Orbital Motion

Historical Background

Johannes Kepler was a German mathematician and astronomer. In 1600 he became the assistant of the Danish astronomer Tycho Brahe (1546-1601). Brahe was the foremost observational astronomer of his day, and had compiled the most accurate star atlas of the day. The remarkable thing about Brahe was that he carried out all his measurements without the benefit of a telescope. Telescopes, of course, don't allow an astronomer to see a star more clearly, or close up, but it does allow him to make more accurate observations than is possible with the naked eye. Brahe got around the limitations of naked eye astronomy by using huge instruments that had scales that could be subdivided down to minutes of arc.

The first person to use a telescope for astronomical observation was Galileo Gallilei (1564-1642) in 1610. Galileo constructed his own telescopes and did not make measurements with them. Instead he used it to make discoveries such as that Jupiter has four moons, that the moon's surface is irregular and mountainous, and that Venus goes through a cycle of phases like those of the Moon.

Kepler's analysis of Brahe's data led him to discover that the orbit of a planet about the Sun is an ellipse, and not circular, as astronomers of his day believed.

He formulated three laws that are now known as Kepler's Laws of planetary motion.

1. the orbit of each planet is an ellipse with the Sun at one focus
2. the line joining the Sun to the planet sweeps out equal areas in equal times.
3. the square of the period of the planet orbital motion is proportional to the cube of its mean distance from the Sun.

Kepler's law's are empirical. That is to say they are based on data alone; they are not derived from a general theory. Kepler's laws can be derived from Isaac Newton's laws of motion and of Universal Gravitation. Newton was born in 1642, and published his ideas about force, motion & gravity in *Principia Mathematica* in 1687. '*Principia*', as it is known, is considered to be the most important scientific book ever published.

Largest objects in Solar System

Name	Orbits	Orbital radius (10^3 km)	Orbital Period	Diameter (km)	Mass(kg)
Sun				1,394,000	1.99×10^{30}
Jupiter	Sun	778,000	11.86 yrs	142,840	1.90×10^{27}
Saturn	Sun	1,429,000	29.46 yrs	120,540	5.69×10^{26}
Uranus	Sun	2,870,990	84 yrs	51,120	8.69×10^{25} *
Neptune	Sun	4,504,300	164.78 yrs	49,530	1.02×10^{26} *
Earth	Sun	149,600	1 yr	12,760	5.98×10^{24}
Venus	Sun	108,200	224.5 days	12,100	4.87×10^{24}
Mars	Sun	227,940	1.88 yrs	6,800	6.42×10^{23}
Ganymede	Jupiter	1,070	7.16 days	5,260	1.48×10^{23}
Titan	Saturn	1,222	15.95 days	5,150	1.35×10^{23}
Mercury	Sun	57,910	88 days	4,880	3.30×10^{23}
Callisto	Jupiter	1,883	16.69 days	4,800	1.08×10^{23}
Io	Jupiter	422	1.77 days	3,630	8.93×10^{22}
Moon	Earth	384	27.3 days	3,480	7.35×10^{22}
Europa	Jupiter	671	3,55 days	3,140	4.80×10^{22}
Triton	Neptune	355	5.88 days	2,710	2.15×10^{22}
Pluto	Sun	5,913,520	248.5 yrs	2,390	1.32×10^{22}
Titania	Uranus	436	8.71 days	1,610	3.5×10^{21}
Oberon	Uranus	583	13.46 days	1,550	3.0×10^{21}
Rhea	Saturn	527	4.52 days	1,530	2.3×10^{21}
Iapetus	Saturn	3,561	79.33 days	1,460	1.6×10^{21}
Charon	Pluto	19.6	6.39 days	1,168	1.9×10^{21}
Umbriel	Uranus	266	4,14 days	1,190	1.2×10^{21}
Ariel	Uranus	191	2.52 days	1,160	1.3×10^{21}
Dione	Saturn	377	2.74 days	1,120	1.1×10^{21}
Tethys	Saturn	295	1.89 days	1,060	0.6×10^{21}
Ceres	Sun	415,000	4.6 yrs	950	8.7×10^{20}

- **Note**: Neptune is slightly denser than Uranus.

For further information:
http://nssdc.gsfc.nasa.gov/planetary/planetfact.html

Solar system data obtained from NASA

Planet	Mass / kg	Radius / m	Density / kg m^{-3}	g / ms^{-2}	g/g_{Earth}
Mercury	3.30×10^{23}	2.44×10^{6}	5.42×10^{3}	3.70	0.38
Venus	4.87×10^{24}	6.05×10^{6}	5.25×10^{3}	8.87	0.90
Earth	5.97×10^{24}	6.37×10^{6}	5.51×10^{3}	9.81	1.00
Mars	6.42×10^{23}	3.39×10^{6}	3.93×10^{3}	3.73	0.38
Jupiter	1.90×10^{27}	6.99×10^{7}	1.33×10^{3}	25.94	2.64
Saturn	5.68×10^{26}	5.82×10^{7}	6.88×10^{2}	11.18	1.14
Uranus	8.68×10^{25}	2.54×10^{7}	1.26×10^{3}	8.97	0.91
Neptune	1.02×10^{26}	2.64×10^{7}	1.32×10^{3}	9.76	1.00
Pluto	1.25×10^{22}	1.14×10^{6}	2.03×10^{3}	0.64	0.07
Moon	7.35×10^{22}	1.74×10^{6}	3.33×10^{3}	1.62	0.17

Asteroid	Mass / kg	Radius / m	Density / kg m^{-3}	g / ms^{-2}	g/g_{Earth}
Ceres	8.70×10^{20}	4.73×10^{5}	1.96×10^{3}	0.26	0.026
Pallas	3.18×10^{20}	2.63×10^{5}	4.18×10^{3}	0.31	0.031
Juno	2.00×10^{19}	1.20×10^{5}	2.76×10^{3}	0.09	0.009
Vesta	3.00×10^{20}	2.65×10^{5}	3.85×10^{3}	0.28	0.029
Chiron	4.00×10^{18}	9.00×10^{4}	1.31×10^{3}	0.03	0.003

You may assume the following values for the introductory problems *unless* specified to the contrary in the question.

- Earth's mass, $M_E = 6.0 \times 10^{24}$ kg
- Earth's radius, $R_E = 6.4 \times 10^{6}$m
- Gravitational acceleration on the Earth's surface, $g_E = 9.8$ ms^{-2}
- Universal gravitational constant, $G = 6.7 \times 10^{-11}$ Nm^2kg^{-1}

ORBITAL MOTION

1 In principle a satellite could orbit 5 km above the Moon's surface, but not 5 km above the Earth's surface.

(a) Explain this.

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(b) Are there any limitations to how close to the lunar surface a satellite can orbit?

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(c) Calculate the orbital period (i.e. time for one orbit) of a satellite's orbit were just to skim the Earth's surface.

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(d) A real satellite might orbit 600 km above the Earth's Surface. Calculate the orbital period of such a satellite.

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(e) Calculate the orbital period of a satellite that just skims the Moon's surface.

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2 Show that the velocity, v, of a satellite in a circular orbit of radius, r, is given by $v = \sqrt{\dfrac{GM}{r}}$

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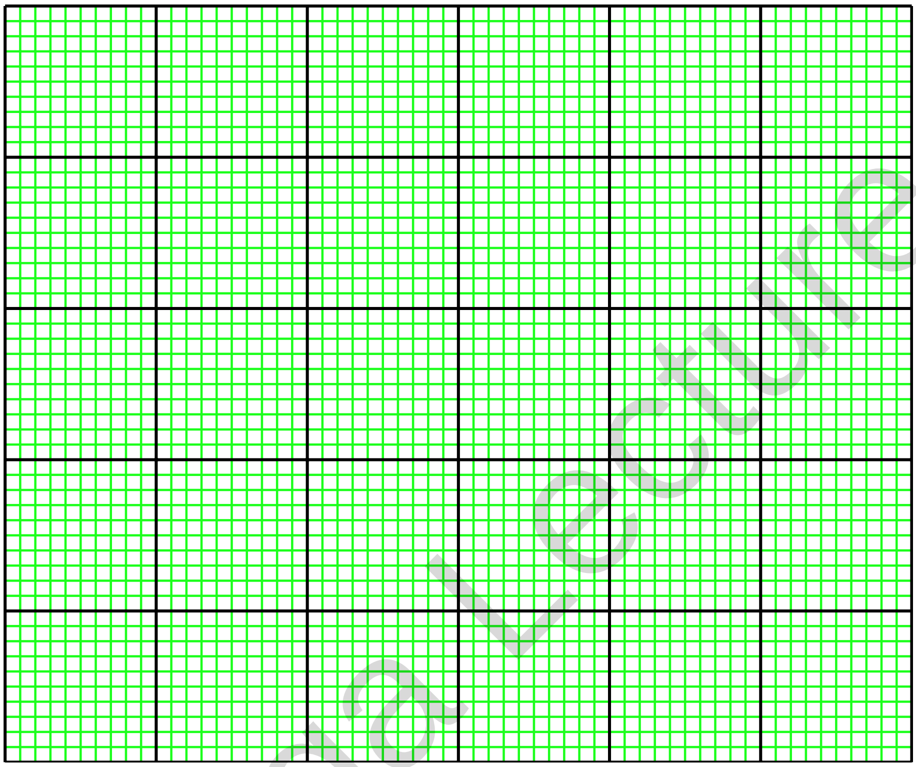
(b) Plot a graph of v against r for r = 0 to 7,000 km.



(c) Two spacecraft are orbiting Earth in the same direction. One is 300 km above the Earth's surface and the other one is 320 km above the Earth's surface. Which one is travelling fastest? Explain.

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3 Suppose you look up one evening and see two satellites passing overhead. As they pass directly overhead you notice that one (call it *A*) overtakes the other (*B*). What can you deduce from your observations about the height of these two satellites above the Earth's surface. Explain your reasoning.

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4 Given that Earth's orbital period is 365.25 days, calculate a value for the mass of the Sun to 3 d.p. The Earth's orbital radius is 1.5×10^8km.

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5 (a) Show that it is **not** necessary to know the mass of a satellite in order to calculate its orbital period about a primary body. Show that it is necessary to know the mass of the primary body. Your answer must include an explanation, in words, of the significance of the expression that you have derived. The formula alone is not enough.

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(b) Four of Jupiter's largest Moon's were discovered by Galileo Gallilei in the winter of 1609/10. The table below gives the mean orbital radius and period for each satellite.
Use this data in a spread sheet to verify the relationship you have derived above.

Name	**Orbital period / days**	**Orbital radius / 10^8 m**		
Io	1.77	4.22		
Europa	3.55	6.71		
Ganymede	7.16	10.7		
Callisto	16.7	18.8		

6 Calculate the mass of Jupiter to 3 d.p. from the orbital data for Europa given in question 5.

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7 What is the orbital radius of a hypothetical satellite of Jupiter that has the same orbital period as our Moon? The mass of Jupiter is 1.90×10^{27} kg. Take the Moon's orbital period as 27.3 days.

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8 (a) Phobos is one of Mars' satellites. Its orbital radius is 9.4×10^6 m and its orbital period is 7 hr 39 min. Calculate a value for the mass of Mars from these figures.

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(b) The diameter of Mars is 6800 km. Using the answer to (a) calculate

(i) a value for the gravitational field strengh on its surface.

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(ii) the weight of someone on the surface of Mars if their mass is 70 kg.

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9 An astronaut has a mass of 85 kg. Calculate the pull of the Earth's gravity on him when he is
(a) on the Earth's surface and

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(b) in orbit 600 km above the Earth's surface.

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(c) Given the answers to (a) and (b) explain why he feels weightless when he is orbit.

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(d) Calculate the orbital period for the astronaut in this orbit.

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To answer these questions you may use G, as well as the mass and radius of Earth, but you may ***not*** *use Earth's gravitational field strength,* g_E.

10 (a) What is a geosynchronous orbit?

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(b) What is the orbital radius of a satellite in a geosynchronous orbit?

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(c) What is the highest latitude from which a geosynchronous satellite is visible from the Earth's surface? Is this of any significance?

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(d) What would the orbital period and radius be of a similar satellite in orbit about (i) Mars, (ii) Jupiter (iii) Pluto?

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(ii) ____

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(iii) ____

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(e) (i) Do any of these planets have moons in geosynchonous orbits? Pay particular attention of Pluto.

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(ii) What would the generic name of such satellites be?

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11 During the Apollo missions to the Moon, the Command Module remained in orbit while the Lunar Module landed on the Moon. The Command Module orbited at an altitude of 100 km above the lunar surface. Calculate the orbital period of the Command Module.

Mass of Moon 7.4×10^{22} kg, radius of Moon = 1.725×10^{6} m.

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12 The mass of Pluto was not known until 1978 when its natural satellite, Charon, was discovered.

(i) What data did this discovery provide that enabled astronomers to calculate the mass of Pluto?

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(ii) Look this data up in the table on page 3, and calculate a mass for Pluto.

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13 A spy satellite is placed in a polar orbit 270 km above the Earth's surface.

(a) Calculate its orbital period.

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(b) How many times does the satellite orbit the Earth in one day?

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(c) The orbital plane of the satellite remains constant with respect to the fixed stars. Explain why.

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(d) Because the orbital plane remains fixed, the Earth rotates beneath the satellite. Calculate the position of a point on the equator directly beneath the satellite after one orbit relative to the previous orbit. Is this to the east or the west of the point beneath the satellite during the previous orbit? Explain your answer.

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14 (a) Derive a relationship between the period of a satellite in close orbit about a planet and the mean density of the planet. Assume that 'close orbit' means that a satellite just skims the planet's surface.

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(b) (i) Use a spreadsheet to complete the following table, using the relationship you have derived in (a).

Density / kg m^{-3}	**1/density / m^3 kg^{-1}**	**Square of period / s^2**
1000		
2000		
3000		
4000		
5000		
6000		

Use the space below for calculations if necessary.

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(ii) Plot a graph of the square of the period (on y-axis) against the inverse of density (on x axis). Start both axes from zero. [4]

(c) Use the graph to answer the following questions

(i) The orbital period for a satellite in close Earth orbit is 5100 seconds. What is Earth's average density?

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(ii) A satellite placed in close orbit about Saturn is observed to have a period of 3.95 hours. What is Saturn's average density?

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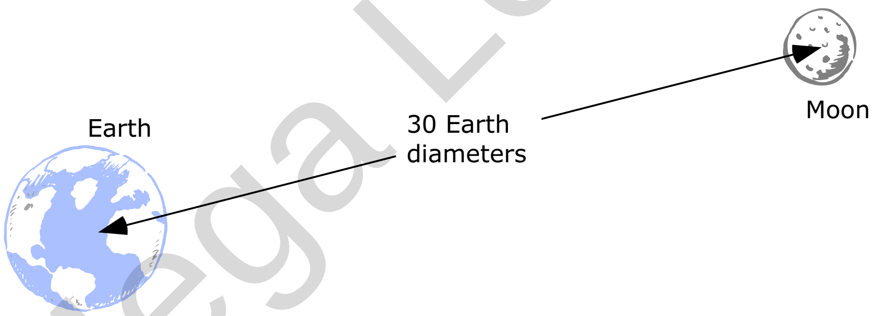
15 To verify whether gravitational attraction is an inverse square law Newton carried out two parallel calculations to determine the Moon's centripetal acceleration. First of all he calculated a value for this based on the Moon's orbital period and the average centre-to-centre distance between it and the Earth. He then calculated a value for the acceleration due to the Earth's gravity at a distance equal to the Moon's orbital radius on the assumption that this would vary as the inverse square of that distance. By comparing the two he hoped to show the correctness of his hunch.



(a) Use the data given at the end of the problem to calculate

(i) the centripetal acceleration of the Moon towards the Earth based upon its orbital period

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(ii) the acceleration of the Moon towards the Earth due to the Earth's gravitational pull based on Newton's guess about the variation of gravitational attraction with distance.

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(b) How could your answers to (i) and (ii) be used to support Newton's idea about gravity?

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(c) Newton was not able to use his gravitational theory to calculate the mass of either the Earth or the Moon. We can! What do we know that Newton did not?

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(d) Show how, *in principle*, the information that Newton lacked can be used to calculate a value for the Earth's mass.

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Moon's Orbital Period: 27.3 days
Earth's radius: 6400 km
Earth-Moon distance (centre to centre): 30 Earth diameters

16 Describe as fully as you can what will happen if an astronaut in orbit about the Earth fires a pistol. Assume that the astronaut is wearing a spacesuit and is not inside a spacecraft. What, if anything, happens to (a) the astronaut, and (b) the bullet? Assume values for speeds and masses.

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17 (a) Show that the velocity, v, of a body in freefall orbit is inversely proportional to the inverse square root of its orbital radius, R. (i.e. that $v \propto \sqrt{1/R}$)

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(b) Orbital rendezvous between spacecraft are complicated by the fact that if a rocket is to catch up with the one ahead of it, the speed of the trailing rocket must be reduced, not increased. Paradoxically, if the speed of the trailing rocket is increased, the distance between it and the lead rocket will actually get larger. Explain. Use a diagram to illustrate your answer.

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18 Is it possible to calculate the altitude of a satellite merely by determining its angular velocity as it moves across the sky by measuring the time it takes to travel from horizon to horizon (or the time it takes to move through a known angle-e.g. the width of the fist of one's outstretched arm)? Support your answer with suitable calculations.

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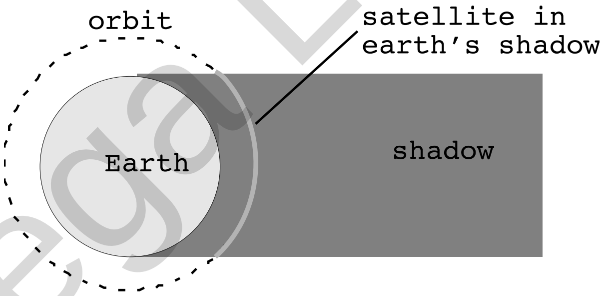
19 Estimate, with the help of a diagram and a simple calculation, the time for which an Earth satellite in within the Earth's shadow. Assume that the satellite orbits 300 km above the Earth's surface, and that sunlight is parallel. The diagram is not to scale.



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Answers to ORBITAL MOTION

1 (a) No atmosphere on the Moon, therefore free fall motion very close to Lunar surface is possible. On Earth, atmospheric drag alone makes this impossible.

(b) Limitations are that the altitude of the satellite must be high enough to clear lunar mountains, and to allow for orbital perturbations due to MASCONS, places beneath lunar surface where density is higher than surrounds.

(c) Orbital period of satellite in close earth orbit =

(d) Orbital period of satellite in earth orbit 600 km above earth's surface =

(e) Orbital period of satellite in close lunar orbit =

2 (a)

(b) The one at 300km altitude

3 A is closer to earth than B because orbital velocity, v = √GM/R. i.e. smaller R, larger v

4 Mass of Sun = 1.997×10^{30} kg

5 (a)

Use this data in a spreadsheet to verify the relationship you have derived above.

Name	Orbital period / days	Orbital radius / 10^8 m	T^2	R^3
Io	1.77	4.22		
Europa	3.55	6.71		
Ganymede	7.16	10.7		
Callisto	16.7	18.8		

Graph of R^3 against T^2 is a straight line

6 Mass of Jupiter = 1.892×10^{27} kg

7 Orbital radius of hypothetical satellite = 2.62×10^6 km.

8 (a) Mass of Mars = 6.453×10^{23} kg

(b) (i) g_{Mars} = 3.74 ms^{-2} , (ii) Weight of 70 kg astronaut = 216.8 N

9 (a) Weight on earth's surface = 833.9 N

(b) Weight in orbit = 697.4 N

(c) Weightlessness………..

(d) Orbital period = 5803.8 s seconds

10 (a) The orbital period of a geosynchronous satellite is equal to that of the Earth's rotational period. A satellite in such an orbit always remains above the same point on the Earth's surface.

(b) Orbital radius of satellite = 42,302 km

(c)

(d) (i) Mars = , (ii) Jupiter = , (iii) Pluto =

(e) (i) Charon is in a geosynchronous orbit about Pluto. (ii) ????

11 Orbital period of lunar module: 6953 seconds = 115.9 min = 1 hr 56 min

12 (a) Orbital period (6.39 days) and orbital radius of satellite (19.6×10^3 km)

(b) Mass of Pluto = 1.45×10^{22} kg.

13 (a) 5402.3 seconds

(b) 16 times per day

(c) Inertia-no force acting perpendicularly to the satellites orbit that would make it rotate with the earth.

(d) 2500 km West

14 (a) $T^2 = 3\pi/GD$

(b) (i)

Density	1/density	T^2
1000	1×10^3	1.41×10^8
2000	0.5×10^3	0.70×10^8
3000	0.33×10^3	0.47×10^8
4000	0.25×10^3	0.35×10^8
5000	0.2×10^3	0.28×10^8
6000	0.17×10^3	0.23×10^8

(ii) density of Earth = 5400 kg m^{-3}

(iii) density of Saturn = 690 kg m^{-3}

15 (i) 2.7×10^{-3} ms^{-2}

(ii) 2.7×10^{-3} ms^{-2}

(iii) Values for Moon's centripetal acceleration arrived at independently of one another

(iv) We know the value of G, Universal gravitational constant.

(v)

16 The relationship between If the speed of a body in a freefall orbit is increased it cannot remain in that orbit. What happens is that it climbs to a higher orbit by converting the increase in k.e. into g.pe.

17 Satellite paradox

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