Name: $\qquad$

## 8 Gravitational fields

Forces due to gravity are a familiar experience. These experiences are formalised in an understanding of the concept of a gravitational field and in Newton's law of gravitation.
Gravitational forces, along with gravitational potential, enable a study to be made of the circular orbits of planets and satellites.

## Learning outcomes

Candidates should be able to:

8.3 Gravitational field of
a point mass
a) derive, from Newtons law of gravitation and the definition of gravitational field strength, the equation $g=\frac{G M}{r^{2}}$ for the gravitational field strength of a point mass
b) recall and solve problems using the)equation $g=\frac{G M}{r^{2}}$ for the gravitational field strength of a point mass
c) show an appreciation that on the surface of the Earth $g$ is approximately constant
8.4 Gravitational potential
a) define potential at a point as the work done per unit mass in bringing a small test mass from infinity to the point
b) solve problems using the equation $\phi=-\frac{G M}{r}$ for the potential in the field of a point mass

- We can represent the Earth's gravitational field by drawing field lines, as shown in Figure.
- The arrows on the field lines show us the direction of the gravitational force on a mass placed in the field.
- The spacing of the field lines indicates the strength of the gravitational field - the further apart they are, the weaker the field.
- This is shown by the greater separation between the field lines.
- The Earth is almost a uniform spherical mass it bulges a bit at the equator.
- The gravitational field of the Earth is as if its entire mass was concentrated at its centre.
- As far as any object beyond the Earth's
 surface is concerned, the Earth behaves as a point mass.
- We describe the Earth's gravitational field as radial, since the field lines diverge (spread out) radially from the centre of the Earth.
- However, on the scale of a building, the gravitational field is uniform, since the field lines are equally spaced.
- Jupiter is a more massive planet than the Earth and so we would represent its gravitational field by showing more closely spaced field lines.


## Newton's law of gravitation:

Any two point masses attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of their separation.
*The force is attractive, so F is in the opposite direction to $r$.


Two point masses separated by distance $r$.

Derive, $\quad F=\frac{G M_{1} M_{2}}{R^{2}}$

- If the distance is doubled, the lines are spread out over four times the surface area, so their concentration is reduced to one-quarter.
- This is called an inverse square law.
- We measure distances from the centre of mass of one body to the centre of mass of the other.
- We treat each body as if its mass

were concentrated at one point.
- The two bodies attract each other with equal and opposite forces, as required by Newton's third law of motion.


## Gravitational field strength $g$ :

- The gravitational field strength at a point is the gravitational force exerted per unit mass on a small object placed at that point.
- To make the meaning of $g$ clearer, we should write it as $9.81 \mathrm{~N} \mathrm{~kg}^{-1}$.
- That is, each 1 kg of mass experiences a gravitational force of 9.81 N .
- Since force is a vector quantity, it follows that gravitational field strength is also a vector.

$$
g=\frac{F}{m}
$$

Derive, $\quad g=\frac{G M}{R^{2}}$


- So the gravitational field strength $g$ at a point depends on the mass $M$ of the body causing the field, and the distance $r$ from its centre.
- Gravitational field strength $g$ also has units $\mathrm{m} \mathrm{s}-2$; it is an acceleration. Another name for $g$ is 'acceleration of free fall'.
- Any object that falls freely in a gravitational field has this acceleration, approximately 9.81 m s -2 near the Earth's surface.


## Energy in a gravitational field

- If we use g.p.e. $=m g \Delta h$, we are assuming that an object's gravitational potential energy is zero on the Earth's surface.
- This is fine for many practical purposes but not, for example, if we are considering objects moving through space, far from Earth.
- For these, there is nothing special about the Earth's surface.
- We start by picturing a mass at infinity, that is, at an infinite distance from all other masses.
- We say that here the mass has zero potential energy.
- This is a more convenient way of defining the zero of g.p.e. than using the surface of the Earth.
- Now we picture moving the mass to the point where we want to know its g.p.e.
- The work done on it is equal to the energy transferred to it, i.e. its g.p.e., and that is how we can determine the g.p.e. of a particular mass.


## Gravitational potential:

- The gravitational potential at a point is the work done per unit mass in bringing a mass from infinity to the point.
- For a point mass $M$, we can write an equation for $\phi$ at a distance $r$ from $M$ :

$$
\phi=-\frac{G M}{R}
$$



- Notice the minus sign; gravitational potential is always negative.
- This is because, as a mass is brought towards another mass, its g.p.e. decreases. Since g.p.e. is zero at infinity, it follows that, anywhere else, g.p.e. and potential are less than zero, i.e. they are negative.


## Important:

- Picture a spacecraft coming from a distant star to visit the solar system.
- The variation of the gravitational potential along path is shown in figure.
- We will concentrate on three
 parts of its journey:

1. 

- As the craft approaches the Earth, it is attracted towards it.
- The closer it gets to Earth, the lower its g.p.e. becomes and so the lower its potential.

2. 

- As it moves away from the Earth, it has to work against the pull of the Earth's gravity.
- Its g.p.e. increases and so we can say that the potential increases.
- The Earth's gravitational field creates a giant 'potential well' in space.
- We live at the bottom of that well.

3. 

- As it approaches the Sun, it is attracted into a much deeper well.
- The Sun's mass is much greater than the Earth's and so its pull is much stronger and the potential at its surface is more negative than on the earth surface.

Fields - terminology:

- Field strength tells us about the force on unit mass at a point;
- Potential tells us about potential energy of unit mass at a point.


## Orbiting under gravity

- For an object orbiting a planet, such as an artificial satellite orbiting the Earth, gravity provides the centripetal force which keeps it in orbit.
- This is a simple situation as there is only one force acting on the satellite the gravitational attraction of the Earth.
- The satellite follows a circular path because the gravitational force is at right angles to its velocity.


## Derive equation for the orbital period, $T^{2}=\left(\frac{4 \pi^{2}}{G M}\right) r^{3}$

Or show that The square of the period is directly proportional to the cube of the radius ( $T^{2} \propto r^{3}$ ).

## Orbiting the Earth

- The Earth has one natural satellite - the Moon - and many thousands of artificial satellites - some spacecraft and a lot of debris.
- Each of these satellites uses the Earth's gravitational field to provide the centripetal force that keeps it in orbit.
- In order for a satellite to maintain a particular orbit, it must travel at the correct speed.
- This is given by the equation,

$$
v^{2}=\frac{G M}{R}
$$

- It follows from this equation that, the closer the satellite is to the Earth, the faster it must travef. If it travels too slowly, it will fall down towards the Earth's surface.
- If it travels too quickly, it will move out into a higher orbit.


## Geostationary orbits

- A special type of orbit is one in which a satellite is positioned so that, as it orbits, the Earth rotates below it at the same rate.
- The satellite remains above a fixed point on the Earth's equator.
- This kind of orbit is called a geostationary orbit.
- Geostationary satellites have a lifetime of perhaps ten years.
- They gradually drift out of the correct orbit, so they need a fuel supply for the rocket motors which return them to their geostationary position, and which keep them pointing correctly towards the Earth.
- Eventually they run out of fuel and need to be replaced.
- Satellites in any other orbits move across the sky so that a tracking system is necessary to communicate with them.

Show that, for a satellite to occupy a geostationary orbit, it must be at a distance of 42300 km from the centre of the Earth and at a point directly above the equator.

1 Two small spheres each of mass 20 g hang side by side with their centres 5.00 mm apart. Calculate the gravitational attraction between the two spheres.

2 It is suggested that the mass of a mountain could be measured by the deflection from the vertical of a suspended mass. Figure 18.13 shows the principle.


Figure 18.13 For End-of-chapter Question 2.
a Copy Figure 18.13 and draw arrows to represent the forces acting on the mass. Label the arrows.
b The whole mass of the mountain, $3.8 \times 10^{12} \mathrm{~kg}$, may be considered to act at its centre of mass. Calculate the horizontal force on the mass due to the mountain.
c Compare the force calculated in b with the Earth's gravitational force on the mass.


Figure 18.14 For End-of-chapter Question 3.
a Copy the diagram and add arrows to show the direction of the field.
b Explain why the formula for potential energy gained $(m g \Delta h)$ can be used to find the increase in potential energy when an aircraft climbs to a height of 10000 m , but cannot be used to calculate the increase in potential energy when a spacecraft travels from the Earth's surface to a height of 10000 km .

4 Mercury, the smallest of the eight recognised planets, has a diameter of $4.88 \times 10^{6} \mathrm{~m}$ and a mean density of $5.4 \times 10^{3} \mathrm{kgm}^{-3}$.
a Calculate the gravitational field at its surface.
b A man has a weight of 900 N on the Earth's surface. What would his weight be on the surface of Mercury?
5 Calculate the potential energy of a spacecraft of mass 250 kg when it is 20000 km from the planet Mars.
(Mass of Mars $=6.4 \times 10^{23} \mathrm{~kg}$, radius of Mars $=3.4 \times 10^{6} \mathrm{~m}$.)

6 Ganymede is the largest of Jupiter's moons, with a mass of $1.48 \times 10^{23} \mathrm{~kg}$. It orbits Jupiter with an orbital radius of $1.07 \times 10^{6} \mathrm{~km}$ and it rotates on its own axis with a period of 7.15 days. It has been suggested that to monitor an unmanned landing craft on the surface of Ganymede a geostationary satellite should be placed in orbit around Ganymede.
a Calculate the orbital radius of the proposed geostationary satellite.
b Suggest a difficulty that might be encountered in achieving a geostationary orbit for this moon.
7 The Earth orbits the Sun with a period of 1 year at an orbital radius of $1.50 \times 10^{11} \mathrm{~m}$. Calculate:
a the orbital speed of the Earth
b the centripetal acceleration of the Earth
c the Sun's gravitational field strength at the Earth.
8 The planet Mars has a mass of $6.4 \times 10^{23} \mathrm{~kg}$ and a diameter of 6790 km .
a i Calculate the acceleration due to gravity at the planet's surface.
ii Calculate the gravitational potential at the surface of the planet.
b A rocket is to return some samples of Martian material to Earth. Write down how much energy each kilogram of matter must be given to escape completely from Mars's gravitational field.
c Use you answer to bto show that the minimum speed that the rocket must reach to escape from the gravitational field is $5000 \mathrm{~m} \mathrm{~s}^{1}$.
d Suggest why it has been proposed that, for a successful mission to Mars, the craft that takes the astronauts to Mars will be assembled at a space station in Earth orbit and launched from there, rather than from the Earth's surface
9 a Explain what is meant by the gravitational potential at a point.
b Figure 18.15 shows the gravitationalpotential near a planet of mass $M$ and radius $R$.


Figure 18.15 For End-of-chapter Question 9.
On a copy of the diagram, draw similar curves:
$i$ for a planet of the same radius but of mass $2 M$ - label this $i$.
ii for a planet of the same mass but of radius $2 R$ - label this ii.
c Use the graphs to explain from which of these three planets it would require the least energy to escape.
d Venus has a diameter of 12100 km and a mass of $4.87 \times 10^{24} \mathrm{~kg}$.
Calculate the energy needed to lift one kilogram from the surface of Venus to a space station in orbit 900 km from the surface.

10 a Explain what is meant by the gravitational field strength at a point.
Figure 18.16 shows the dwarf planet, Pluto, and its moon, Charon. These can be considered to be a double planetary system orbiting each other about their joint centre of mass.


Figure 18.16 For End-of-chapter Question 10.
b Calculate the gravitational pull on Charon due to Pluto.
c Use your result to b to calculate Charon's orbital period.
d Explain why Pluto's orbital period must be the same as Charon's.
11 Figure 18.17 shows the variation of the Earth's gravitational field strength with distance from its centre.

a Determine the gravitational field strength at a height equal to $2 R$ above the Earth's surface, where $R$ is the radius of the Earth.
b A satellite is put into an orbit at this height. State the centripetal acceleration of the satellite.
c Calculate the speed at which the satellite must travel to remain in this orbit.
d i Frictional forces mean that the satellite gradually slows down after it has achieved a circular orbit. Draw a diagram of the initial circular orbital path of the satellite, and show the resulting orbit as frictional forces slow the satellite down.
ii Suggest and explain why there is not a continuous bombardment of old satellites colliding with the Earth.

