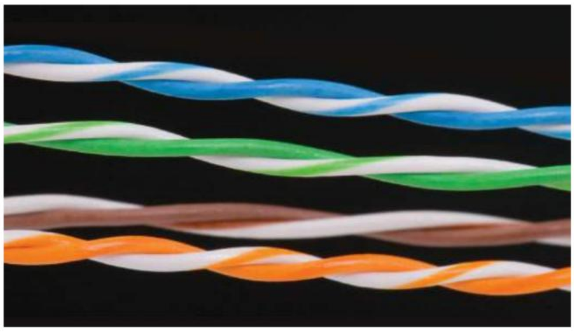


Figure 20.14 Twisted wire-pairs in a computer network.

  - ➢ A wire-pair is the cheapest transmission medium
  - ➢ Has a small bandwidth

- Reflections occur due to poor impedance matching
- Wire-pairs are easily 'tapped' hence low security
- Suffers from **cross-linking** where signal in one pair is picked up by a neighbouring wire pair, resulting to crosstalk
- Large attenuation / energy loss due to the changing currents in the wires produce electromagnetic (EM) fields, hence acting as aerials – requires energy EM waves might pass from one wire-pair to another, leading to crosstalk

❖ Coaxial cables:

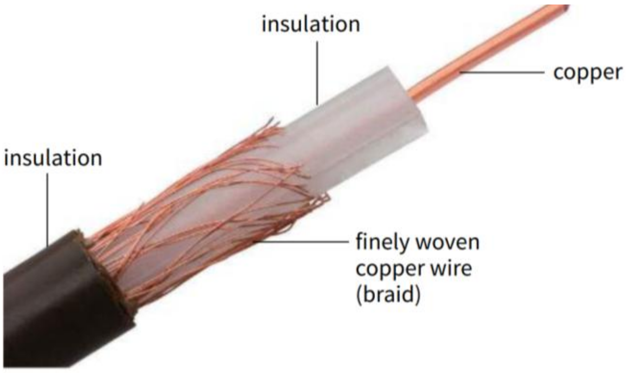


Figure 20.15 Coaxial cable.

- Application: connecting an aerial to a television
- The copper core and the finely woven copper braid are the two conductors that transmit the signal
- Less cross-linking (crosstalk) than wire-pair, as copper wire braid is earthed and shields the core from noise / external signals
    - The copper braid acts as 'return' conductor for signal, shielding from noise/crosstalk/interference
- Greater bandwidth than wire-pair
- Has less attenuation per unit length than a wire-pair, as it prevents any emission of EM waves
- Can transmit data faster, over longer distances, with less electrical interference / noise
- More expensive than a wire-pair

❖ Radio waves:

- Electromagnetic waves covering a vast range of wavelengths; used in a variety of communication depending on frequency, there are three types:

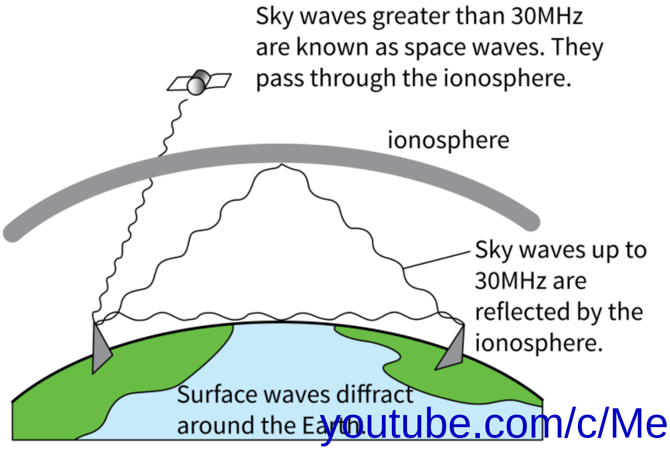
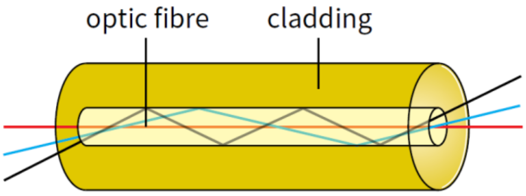


Figure 20.16 Radio wave transmissions.

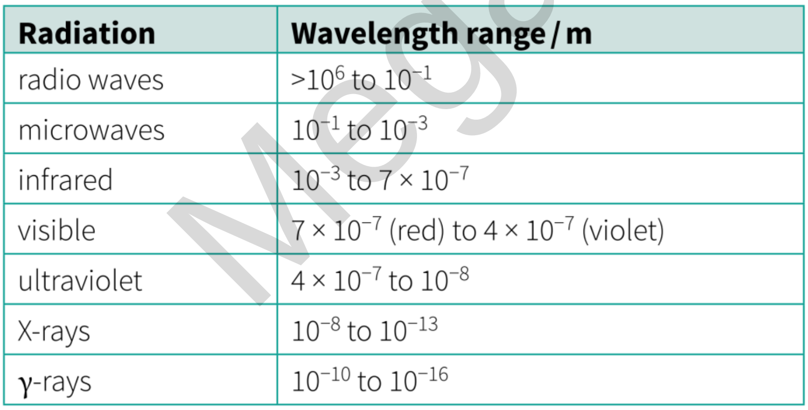
    - Surface waves travel close to the surface of the earth, low frequencies of up to 3 MHz (medium-wave and long-wave bands), diffract around the surface of the Earth due to their long wavelengths, giving them a long range of travel; used in AM radio transmissions
        - Adv: very large range
        - Dis: attenuation and distortion significant
    - Sky waves, above 3 MHz (high-frequency band), travels almost in straight lines, less diffracted, hence travels shorter distances; using total internal reflection from a layer of charged particles in the atmosphere known as ionosphere; used in short-wave radio broadcasts
        - Adv: large range
        - Dis: substantial attenuation in the ionosphere; unreliable due to unpredictable ionosphere conditions and interference
    - Space waves are sky waves, above 30 MHz (very-high-frequency and ultra-high-frequency bands), which pass through the ionosphere; the transmission is line-of-sight, o, if the receiver and transmitter are on the Earth's surface, there must be a clear line between the receiver and the transmitter; used in satellite television transmissions
        - Adv: ionosphere has no effect, so more reliable; can be used by satellites
        - Dis: small range (for ground-based stations)

- Microwave links, above 1 GHz, able to pass through the ionosphere to reach satellites in space
    - Application: linking a ground station to a satellite
    - The satellite receives a space wave from a transmitter on Earth, the uplink, with a carrier frequency in the microwave region; due to significant attenuation between a geostationary satellite and Earth, the uplink and downlink frequencies must be different, as the signal must be amplified greatly before transmission back to Earth, and if the signals are the same, the uplink signal would be swamped by downlink signal
    - The use of ionospheric reflection of radio-waves for long-distance communication been replaced by satellite communication due to:
        - Unreliable communication because the ion layers vary in height/density
        - Cannot carry all information required, as bandwidth is too narrow/low
        - Coverage is limited due to poor reception in hilly areas
- For the satellite dish to always point towards the satellite, geostationary satellite is used
- Polar-orbit satellites used for surface observation and as weather satellites
- Compared to a geostationary satellite, polar-orbits satellite:
    - Travels from pole to pole, with a shorter period of orbit
    - Os at a smaller height above the Earth and can detect objects of smaller detail

  - Is not always in the same position relative to the Earth, hence dishes must be moved
  - Has smaller time delays
- Optic fibres:
  - Consists of a thin glass core surrounded by a material of slightly lower refractive index called the cladding, to cause total internal reflection
  - Advantages of coaxial cables for the transmission of data:
    - Large bandwidth / carries more information
    - Low attenuation of signal, so repeater and regeneration amplifiers can be further apart
    - Low cost than the same length of copper wire
    - Smaller diameter, easier handling, easier storage, less weight
    - High security / no crosstalk
    - Low noise / no EM interference



- Method to convert decimal into binary number: internet / past papers
- The bit on the left-hand side of a binary number is the **most significant bit** (MSB) and has the highest value



Radiation	Wavelength range / m
radio waves	$>10^{6}$ to 10^{-1}
microwaves	10^{-1} to 10^{-3}
infrared	10^{-3} to 7×10^{-7}
visible	7×10^{-7} (red) to 4×10^{-7} (violet)
ultraviolet	4×10^{-7} to 10^{-8}
X-rays	10^{-8} to 10^{-13}
γ-rays	10^{-10} to 10^{-16}

Table 13.3 Wavelengths (in a vacuum) of the electromagnetic spectrum.

Medical imaging (Chapter 32 TB):

- Principles of the production of X-rays by electron bombardment of a metal target:

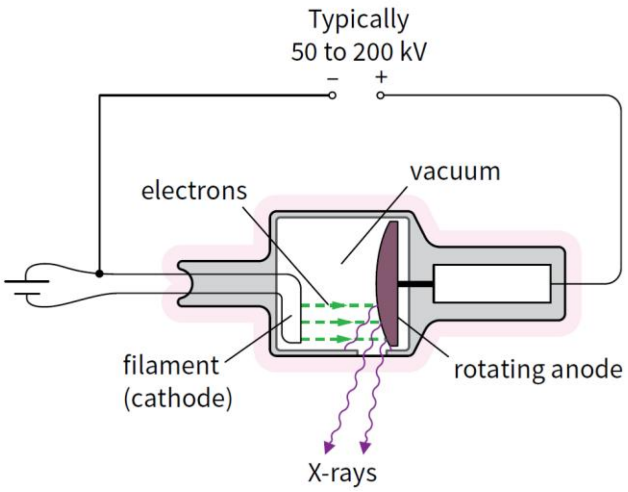


Figure 32.4 A simplified diagram of an X-ray tube.

- Contains two electrodes:
    - Cathode – the heated filament from which electrons are emitted
    - Anode – the rotating anode (to avoid overheating) made of a hard target metal (tungsten)
- External power supply produces a voltage between the two electrodes, resulting to the acceleration of a beam of electrons across the gap between the cathode and the anode, bombarding the anode (metal target) at high speed, stopping the electrons, hence losing a small amount of their kinetic energy in the form of X-ray photons which emerge in all directions
- The window, is thinner than the rest allowing X-rays to emerge into the space outside the tube; the width of the X-ray beam can be controlled using metal tubes beyond the window to absorb X-rays, which produces a parallel-sided beam called a collimated beam

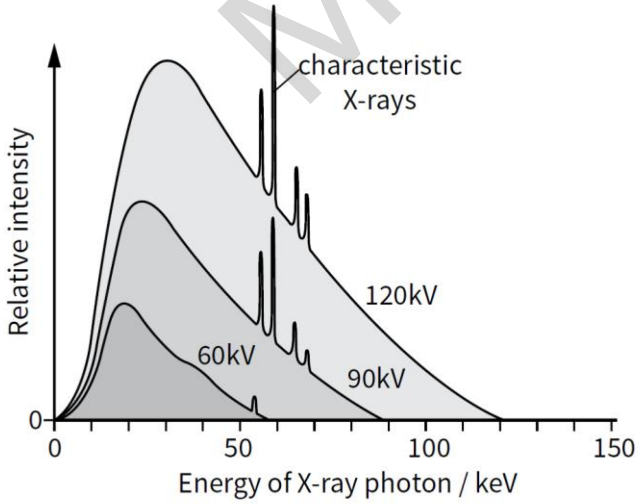


Figure 32.5 X-ray spectra for a tungsten target with accelerating voltages of 60 kV, 90 kV and 120 kV. The continuous curve shows the braking radiation while the sharp spikes are the characteristic X-rays.

- X-ray photons produced when electrons is accelerated have range of accelerations when hitting the target metal, hence the distributions of wavelengths forming the broad background **braking radiation**; the **characteristic radiation** are due to the de-excitation of orbital electrons in the target metal (anode); the sharp cut-off at short wavelength is because an electron gives all of its energy to a single photon and is stopped in a single collision, as well as where its minimum wavelength gives the maximum energy

$f_{\max} = \frac{eV}{h}$ or $\lambda_{min} = hc / E_{max}$

- **Hardness** of an X-ray beam: the measure of the penetration of the beam; the greater the hardness, the greater the penetration / shorter wavelength / higher frequency / higher photon energy
  - Increasable by increasing the accelerating potential difference (p.d. between anode and cathode)
  - Soft X-rays are less penetrative (long wavelengths), so it is more likely to be absorbed in the body, hence it possesses a greater health hazard than short-wavelength radiation; minimised by using aluminium sheet filter placed in the X-ray beam from tube
- The gradual decrease in the intensity of X-rays as it passes through matter is called **attenuation**
- Intensity is the power / rate of energy transfer per unit cross-sectional area; unit: W m^{-2}

$$I = \frac{P}{A}$$

- Attenuation of X-rays passing through a uniform material given by:

$$I = I_0 e^{-\mu x}$$

  - I_0 is the initial intensity
  - x is the thickness of the material
  - I is the transmitted intensity
  - μ is the attenuation coefficient; unit: m^{-1} or cm^{-1}, etc

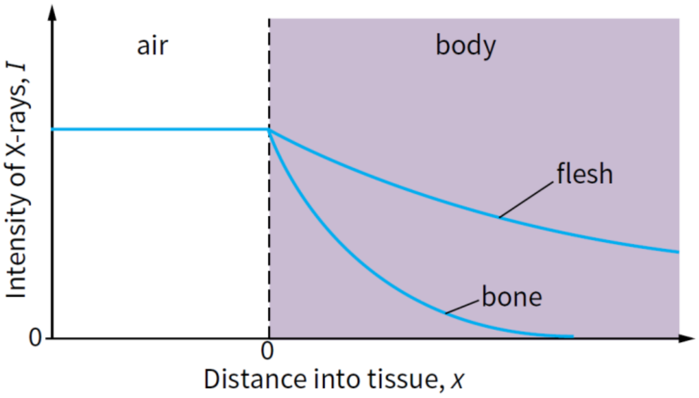


Figure 32.6 The absorption of X-rays follows an exponential pattern.

- Half-thickness of an absorbing material given by:

$$x_{1/2} = \frac{\ln 2}{\mu}$$

- **Sharpness**: ease with which edges of structures can be seen
  - Determined by the width of the X-ray beam (sharp image achieved by a narrow beam of parallel X-rays) controlled by:
    - Width of the electron beam and the metal target
    - Size of aperture at the exit window (reduced by adjustable lead plates)
    - Collimation of beam – ensuring parallel-sided beams when passed through the lead slits

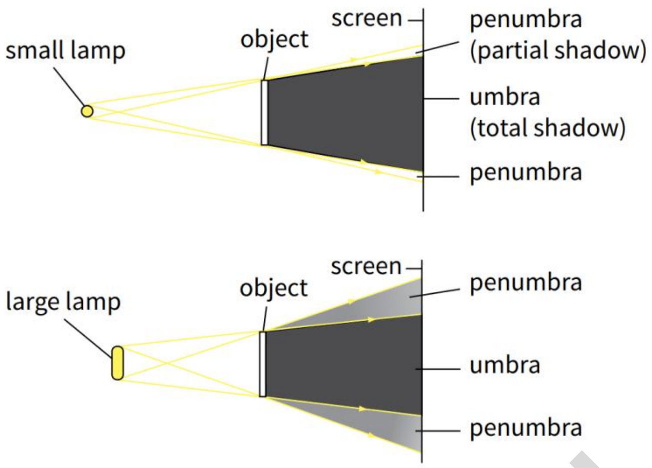


Figure 32.9 The small lamp casts a smaller penumbra and this improves the sharpness of the shadow.

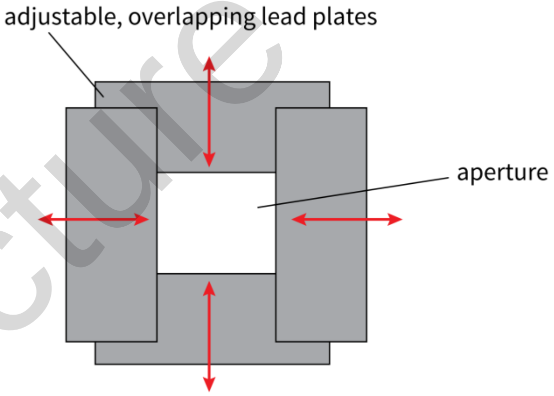


Figure 32.11 The smaller the aperture, the narrower the X-ray beam.

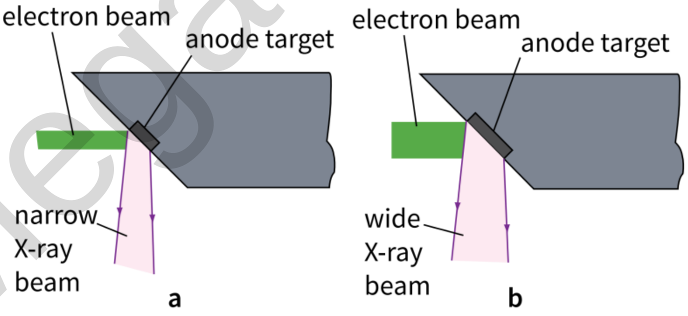


Figure 32.10 A wide anode target results in a wide X-ray beam, giving fuzzy edges to the shadow image.

- **Contrast**: difference in degree of blackening between structures
  - Good contrast when the ratio between the intensity of the incident and the emergent X-ray beam of different organs are far apart
- Principles of computed tomography or CT scanning:
  - X-ray images of one slice is taken from different angles, all in the same plane
  - Combined to produce a 2D image of one slice
  - Repeated for many slices and images combined
  - To build up 3D image of whole body structure using the computer that can be rotated and viewed from different angles

- Advantages of CT scan over the standard X-ray image: image gives depth / 3D image formed / final image can be rotated viewed from any angle
- Disadvantages of CT scan over the standard X-ray image: greater exposure to X-ray radiation / more expensive / higher health risks / person must remain stationary#
- The image of an 8-voxel cube can be developed using CT scanning:

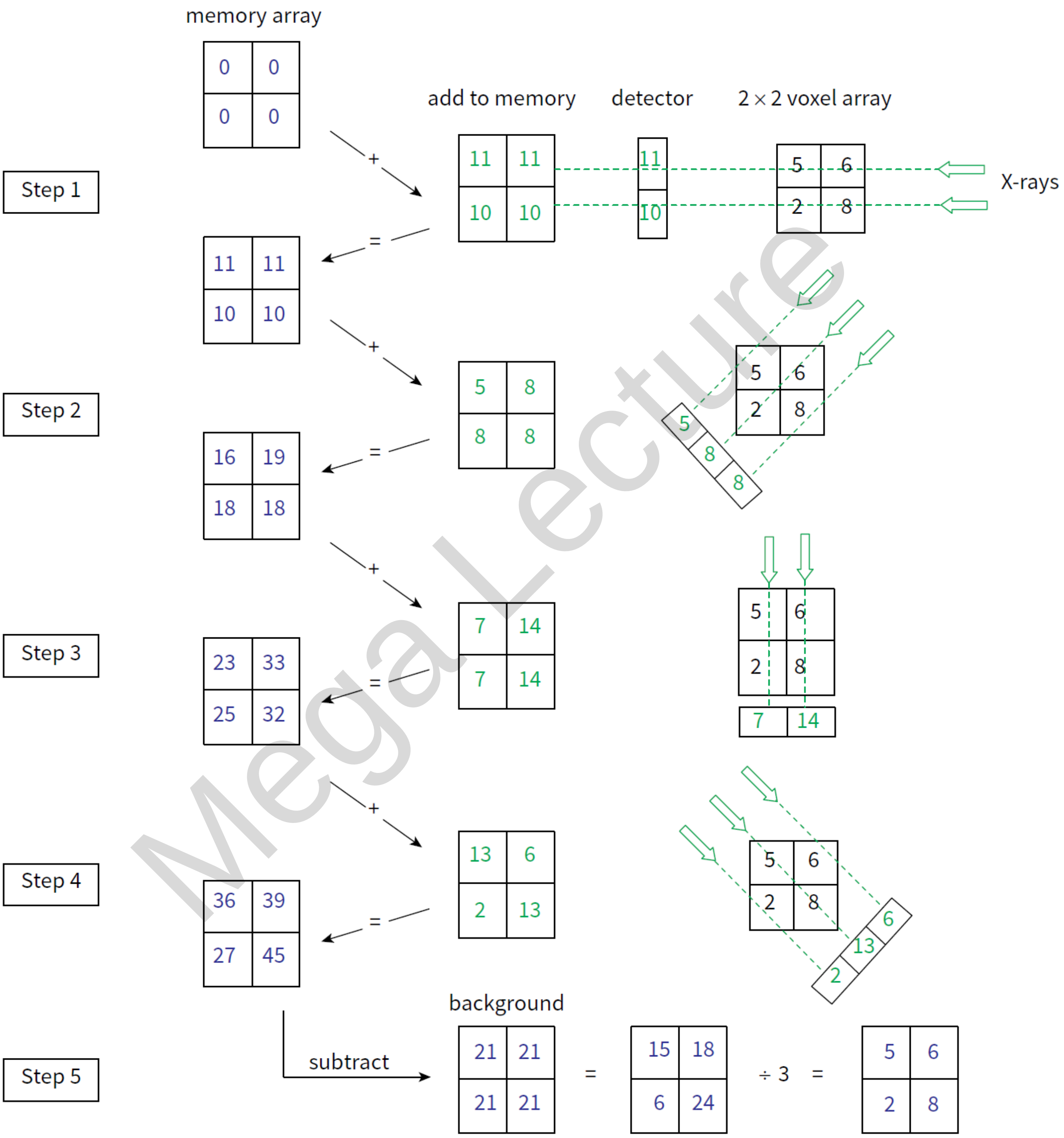


Figure 32.19 Data is built up from a CT scan of a 2 × 2 voxel array, and then processed to deduce the original array.

- Principles of the generation and detection of ultrasonic waves using piezo-electric transducers:

  - Potential difference is applied across the piezo-electric transducer (quartz crystal) causing it to change shape (distort)
  - Applying alternating p.d. causes oscillations/vibrations
  - When applied frequency is natural frequency, crystal resonates
  - Natural frequency of crystal is in the ultrasound range (hence usage of ultrasound waves), alternating p.d. produced across the crystal
- Principles behind the use of ultrasound to obtain diagnostic information about internal structures:
  - Pulse of ultrasound produced by quartz crystal / piezo-electric crystal
  - Gel/coupling medium on the skin to reduce reflection at skin
  - Reflected from the boundaries between media
  - Reflected pulse/wave detected by ultrasound transmitter
  - Reflected wave processed and displayed
  - Intensity of reflected pulse/wave gives information about boundary
  - Time delay gives information about the depth of boundary
- Ultrasound at higher frequencies allows smaller structures to be observed/resolved
- **Specific acoustic impedance** Z of a medium: product of medium density and the speed of sound in the medium; unit: kg m^{-2} s^{-1}

$$Z = \rho c$$

- The intensity reflection coefficient α is given by the expression:

$$\alpha = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} \quad \text{or} \quad \frac{I_r}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

  - α is the ratio of the reflected intensity to incident intensity
  - Z_1 and Z_2 are the specific acoustic impedances of media on each of the boundaries
  - The greater the difference in acoustic impedances, the greater the reflected fraction of the ultrasound waves
- The intensity I decreases with as it passes through the body of distance x, hence attenuation of ultrasound in matter is given by:

$$I = I_0 e^{-\mu x}$$

- Ultrasound relies on the reflection of ultrasound at the boundaries between different tissues, hence the thickness of an organ is given by:

$$\text{thickness of bone} = \frac{\text{distance travelled by ultrasound}}{2} = \frac{c\Delta t}{2}$$

- Principles behind the use of nuclear magnetic resonance imaging (NMRI) to obtain diagnostic information about internal structures:
  - Strong uniform magnetic field is applied
  - Nuclei precess about the field direction

  - Radio frequency pulse is applied, at the Larmor frequency, causing resonance (nuclei absorbs the energy)
  - On relaxation, nuclei emit the radio frequency pulse
  - Emitted pulse is detected and processed
  - Non-uniform field superimposed on uniform field
  - Allows the position of resonating nuclei to be determined
  - Allows for the position of detection to be changed (different slices to be studied)
- The function of the non-uniform magnetic field, superimposed on the large constant magnetic field, in diagnosis using NMRI:
  - Strong uniform magnetic field aligns the nuclei / gives rise to Larmor frequency in the r.f. region
  - Non-uniform magnetic field enables the nucli to be located / changes the Larmor frequency