Name: ____

7 Motion in a circle

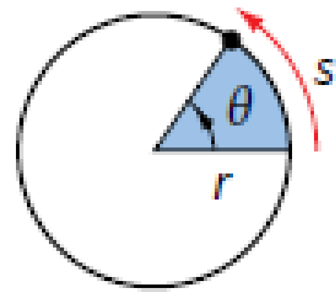
The turning effect of forces is introduced in Topic 5. In this topic, rotational motion, confined to motion in a circle, is studied.

Radian measure is introduced and equations for circular motion are developed, in terms of both angular and linear speeds.

	Learning outcomes Candidates should be able to:
7.1 Kinematics of uniform circular motion	**a) define the radian and express angular displacement in radians** **b) understand and use the concept of angular speed to solve problems** **c) recall and use $v = r\omega$ to solve problems**
7.2 Centripetal acceleration and centripetal force	**a) describe qualitatively motion in a curved path due to a perpendicular force, and understand the centripetal acceleration in the case of uniform motion in a circle** **b) recall and use centripetal acceleration equations $a = r\omega^2$ and $a = \frac{v^2}{r}$** **c) recall and use centripetal force equations $F = mr\omega^2$ and $F = \frac{mv^2}{r}$**

Circular motion:

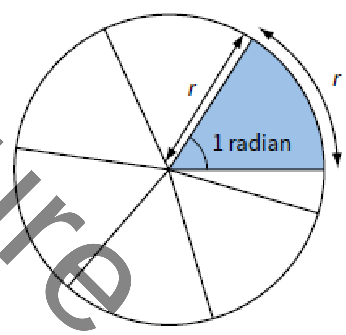
- The angle θ through which the object has moved is known as its angular displacement.
- To know how far an object has moved round the circle, we need to know the angle θ.
- If an object moves a distance ***S*** around a circular path of radius ***r***, its angular displacement θ in radians is defined as follows:



$$Angular\ Displacement\ (in\ Radians) = \frac{Arc\ Length}{Radius\ of\ Circular\ Path}$$

$$\theta = \frac{S}{r}$$

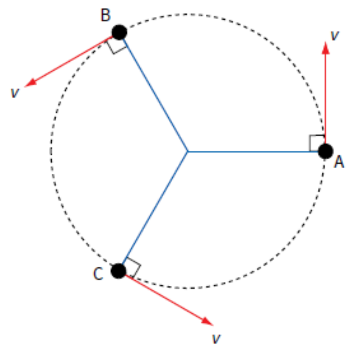
- One radian is the angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.
- An angle of 360° is equivalent to an angle of 2π radians. Or it can be said that there are 2π radians in a full circle.
- 1 radian ≈ 57.3°



Angular Velocity:

- There is an important distinction between speed and velocity: speed is a scalar quantity which has magnitude only, whereas velocity is a vector quantity, with both magnitude and direction.
- The velocity *v* of an object changes direction as it moves along a circular path.

$$Angular\ Velocity = \frac{Angular\ Displacement}{time}$$



$$\omega = \frac{\Delta\theta}{\Delta t}$$

- As the hands of a clock travel steadily around the clock face, their velocity is constantly changing. The minute hand travels round 360° or 2π radians in 3600 seconds. Although its velocity is changing, we can say that its angular velocity is constant, because it moves through the same angle each second.
- For the minute hand of a clock, we have $\omega \approx 0.001\ 75$ rad s^{-1}.

$$Linear\ Speed = Angular\ velocity * \ Radius$$

$$v = r\ \omega$$

- Newton's first law states that an object remains at rest or in a state of uniform motion (at constant speed in a straight line) unless it is acted on by an external force.
- In the case of an object moving at steady speed in a circle, we have a body whose velocity is not constant; therefore, there must be a resultant (unbalanced) force acting on it.
- This force is what we call centripetal force.
- The Centripetal force on the object is always directed towards the centre of the circle.

Example 01

- Consider a rubber bung on the end of a string. Imagine whirling it in a horizontal circle above your head.
- To make it go round in a circle, you have to pull on the string.
- The pull of the string on the bung is the unbalanced force, which is constantly acting to change the bung's velocity as it orbits your head.
- If you let go of the string, suddenly there is no tension in the string and the bung will fly off at a tangent to the circle.

Example 02

- Similarly, as the Earth orbits the Sun, it has a constantly changing velocity.
- Newton's first law suggests that there must be an unbalanced force acting on it.
- That force is the gravitational pull of the Sun.
- If the force disappeared, the Earth would travel off in a straight line.

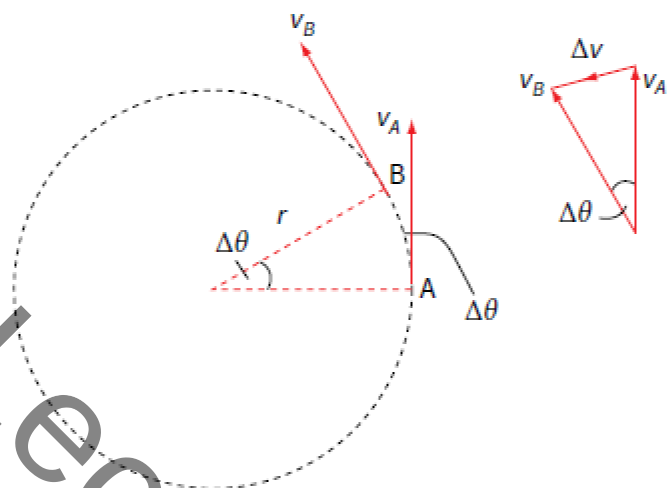
- **The word centripetal is an adjective.**
- We use it to describe a force that is making something travel along a circular path.
- It does not tell us what causes this force, which might be gravitational, electrostatic, magnetic, frictional or whatever.
- If the force is to make the object change its speed, it must have a component in the direction of the object's velocity; it must provide a push in the direction in which the object is already travelling.
- However, here we have a force at 90° to the velocity, so it has no component in the required direction. (Its component in the direction of the velocity is $F \cos 90° = 0$.)
- It acts to pull the object around the circle, without ever making it speed up or slow down.

- The work done by a force is equal to the product of the force and the distance moved by the object in the direction of the force.
- The distance moved by the object in the direction of the centripetal force is zero; hence the work done is zero.
- If no work is done on the object, its kinetic energy must remain the same and hence its speed is unchanged.

Derivation for Centripetal Acceleration and Centripetal Force:

$$a = \frac{v^2}{r}$$

$$a = \omega^2 r$$



Show that orbital speed around earth is nearly 8kms^{-1}:

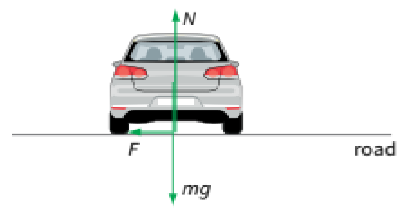
Given: Radius of earth is 6400 km

The origins of centripetal forces:

An object moving along a circular path is not in equilibrium and the resultant force acting on it is the centripetal force.

1. **Consider a car cornering on a level road.**

- Here the road provides two forces.
- The force N is the normal contact force which balances the weight **mg** of the car.
- The car has no acceleration in the vertical direction.
- The second force is the force of friction F between the tyres and the road surface. This is the unbalanced, centripetal force.
- If the road or tyres do not provide enough friction, the car will not go round the bend along the desired path.
- The friction between the tyres and the road provides the centripetal force necessary for the car's circular motion.



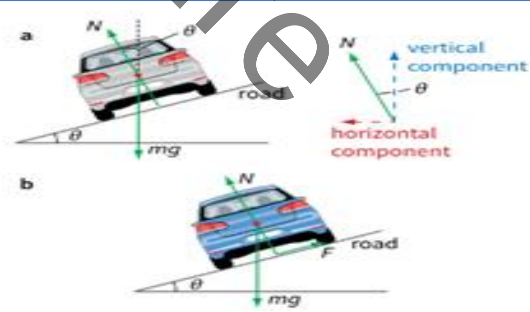
2. **Consider a car cornering on a banked road.**
   - Here, the normal contact force N has a horizontal component which can provide the centripetal force.
   - The vertical component of N balances the car's weight. Therefore:

Vertically,

$$N \cos \theta = mg$$

Horizontally,

$$N \sin \theta = \frac{mv^2}{r}$$



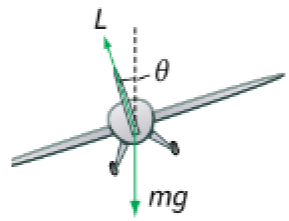
a On a banked road, the horizontal component of the normal contact force from the road can provide the centripetal force needed for cornering. b For a slow car, friction acts up the slope to stop it from sliding down.

- Where r is the radius of the circular corner and v is the car's speed.

- If a car travels around the bend too slowly, it will tend to slide down the slope and friction will act up the slope to keep it on course.
- If it travels too fast, it will tend to slide up the slope.
- If friction is insufficient, it will move up the slope and come off the road.

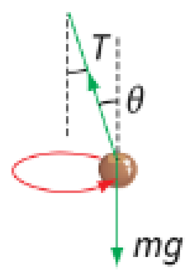
3. <u>An aircraft banking.</u>

- To change direction, the pilot tips the aircraft 's wings.
- The vertical component of the lift force L on the wings balances the weight.
- The horizontal component of L provides the centripetal force.



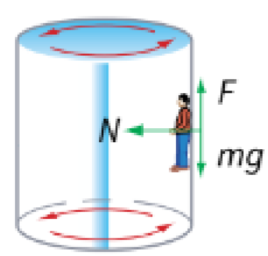
4. <u>A stone being whirled in a horizontal circle on the end of a string.</u>

- This arrangement is known as a conical pendulum.
- The vertical component of the tension T is equal to the weight of the stone.
- The horizontal component of the tension provides the centripetal force for the circular motion.



5. <u>At the fairground, as the cylinder spins, the floor drops away.</u>

- Friction balances your weight.
- The normal contact force of the wall provides the centripetal force.
- You feel as though you are being pushed back against the wall; what you are feeling is the push of the wall on your back.



Question:

Explain why it is impossible to whirl a bung around on the end of a string in such a way that the string remains perfectly horizontal.

Answer:

The tension in the string must have a vertical component to balance the weight of the bung.

Question:

Explain why an aircraft will tend to lose height when banking, unless the pilot increases its speed to provide more lift.

Answer:

In level flight, lift balances the weight. During banking, the vertical component of the lift is less than the weight, so the airplane loses height unless the speed can be increased to provide more lift.

Question:

If you have ever been down a water-slide (a flume) you will know that you tend to slide up the side as you go around a bend. Explain how this provides the centripetal force needed to push you around the bend. Explain why you slide higher if you are going faster.

Answer:

The normal contact force of the wall of the slide has a horizontal component, which provides the centripetal force. If you are going fast, you need a bigger force, so the horizontal component must be greater. This happens as you move up the curved wall of the slide.

Question:

An object follows a circular path at a steady speed. Describe how each of the following quantities changes as it follows this path: speed, velocity, kinetic energy, momentum, centripetal force, centripetal acceleration. (Refer to both magnitude and direction, as appropriate.)

Answer:

Speed and kinetic energy are scalar quantities, the others are vectors. Speed is constant; velocity has a constant magnitude but continuously changing direction (the direction is tangential to the circle); kinetic energy is constant; momentum has a constant magnitude but continuously changing direction (the direction is tangential to the circle); the centripetal force has a constant magnitude but continuously changing direction (the direction is always towards the centre of the circle); the centripetal acceleration behaves in the same way as the centripetal force.

Question:

a By how many degrees does the angular displacement of the hour hand of a clock change each hour?

b A clock is showing 3.30. Calculate the angular displacements in degrees from the 12.00 position of the clock to:

i the minute hand

ii the hour hand.

Answer:

a

Full circular face of clock = 360°

Clock face divided into twelve sections, so angular displacement of hour hand per hour = 360°/12 = 30°

b

i Angular displacement of minute hand = half the clock face = 180°

ii Angular displacement of hour hand = 3.5 × 30° = 105°

Mass swinging in vertical and horizontal circle:

1 **a** Explain what is meant by a **radian**. [1]

b A body moves round a circle at a constant speed and completes one revolution in 15 s. Calculate the angular velocity of the body. [2]

2 Figure 17.17 shows part of the track of a roller-coaster ride in which a truck loops the loop. When the truck is at the position shown, there is no reaction force between the wheels of the truck and the track. The diameter of the loop in the track is 8.0 m.

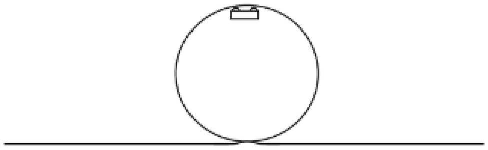


Figure 17.17 For End-of-chapter Question 2.

a Explain what provides the centripetal force to keep the truck moving in a circle. [1]

b Given that the acceleration due to gravity g is $9.8\,\mathrm{m\,s^{-2}}$, calculate the speed of the truck. [3]

3 **a** Describe what is meant by **centripetal force**. [1]

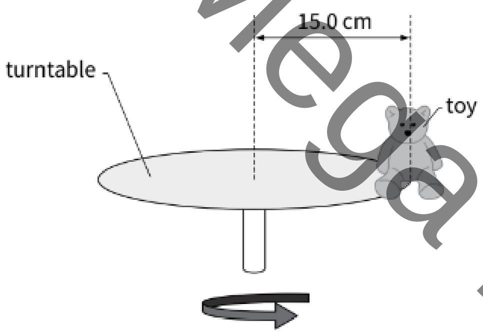


Figure 17.18 For End-of-chapter Question 3.

b Figure 17.18 shows a toy of mass 60 g placed on the edge of a rotating turntable.

i The diameter of the turntable is 15.0 cm. The turntable rotates, making 20 revolutions every minute. Calculate the centripetal force acting on the toy. [4]

ii Explain why the toy falls off when the speed of the turntable is increased. [2]

4 One end of a string is secured to the ceiling and a metal ball of mass 50 g is tied to its other end. The ball is initially at rest in the vertical position. The ball is raised through a vertical height of 70 cm (see Figure 17.19). The ball is then released. It describes a circular arc as it passes through the vertical position.

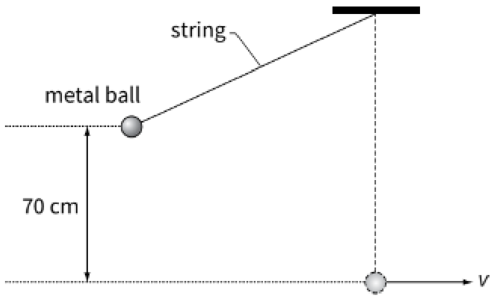


Figure 17.19 For End-of-chapter Question 4.

The length of the string is 1.50 m.

a Ignoring the effects of air resistance, determine the speed *v* of the ball as it passes through the vertical position. [2]

b Calculate the tension *T* in the string when the string is vertical. [4]

c Explain why your answer to **b** is not equal to the weight of the ball. [2]

5 A car is travelling round a bend when it hits a patch of oil. The car slides off the road onto the grass verge. Explain, using your understanding of circular motion, why the car came off the road. [2]

6 Figure 17.20 shows an aeroplane banking to make a horizontal turn. The aeroplane is travelling at a speed of 75 m s^{-1} and the radius of the turning circle is 80 m.

a Copy the diagram. On your copy, draw and label the forces acting on the aeroplane. [2]

b Calculate the angle which the aeroplane makes with the horizontal. [4]

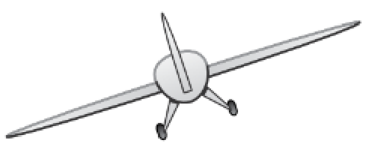


Figure 17.20 For End-of-chapter Question 6.

7 **a** Explain what is meant by the term **angular velocity**. [2]

b Figure 17.21 shows a rubber bung, of mass 200 g, on the end of a length of string being swung in a horizontal circle of radius 40 cm. The string makes an angle of 56° with the vertical.

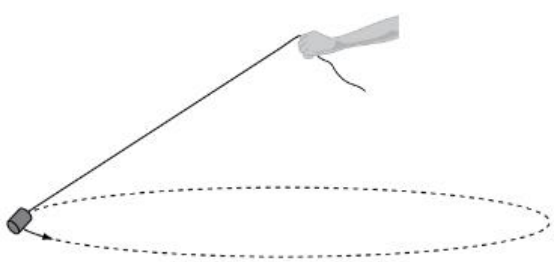


Figure 17.21 For End-of-chapter Question 7.

Calculate:

i the tension in the string [2]

ii the angular velocity of the bung [3]

iii the time it takes to make one complete revolution. [1]

8 **a** Explain what is meant by a **centripetal force**. [2]

b A teacher swings a bucket of water, of total mass 5.4 kg, round in a vertical circle of diameter 1.8 m.

i Calculate the minimum speed which the bucket must be swung at so that the water remains in the bucket at the top of the circle. [3]

ii Assuming that the speed remains constant, what will be the force on the teacher's hand when the bucket is at the bottom of the circle? [2]