



Candidate's Name: \_\_\_\_\_

## 22 Magnetic fields

The concept of a magnetic field is developed by studying the force on current-carrying conductors and on charged particles in magnetic fields.

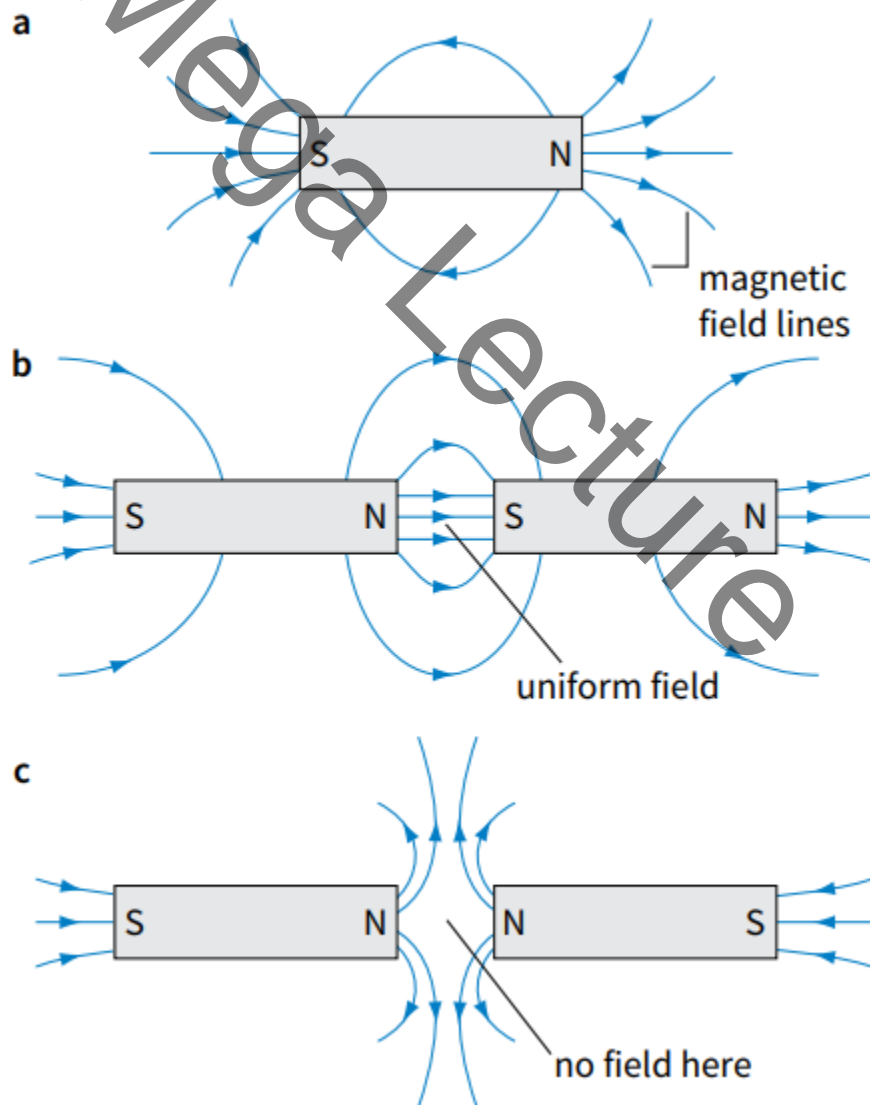
The Hall effect and nuclear magnetic resonance imaging are studied as examples of the use of magnetic fields.

### Learning outcomes

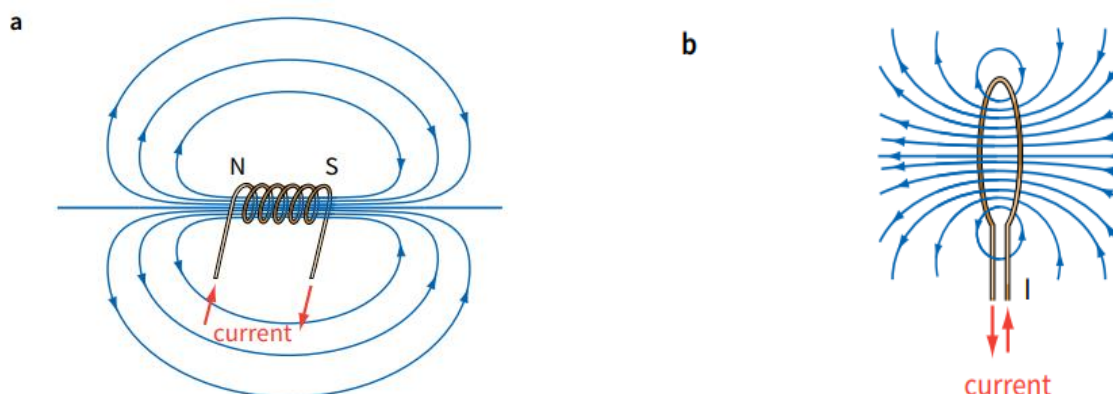
Candidates should be able to:

- |   |  |
|---|--|
| <b>22.1 Concept of magnetic field</b>             | <ul style="list-style-type: none"> <li>a) understand that a magnetic field is an example of a field of force produced either by current-carrying conductors or by permanent magnets</li> <li>b) represent a magnetic field by field lines</li> </ul>   |
| <b>22.2 Force on a current-carrying conductor</b> | <ul style="list-style-type: none"> <li>a) appreciate that a force might act on a current-carrying conductor placed in a magnetic field</li> <li>b) recall and solve problems using the equation <math>F = BIL \sin \theta</math>, with directions as interpreted by Fleming's left-hand rule</li> <li>c) define magnetic flux density and the tesla</li> <li>d) understand how the force on a current-carrying conductor can be used to measure the flux density of a magnetic field using a current balance</li> </ul>  |
| <b>22.3 Force on a moving charge</b>              | <ul style="list-style-type: none"> <li>a) predict the direction of the force on a charge moving in a magnetic field</li> <li>b) recall and solve problems using <math>F = BQv \sin \theta</math></li> <li>c) derive the expression <math>V_H = \frac{BI}{ntq}</math> for the Hall voltage, where <math>t</math> = thickness</li> <li>d) describe and analyse qualitatively the deflection of beams of charged particles by uniform electric and uniform magnetic fields</li> <li>e) explain how electric and magnetic fields can be used in velocity selection</li> <li>f) explain the main principles of one method for the determination of <math>v</math> and <math>\frac{e}{m_e}</math> for electrons</li> </ul> |
| <b>22.4 Magnetic fields due to currents</b>       | <ul style="list-style-type: none"> <li>a) sketch flux patterns due to a long straight wire, a flat circular coil and a long solenoid</li> <li>b) understand that the field due to a solenoid is influenced by the presence of a ferrous core</li> <li>c) explain the forces between current-carrying conductors and predict the direction of the forces</li> <li>d) describe and compare the forces on mass, charge and current in gravitational, electric and magnetic fields, as appropriate</li> </ul>  |

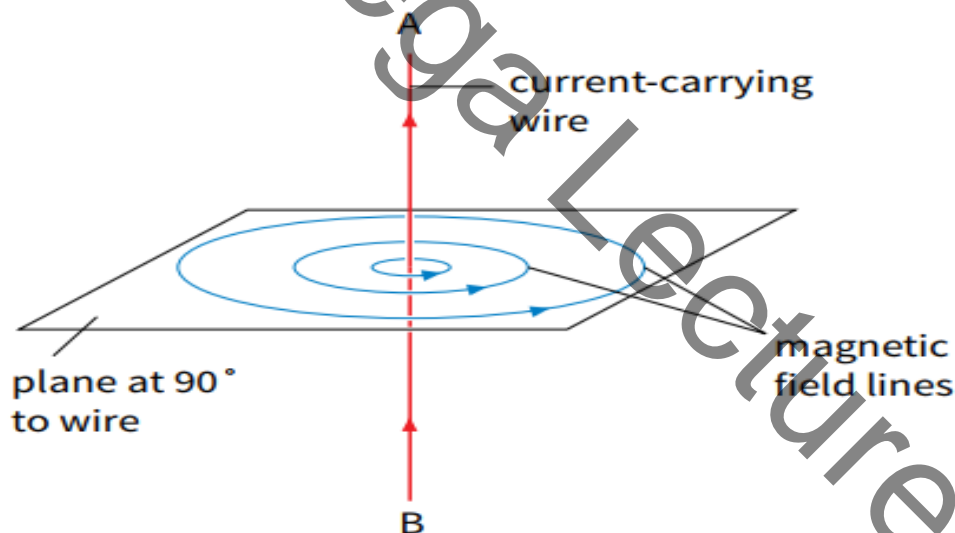
- A magnetic field exists wherever there is force on a magnetic pole.
- You can make a magnetic field in two ways: using a permanent magnet, or using an electric current.
- We represent magnetic field patterns by drawing magnetic field lines.
- The magnetic field lines come out of north poles and go into south poles.
- The direction of a field line at any point in the field shows the direction of the force that a 'free' magnetic north pole would experience at that point.
- The field is strongest where the field lines are closest together.
- The idea that magnetic field lines emerge from north poles and go into south poles is simply a convention.



**Magnetic field patterns for (a) a solenoid, and (b) a flat circular coil:**

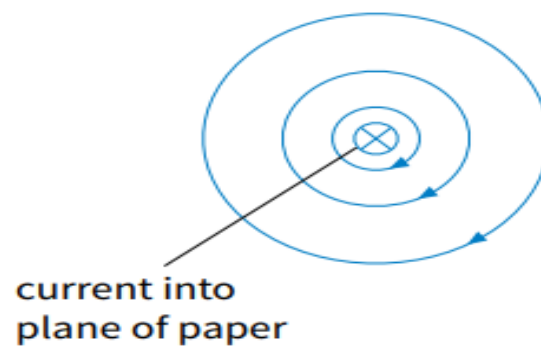
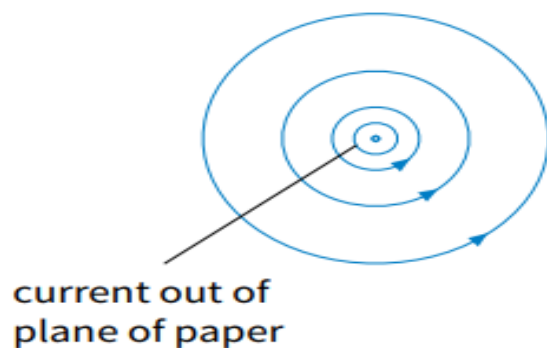


**The magnetic field pattern around a current carrying wire:**



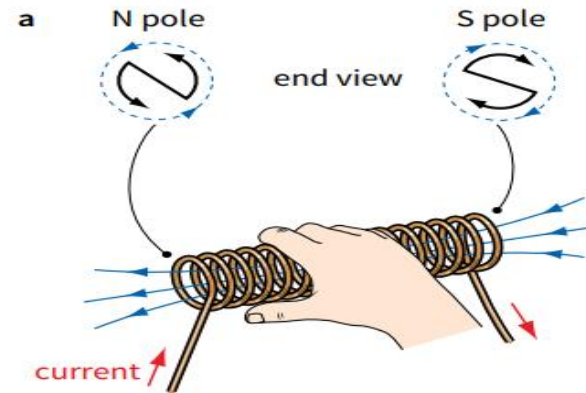
View from A – anticlockwise

View from B – clockwise



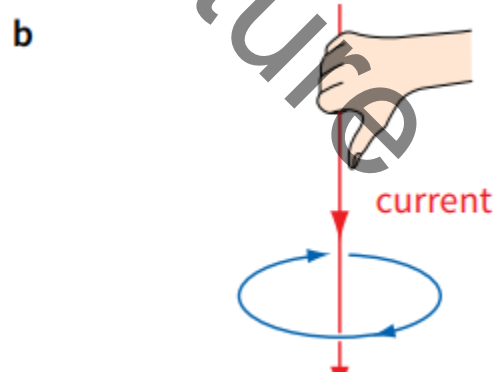
**The right-hand grip rule gives the direction of magnetic field lines in an electromagnet.**

“Grip the coil so that your fingers go around it following the direction of the current. Your thumb now points in the direction of the field lines inside the coil, i.e. it points towards the electromagnet’s North Pole.”



**The circular field around a wire carrying a current does not have magnetic poles. To find the direction of the magnetic field you need to use another rule, the right hand rule.**

“Grip the wire with your right hand, pointing your thumb in the direction of the current. Your fingers curl around in the direction of the magnetic field.”





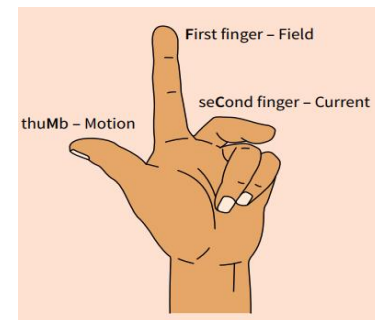
Mega Lecture

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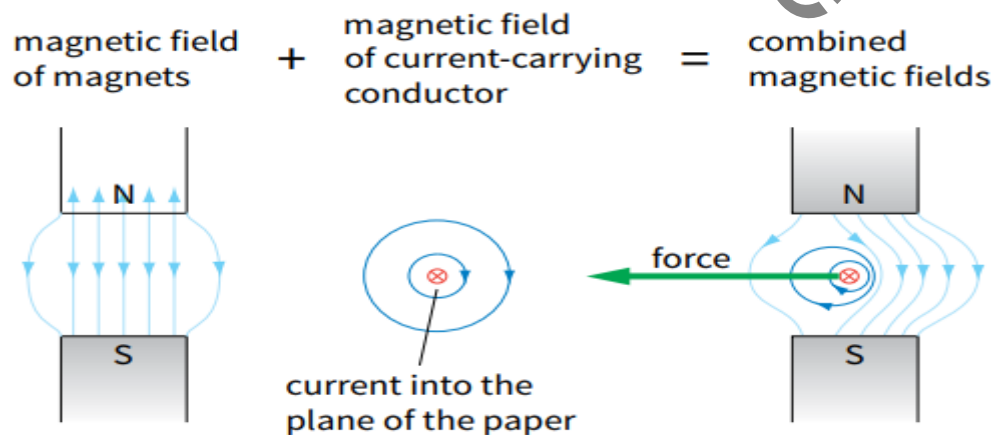
### Magnetic force:

- A current-carrying wire is surrounded by a magnetic field.
- This magnetic field will interact with an external magnetic field, giving rise to a force on the conductor, just like the fields of two interacting magnets.
- We use Fleming's left-hand (motor) rule to predict the direction of the force on the current-carrying conductor, placed in a magnetic field

"If the thumb and first two fingers of the left hand are held at right angles to one another, with the First finger pointing in the direction of the Field and seCond finger in the direction of the Current, then the thuMb points in the direction of the Motion or force."



- If you think of the magnetic field lines as elastic bands then you can see why the wire is pushed out in the direction shown.
- The production of this force is known as the motor effect, because this force is used in electric motors.



### **Magnetic flux density**

- The strength of a magnetic field is known as its magnetic flux density, with symbol B.

**“The magnetic flux density (B) at a point in space is the force experienced per unit length by a long straight conductor carrying unit current and placed at right angles to the field at that point.”**

$$B = \frac{F}{IL}$$

- The unit for magnetic flux density is the tesla, T.
- It follows from the equation for B that

$$1 \text{ T} = 1 \text{ NA}^{-1}\text{m}^{-1}.$$

The tesla is defined as follows:

**“The magnetic flux density is 1T when a wire carrying a current of 1A placed at right angles to the magnetic field experiences a force of 1N per metre of its length.”**

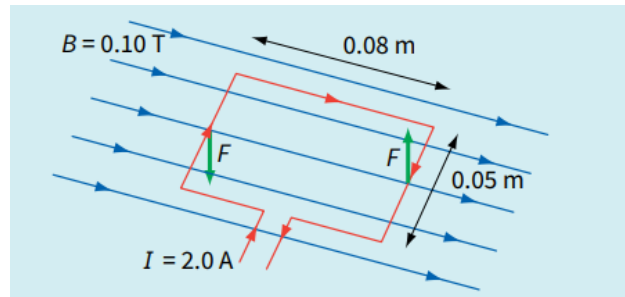
The force on the conductor is given by the equation:

$$F = BIL$$

- Note that you can only use this equation when the field is at right angles to the current.
- The force on a current-carrying conductor depends on the angle it makes with the magnetic field lines. At an angle other than 90°, to calculate the force, we need to find the component of the magnetic flux density B at right angles to the current. We use the following equation

$$F = BIL\sin\theta$$

Q: An electric motor has a rectangular loop of wire with the dimensions shown in Figure given. The loop is in a magnetic field of flux density  $0.10\text{ T}$ . The current in the loop is  $2.0\text{ A}$ . Calculate the torque that acts on the loop in the position shown.

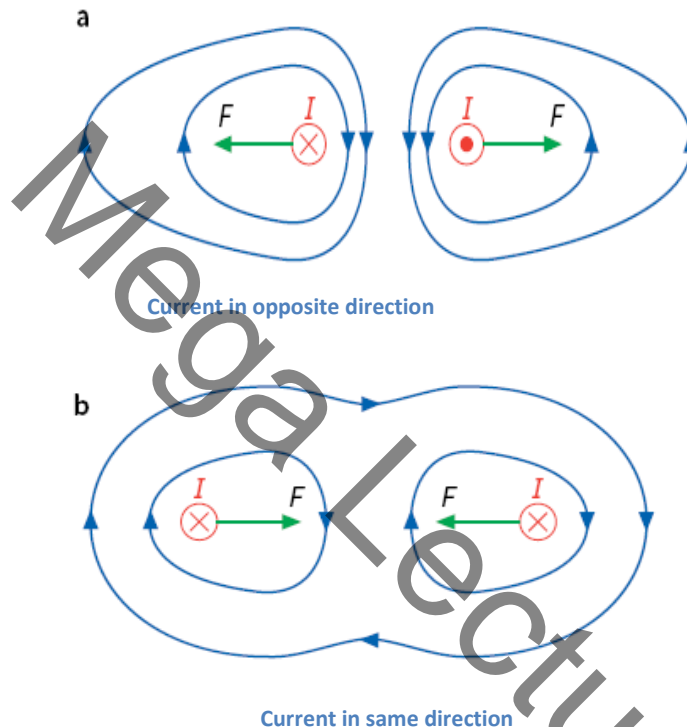


Mega Lecture

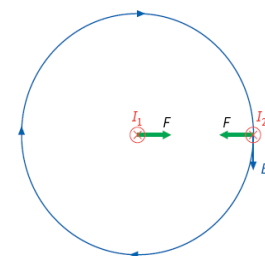


### Forces between currents:

- There are two ways to understand the origin of the forces between current-carrying conductors.
- In the first, we draw the magnetic fields around two current-carrying conductor.
- The diagram shows the resultant field in both the cases i.e current in same and opposite direction.



- The other way to explain the forces between the currents is to use the idea of the motor effect.
- Figure on right again shows two like currents,  $I_1$  and  $I_2$ , but this time we only consider the magnetic field of one of them,  $I_1$ .
- The second current  $I_2$  is flowing across the magnetic field of  $I_1$ ; from the diagram, you can see that  $B$  is at right angles to  $I_2$ .
- Hence there will be a force on  $I_2$  (the  $BIL$  force), and we can find its direction using Fleming's left-hand rule.
- The arrow shows the direction of the force, which is towards  $I_1$ . Similarly, there will be a  $BIL$  force on  $I_1$ , directed towards  $I_2$ .
- These two forces are equal and opposite to one another.
- They are an example of an action and reaction pair, as described by Newton's third law of motion.



**Comparing forces in magnetic, electric and gravitational fields:**

Gravitational and electric fields are defined in terms of placing a test mass or a test charge at a point to measure the field strength. Similarly, a test wire carrying a current can be placed at a point to measure the magnetic field strength. Therefore all fields are defined in terms of the force on a unit mass, charge or current.

- Action at a distance, between masses, between charges or between wires carrying currents.
- Decreasing strength with distance from the source of the field.
- Representation by field lines, the direction of which show the direction of the force at points along the line; the density of field lines indicates the relative strength of the field.
- The force between two 1 kg masses 1 m apart =  $6.7 \times 10^{-11}$  N
- The force between two charges of 1 C placed 1 m apart =  $9.0 \times 10^9$  N
- The force per metre on two wires carrying a current of 1 A placed 1 m apart =  $2.0 \times 10^{-7}$  N
- For an electron, or any other small charged object, electric forces are the most significant.
- However, over larger distances and with objects of large mass, the gravitational field becomes the most significant.

### The magnetic force on a moving charge

The magnetic force  $F$  on a moving particle at right angles to a magnetic field is given by the equation:

$$F = BQv$$

$B$  = the magnetic flux density (strength of the magnetic field)

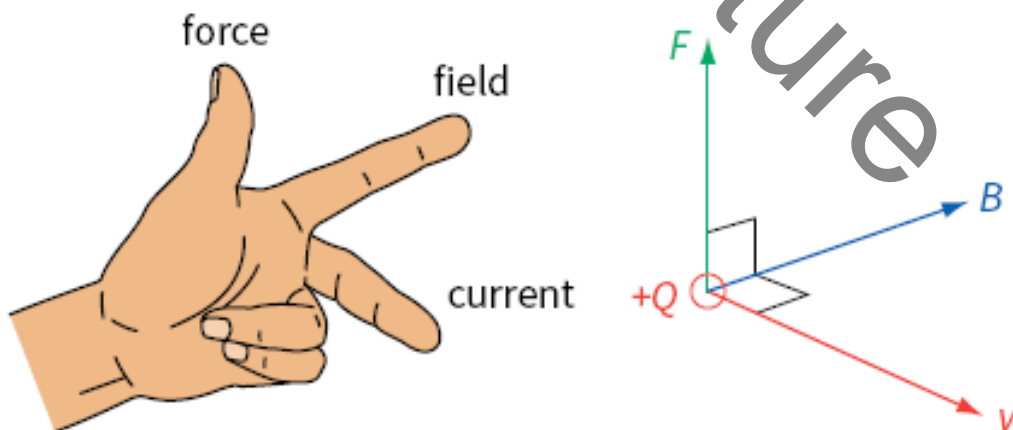
$Q$  = the charge on the particle

$v$  = the speed of the particle.

If the charged particle is moving at an angle  $\vartheta$  to the magnetic field, the component of its velocity at right angles to  $B$  is  $v \sin \vartheta$ .  
Hence the equation becomes:

$$F = BQv \sin \theta$$

- The direction of the force can be determined from Fleming's left-hand rule.
- The force  $F$  is always at  $90^\circ$  to the velocity of the particle.



Fleming's left-hand rule, applied to a moving positive charge.

### Orbiting charges

- Consider a charged particle moving at right angles to a uniform magnetic field.
- It will describe a circular path because the magnetic force  $F$  is always perpendicular to its velocity.
- We can describe  $F$  as a centripetal force, because it is always directed towards the centre of the circle.
- The fact that the  $Bev$  force acts as a centripetal force gives us a clue as to how we can calculate the radius of the orbit of a charged particle in a uniform magnetic field.

The centripetal force on the charged particle is given by:

$$\text{centripetal force} = \frac{mv^2}{r}$$

The centripetal force is provided by the magnetic force  $Bev$ . Therefore:

$$Bev = \frac{mv^2}{r}$$

Cancelling and rearranging to find  $r$  gives:

$$r = \frac{mv}{Be}$$

The above equation shows that:

- faster-moving particles move in bigger circles ( $r \propto v$ )
- particles with greater masses also move in bigger circles (they have more inertia:  $r \propto m$ )
- Stronger field makes the particles move in tighter circles ( $r \propto 1/B$ ).

**The charge-to-mass ( $e/m$ ) ratio of an electron:**

- This ratio is also known as the specific charge on the electron
- the word 'specific' here means 'per unit mass'.
- the equation for an electron travelling in a circle in a magnetic field, we have

$$\frac{e}{m} = \frac{v}{Br}$$

Practically, there are difficulties in measuring  $B$  and  $r$ .

**Method 01:**

- One way is to use the cathode–anode voltage  $V$ .
- This p.d. causes each electron to accelerate as it moves from the cathode to the anode.
- If an individual electron has charge  $-e$  then an amount of work  $e \times V$  is done on each electron.
- This is its kinetic energy as it leaves the anode. Hence,

$$e V = \frac{1}{2} m v^2$$

- here  $m$  is electron mass and  $v$  is the speed of the electron.
- Eliminating  $v$  from the two equations gives,

$$\frac{e}{m} = \frac{2V}{r^2 B^2}$$

- Hence, if we measure  $V$ ,  $r$  and  $B$ , we can calculate  $e/m$ .

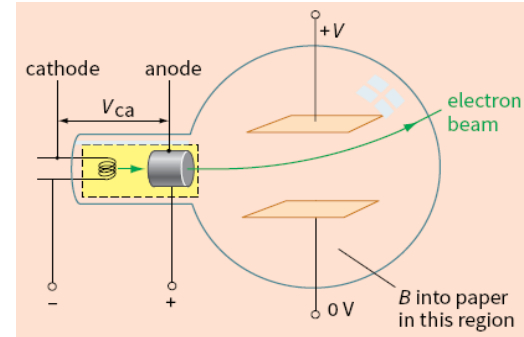
**Q:** An electron is travelling at right angles to a uniform magnetic field of flux density 1.2 mT. The speed of the electron is  $8.0 \times 10^6 \text{ m s}^{-1}$ . Calculate the radius of circle described by this electron.

(For an electron, charge  $e = 1.60 \times 10^{-19} \text{ C}$  and mass  $m = 9.11 \times 10^{-31} \text{ kg}$ .)  
[3.8 cm]

Q: If the electron charge is  $1.60 \times 10^{-19} \text{ C}$  and the charge-to-mass ratio  $e/m$  is  $1.76 \times 10^{11} \text{ C kg}^{-1}$ , calculate the electron mass.

### The deflection tube:

- A deflection tube is designed to show a beam of electrons passing through a combination of electric and magnetic fields.
- By adjusting the strengths of the electric and magnetic fields, you can balance the two forces on the electrons, and the beam will remain horizontal.
- If the electron beam remains straight, it follows that the electric and magnetic forces on each electron must have the same magnitude and act in opposite directions.



electric force = magnetic force  
(upward) (downward)

$$eE = Bev$$

- $E$  is the electric field strength between the parallel plates with a p.d of  $V$ .
- The speed  $v$  of the electrons is simply related to the strengths of the two fields. That is:

$$v = \frac{E}{B}$$

The electric field strength is given by:

$$E = \frac{V}{d}$$

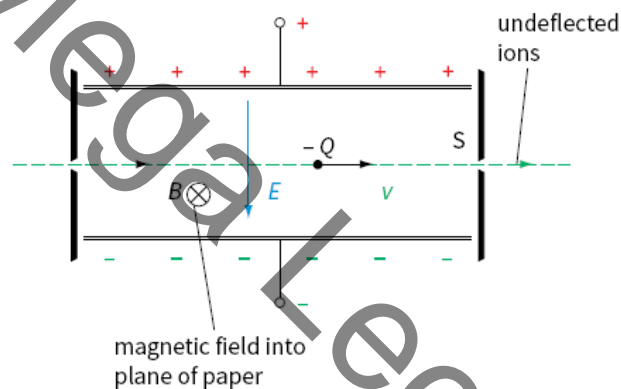
therefore:

$$v = \frac{V}{Bd}$$

What happens when an electron beam passes through an electric field and a magnetic field at the same time?

**Velocity selection:**

- Balancing the effects of electric and magnetic fields is also used in a device called a velocity selector.
- The construction of a velocity selector is shown in Figure
- The apparatus is very similar to the deflection tube
- Two parallel plates are situated in an evacuated chamber.
- They provide a uniform electric field of strength  $E$ .



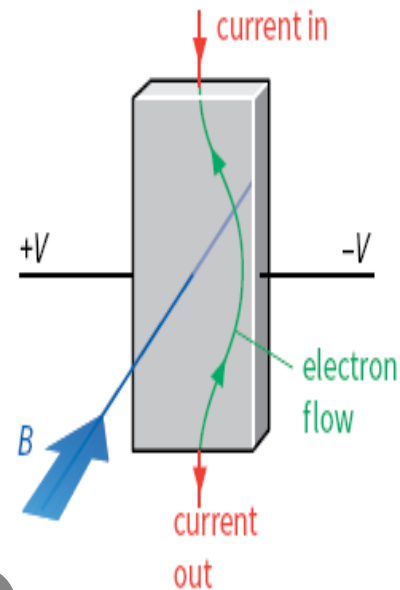
A velocity selector – only particles with the correct combination of charge, mass and velocity will emerge through the slit S.

- The region between the plates is also occupied by a uniform magnetic field of flux density  $B$  which is at right angles to the electric field.
- Charged particles (electrons or ions) enter from the left.
- They all have the same charge and mass but are travelling at different speeds.
- The electric force  $Ee$  will be the same on all particles as it does not depend on their speed; however, the magnetic force  $Bev$  will be greater on those particles which are travelling faster.
- Hence, for particles travelling at the desired speed  $v$ , the electric and magnetic forces balance and they emerge undeflected from the slit S.
- If a negative ion has a speed greater than  $v/Bd$  the downward magnetic force on it will be greater than the upward electric force.
- Thus it will be deflected downwards and it will hit below slit S.



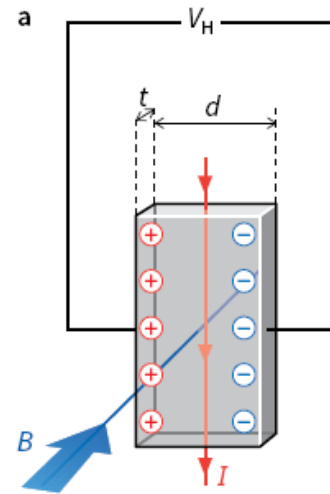
## The Hall effect

- The Hall effect is another mechanism in which the magnetic and electric forces on a moving charged particle are balanced.
- The probe itself is made of semiconductor. This material is used because the electrons move much faster in a semiconductor than in a metal for a given current, and so the effect is much greater.
- The mean drift velocity of free electrons in a semiconductor is perhaps a million times greater than in a metal because there are many fewer free electrons in a semiconductor.
- A small current flows through the probe from one end to the other.
- When a magnetic field is applied, the electrons are pushed sideways by the magnetic force, so that they accumulate along one side of the probe (the right-hand side in Figure.)
- This is the **Hall effect**.
- The charge is detected as a small voltage across the probe, known as the **Hall voltage**.
- The greater the flux density of the field, the greater the Hall voltage
- If the direction of the magnetic field is reversed, the electrons are pushed in the opposite direction and so the Hall voltage is reversed.



**Equation for the Hall voltage:**

- The Hall voltage is the voltage which appears between the two opposite sides of the slice.
- The voltage arises because electrons accumulate on one side of the Hall probe.
- There is a corresponding lack of electrons on the opposite side, i.e. a positive charge.
- As a result, there is an electric field between the two sides.
- The electric field strength  $E$  is related to the Hall voltage  $V_H$  by:

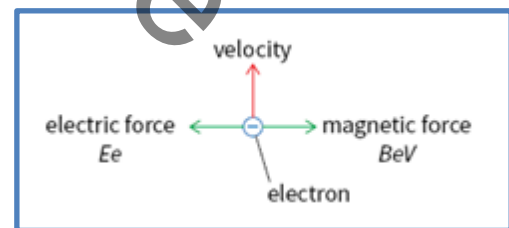


$$E = \frac{V_H}{d}$$

$d$  is the width of the slice.

- When the current first starts to flow, there is no Hall voltage and so an electron will be pushed to the right by the magnetic force.
- However, as the charge on the right-hand side builds up, so does the electric field and this pushes the electron in the opposite direction to the magnetic force.
- Soon an equilibrium is reached.
- The resultant force on this moving electron is zero so that no more charge accumulates.

Now we can equate the two forces:



$$eE = Bev$$

Substituting for  $E$  we have:

$$\frac{eV_H}{d} = Bev$$

- But current  $I$  is related to the mean drift velocity of electrons by

$$I = nAve$$

- $A$  is the cross-sectional area of the conductor
- $n$  is the number density of conducting particles (in this case electrons).

So we can substitute for  $v$  to get:

$$\frac{eV_H}{d} = \frac{BeI}{nAe}$$

Or

$$V_H = \frac{BId}{nAe}$$

- But the area of the side face of the conductor  $A = d \times t$
- $t$  is the thickness of the slice.

Substituting and cancelling gives:

$$V_H = \frac{BI}{nte}$$

- This equation for the Hall voltage shows that  $V_H$  is directly proportional to the magnetic flux density  $B$ .
- That is what makes the Hall effect so useful for measuring  $B$  fields.