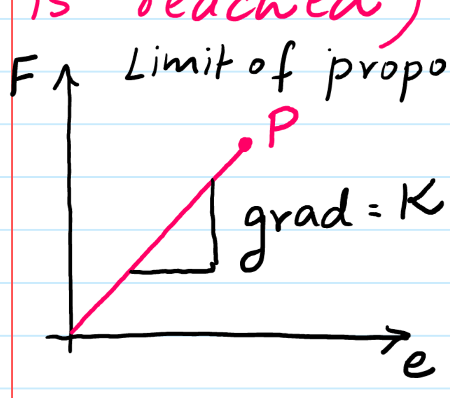
Deformation of solids

08 December 2020 08:24

Hooke's Law.

$F \propto e$ (until the limit of proportionality (P) is reached)



$F \propto e$

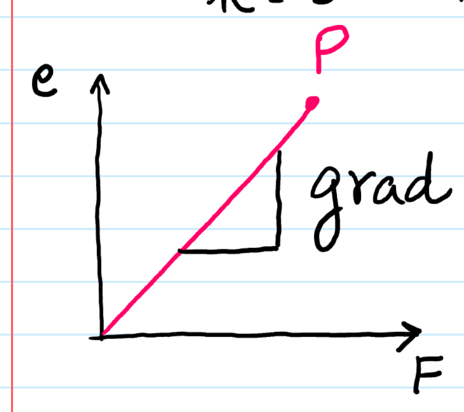
$\boxed{F = Ke}$ where k is a constant which is known as force Constant or Spring Constant *

- k defination :: $k = \frac{F}{e}$ [force per unit extension]
  units :: Nm^{-1}

* $k = 50\,Ncm^{-1}$ How to convert into Nm^{-1}

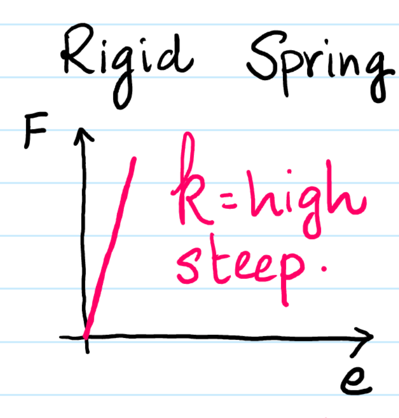
multiply by 100 [to Convert Ncm^{-1} into Nm^{-1}].

$k = 5000\,Nm^{-1}$



grad = $\frac{1}{k}$.

① Rigid Spring



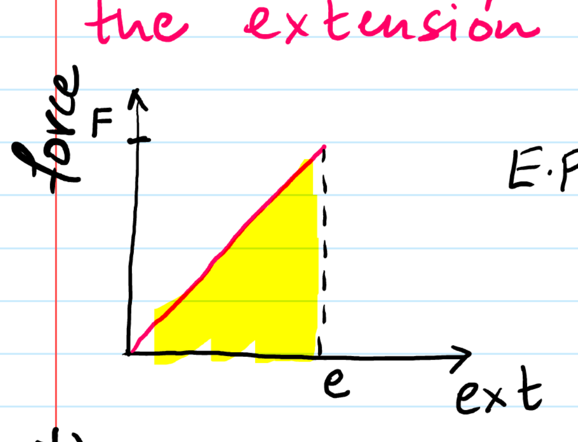
③ Flexible Spring.

Concept of "Energy stored by a Spring"

Whenever a spring is either stretched or compressed, it stores energy.

This energy is called Elastic Potential Energy, Strain Energy or Work done by the spring.

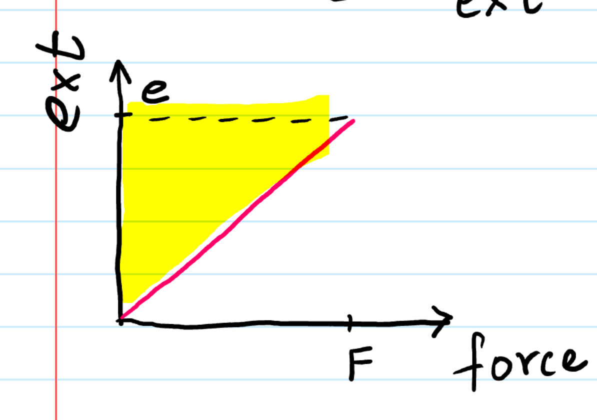
This energy can be obtained from the Area b/w the graph and the extension Axis



E.P.E / Strain Energy / W.done by the Spring

$$\boxed{EPE = \frac{1}{2} \cdot F \cdot e}$$

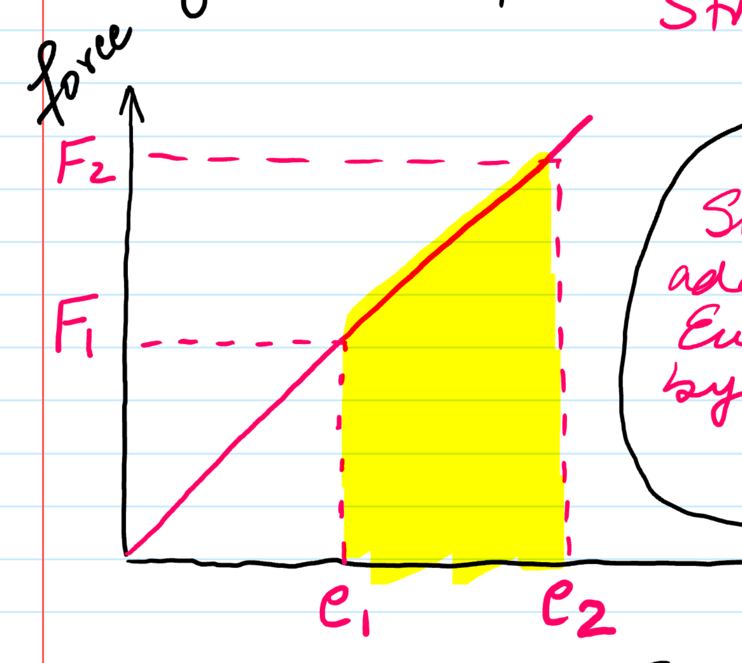
(ke) Since $F = Ke$



$$\boxed{EPE = \frac{1}{2}ke^2.}$$

Concept of "Additional Strain Energy"

Consider a material which has undergone an initial extension e_1 when a force of F_1 has been applied. If the force increases to F_2 & the corresponding extension is represented by e_2, than the energy stored ONLY during the SECOND STAGE is given a special name i.e Additional Strain Energy.



Show that additional Strain Energy is given by the formula $\frac{1}{2}k(e_2^2 - e_1^2)$

Additional Strain Energy = $A_{Trapezium}$

$$\frac{1}{2} \times (F_1 + F_2) \times (e_2 - e_1)$$

Since $F = ke$ (Hooke's Law) $\therefore$ $F_1 = ke_1$ & $F_2 = ke_2$ replace

$$\frac{1}{2} \times (ke_1 + ke_2) \times (e_2 - e_1)$$

$$\frac{1}{2}k(e_1 + e_2)(e_2 - e_1)$$ Simplify the brackets to get

$$\boxed{\text{Additional Strain Energy} = \frac{1}{2}k(e_2^2 - e_1^2)}$$

Ex.1 Spring $k = 30\,Ncm^{-1}$ [$k = 3000\,Nm^{-1}$)

$e = 5cm$ to $e = 7cm$
$e = 0.05m$ $e = 0.07m$

Cal Additional Strain Energy

$$\frac{1}{2}k(e_2^2 - e_1^2)$$

$$\frac{1}{2}(3000)(0.07^2 - 0.05^2)$$

$$= +3.6\,J$$ Energy Gain

Ex. 2 Spring $k = 40\,Ncm^{-1}$

$e = 6cm$ to $e = 4cm$

Can we still use the term additional Strain Energy?

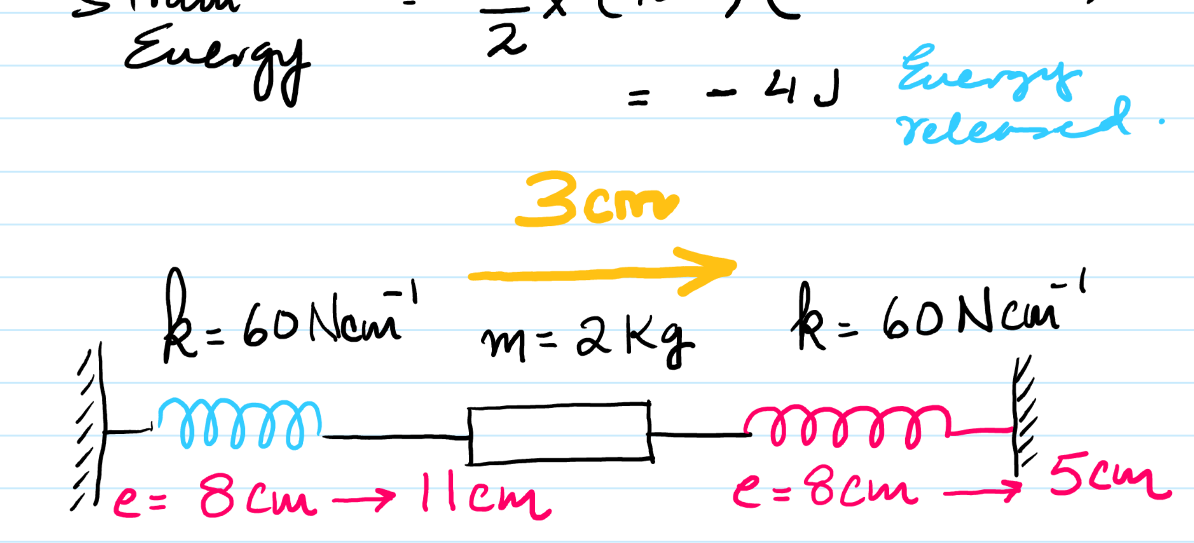
Yes, although keep in mind that your answer will turn out to be negative?

What is the significance of negative answer?

Additional Strain Energy $= \frac{1}{2} \times (4000)(0.04^2 - 0.06^2)$

$= -4\,J$ Energy released.

Q



The mass is pulled by 3cm to the right & than released.

Cal. the Total Change in E.P.E?

BLUE	RED
$\frac{1}{2} \times 6000 \times (0.11^2 - 0.08^2)$	$\frac{1}{2} \times 6000 \times (0.05^2 - 0.08^2)$
$= +17.1\,J$	$= -11.7$

Net Gain in E.P.E = 5.4 J

Given that all of this energy is converted into the K.E of the block, cal the initial speed with which this block begins to move

$$\frac{1}{2}mv^2 = 5.4\,J$$

$$\frac{1}{2}(2)v^2 = 5.4$$

$$v = 2.3\,m/s.$$

Describe the motion of this object?

Oscillatory motion about its mean position.

Q::

Spring P	Spring Q
F	F
k	3k
$e = \frac{F}{k}$	$e = \frac{F}{3k}$

find Ratio of.

$$\frac{\text{Strain Energy in P}}{\text{Strain Energy in Q}} = \frac{\frac{1}{2} \times F \times \frac{F}{k}}{\frac{1}{2} \times F \times \frac{F}{3k}}$$

Strain Energy = $\frac{1}{2}F \cdot e$ or $\frac{1}{2}ke^2$

$$= \frac{3}{1}$$ Ans.

Q

Spring P	Spring Q
5F	7F
3K, $e = \frac{5F}{3K}$	$e = \frac{7F}{8K}$, 8K

Ratio of $\frac{\text{Strain Energy in P}}{\text{Strain Energy in Q}} = \frac{\frac{1}{2} \times 5F \times \frac{5F}{3K}}{\frac{1}{2} \times 7F \times \frac{7F}{8K}}$

either use $\frac{1}{2}F.e$ or $\frac{1}{2}ke^2$

$$= \frac{25}{6} \times \frac{16}{49}$$

$$= \frac{200}{147}$$ Ans.