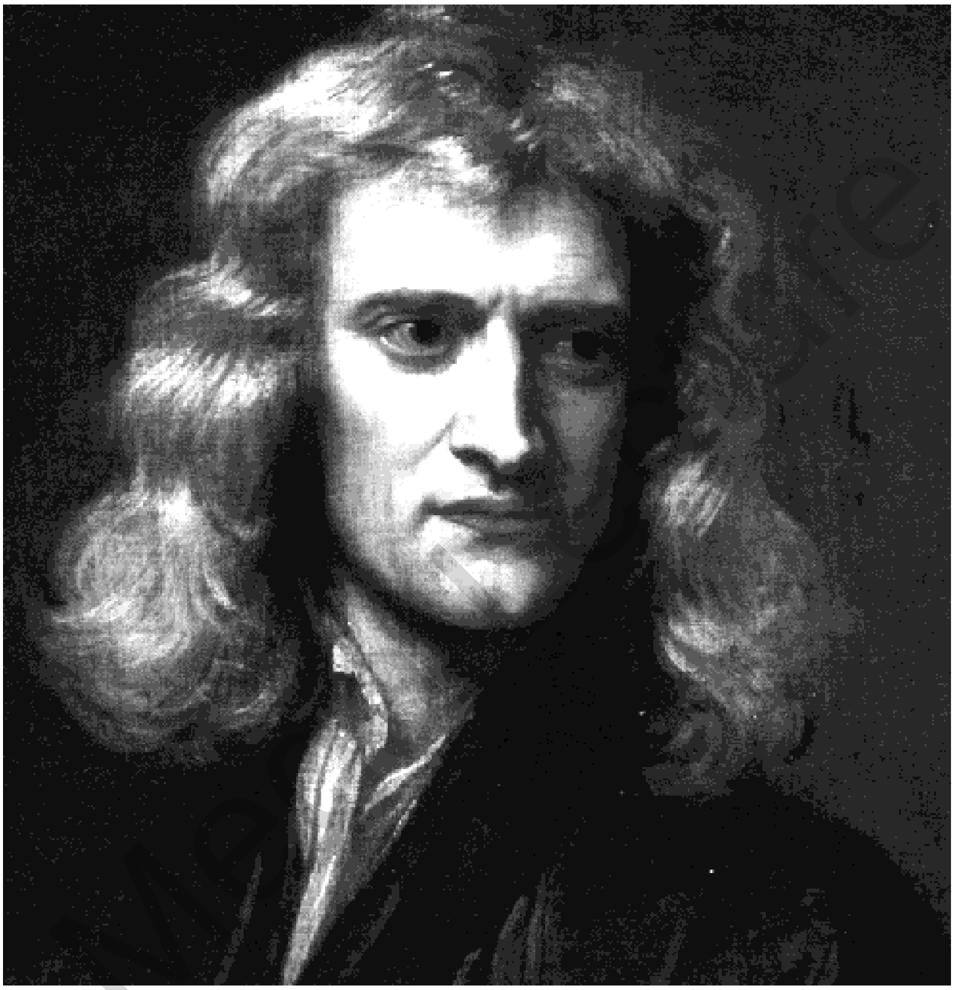
GRAVITATIONAL FORCES



ISAAC NEWTON
1642 - 1727

You will need this data to answer some of the questions in this section.

Largest objects in Solar System

Name	Orbits	Orbital radius (10^3 km)	Orbital Period	Diameter (km)	Mass(kg)
Sun	n.a.	n.a.	n.a.	1,394,000	1.99×10^{30}
Jupiter	Sun	778,000	11.86 yrs	142,840	1.90×10^{27}
Saturn	Sun	1,429,000	29.46 yrs	120,540	5.69×10^{26}
Uranus	Sun	2,870,990	84 yrs	51,120	8.69×10^{25} *
Neptune	Sun	4,504,300	164.78 yrs	49,530	1.02×10^{26} *
Earth	Sun	149,600	1 yr	12,760	5.98×10^{24}
Venus	Sun	108,200	224.5 days	12,100	4.87×10^{24}
Mars	Sun	227,940	1.88 yrs	6,800	6.42×10^{23}
Ganymede	Jupiter	1,070	7.16 days	5,260	1.48×10^{23}
Titan	Saturn	1,222	15.95 days	5,150	1.35×10^{23}
Mercury	Sun	57,910	88 days	4,880	3.30×10^{23}
Callisto	Jupiter	1,883	16.69 days	4,800	1.08×10^{23}
Io	Jupiter	422	1.77 days	3,630	8.93×10^{22}
Moon	Earth	384	27.3 days	3,480	7.35×10^{22}
Europa	Jupiter	671	3,55 days	3,140	4.80×10^{22}
Triton	Neptune	355	5.88 days	2,710	2.15×10^{22}
Pluto	Sun	5,913,520	248.5 yrs	2,390	1.32×10^{22}
Titania	Uranus	436	8.71 days	1,610	3.5×10^{21}
Oberon	Uranus	583	13.46 days	1,550	3.0×10^{21}
Rhea	Saturn	527	4.52 days	1,530	2.3×10^{21}
Iapetus	Saturn	3,561	79.33 days	1,460	1.6×10^{21}
Charon	Pluto	19.6	6.39 days	1,168	1.9×10^{21}
Umbriel	Uranus	266	4,14 days	1,190	1.2×10^{21}
Ariel	Uranus	191	2.52 days	1,160	1.3×10^{21}
Dione	Saturn	377	2.74 days	1,120	1.1×10^{21}
Tethys	Saturn	295	1.89 days	1,060	0.6×10^{21}
Ceres	Sun	415,000	4.6 yrs	950	8.7×10^{20}

- **Note**: Neptune is slightly denser than Uranus.

For further information: http://nssdc.gsfc.nasa.gov/planetary/planetfact.html

Solar system data obtained from NASA

Planet	Mass / kg	Radius / m	Density / kg m^{-3}	g / ms^{-2}	g/g$_{Earth}$
Mercury	3.30×10^{23}	2.44×10^{6}	5.42×10^{3}	3.70	0.38
Venus	4.87×10^{24}	6.05×10^{6}	5.25×10^{3}	8.87	0.90
Earth	5.97×10^{24}	6.37×10^{6}	5.51×10^{3}	9.81	1.00
Mars	6.42×10^{23}	3.39×10^{6}	3.93×10^{3}	3.73	0.38
Jupiter	1.90×10^{27}	6.99×10^{7}	1.33×10^{3}	25.94	2.64
Saturn	5.68×10^{26}	5.82×10^{7}	6.88×10^{2}	11.18	1.14
Uranus	8.68×10^{25}	2.54×10^{7}	1.26×10^{3}	8.97	0.91
Neptune	1.02×10^{26}	2.64×10^{7}	1.32×10^{3}	9.76	1.00
Pluto	1.25×10^{22}	1.14×10^{6}	2.03×10^{3}	0.64	0.07
Moon	7.35×10^{22}	1.74×10^{6}	3.33×10^{3}	1.62	0.17

Asteroid	Mass / kg	Radius / m	Density / kg m^{-3}	g / ms^{-2}	g/g$_{Earth}$
Ceres	8.70×10^{20}	4.73×10^{5}	1.96×10^{3}	0.26	0.026
Pallas	3.18×10^{20}	2.63×10^{5}	4.18×10^{3}	0.31	0.031
Juno	2.00×10^{19}	1.20×10^{5}	2.76×10^{3}	0.09	0.009
Vesta	3.00×10^{20}	2.65×10^{5}	3.85×10^{3}	0.28	0.029
Chiron	4.00×10^{18}	9.00×10^{4}	1.31×10^{3}	0.03	0.003

You may assume the following values for the introductory problems *unless* specified to the contrary in the question.

- Earth's mass, $M_E = 6.0 \times 10^{24}$ kg
- Earth's radius, $R_E = 6.4 \times 10^{6}$ m
- Gravitational acceleration on the Earth's surface, $g_E = 9.8$ ms^{-2}
- Universal gravitational constant, $G = 6.7 \times 10^{-11}$ Nm^{2}kg^{-1}

GRAVITATIONAL ATTRACTION

1 Calculate the gravitational attraction between two identical lead spheres each of mass 10 kg if their centres are 0.2 m apart.

[2]

2 Calculate the gravitational force between the Earth and the Moon. Take the mass of the Moon to be 7.3×10^{22}kg. Their centre-to-centre distance is 3.9×10^{5}km.

[2]

3 Calculate the gravitational force between the Earth and the Sun. Take the mass of the Sun to be 2.0×10^{30} kg. Their centre-to-centre distance is 1.5×10^{8} km.

[2]

4 (i) Calculate the gravitational force between you and the Earth,

[2]

(ii) Draw a free-body diagram to show the forces acting between you and the Earth. [2]

5 In a Cavendish apparatus, used to determine the value of G, the large sphere has a mass of 10 kg, and the small one has a mass of 10 grams. They reach equilibrium when their centres are 6 cm apart.

(a) What is the magnitude of the gravitational force between them?

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(b) Use your answer to explain why is G so difficult to measure accurately.

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6 In elementary questions on orbital motion, it is usually assumed that orbits are circular. In fact they are elliptical. The Moon's orbit, for example, varies from 356,400 km at perigee to 406,700 km at apogee.

(a) Use the data on page 4 to calculate the force exerted by the Earth on the Moon

(i) at perigee ____

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____ [2]

(ii) at apogee ____

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(b) Calculate the percentage increase as the Moon goes from apogee to perigee.

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(c) How does the force that the Moon exerts on the Earth compare to that which the Earth exerts on the Moon as the Moon goes from apogee to perigee? Explain your answer.

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7 (a) (i) Derive an expression for the Earth's mass, M_e, in terms of g_E, G and R_E

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(ii) Use the values of given for g_E, G and R_E on page 3 to calculate a value for the Earth's mass.

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____ [2]

(iii) Use the answer to (ii) to calculate a value for the Earth's average density.

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(iv) The average density of the Earth's mantle is about half the value that you obtained in the previous calculation. Explain why the Earth's average density is approximately half of the density of rocks found on its surface.

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(b) (i) If the radius of the Earth were to halve, its mass remaining the same, how, if at all, would your weight change? Calculation required to back up your answer.

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(ii) What would happen if the Earth's radius halved while its density remained the same? Calculation required to support your answer.

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8 Which would you choose to have: a lump of gold that weighs 1 N on Earth or one that weights 1 N on the Moon? Explain your answer. Be cynical: assume normal human greed.

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9 Gravitational attraction depends on mass. The mass of the Earth is 81 times that of the Moon. Yet the Earth's surface gravitational field strength is only 6 times that of the Moon. Why isn't the Earth's surface gravitational field strength larger than this? Give a *qualitative* explanation.

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10 (i) Derive an expression for the gravitational field strength on the Moon's surface, g_m, in terms of the Earth's gravitational field strength, g_e, given that the mass of the Moon, M_m, is approximately 1% of the Earth's mass, M_e, and that the Moon's radius, R_m, is approximately one quarter that of the Earth's radius, R_e.

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(ii) Calculate a value for the gravitational field strength on the Moon's surface using vales for R_e and g_e given on page 4. *You may* ***not*** *use G or M_E to solve this problem.*

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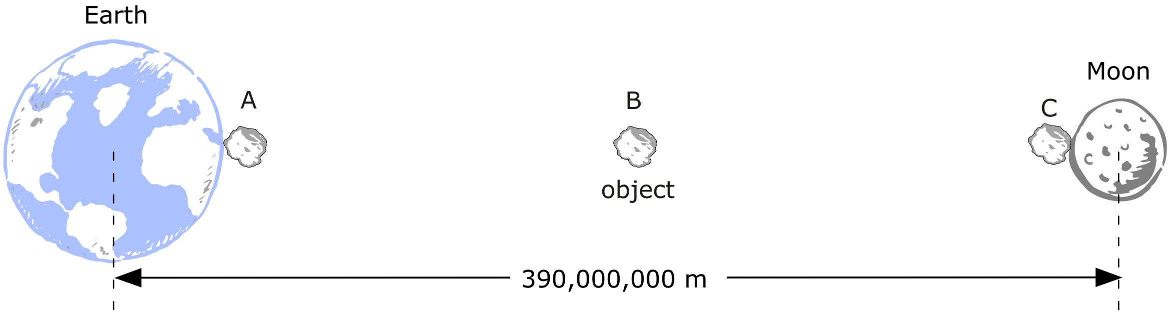
____ [2]

11 Calculate the gravitational pull on a body of mass 1 kg by (i) the Earth and (ii) the Moon in each of the following situations A, B & C

Distance between the Moon and the Earth = 3.9×10^8 m and the mass of the Moon as 7.3×10^{22} kg. *Take* $R_E = 6.4 \;\; 10^6$ *m and assume* $R_M = R_E/4$. *Do* **not** *use* g_E *to solve this problem.*



(a) When the body is on the Earth's surface.

(i) ____

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____ [2]

(ii) ____

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____ [2]

(b) When it is half way between the Earth and the Moon.

(i) ____

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____ [2]

(ii) ____

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____ [2]

(c) When it is on the Moon's surface.

(i) ____

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____ [2]

(ii) ____

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____ [2]

(d) In Jules Vernes' novel '*Journey to the Moon'*, the travellers become temporarily weightless when they reach a point in their voyage somewhere between the Earth and the Moon.

(i) Can you explain why Verne thought this would happen?

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(ii) Where did Verne believe that the travellers become weightless? A diagram and a calculation is required.

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____ [2]

(iii) Explain what really happens.

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____ [2]

12 (a) (i) Derive an expression for the value of g_E on the Earth's surface in terms of M_E and R_E.

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____ [1]

(ii) Derive an expression for the density, D_E, of the Earth in terms of M_E and R_E.

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____ [2]

(iii) Derive an expression for the Earth's gravitational field strength, g_E, in terms of D_E and R_E.

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____ [2]

(b) The radius of planet X is half that of the Earth. If its mass is half that of the Earth calculate:

(i) the ratio of the acceleration due to gravity on the planet's surface to that on the Earth's surface.

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____ [2]

(ii) the ratio of density of planet X to that of the Earth.

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____ [3]

(c) (i) Calculate the gravitational field strength at the surface of a planet that has the same density as the Earth but with a radius that is 3.5 times less than the Earth's .

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____ [2]

(ii) In fact the Moon's radius is 3.5 times less than that of the Earth, but the gravitational filed strength at its surface is 1/6 that of the Earth. What does this tell you about the composition of the Moon?

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____ [2]

13 The mass of a typical neutron star is 2.0×10^{30} kg, the same as that of the Sun. However, the radius of a neutron star is only 10 km. Calculate a value for (a) the density and (b) the gravitational field strength on the surface of the neutron star.

(i) ____

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(i) ____

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____ [2]

14 The value of the Universal Gravitational Constant is assumed to be the same throughout space and time. What would be the consequences - if any - of an increase or a decrease in its magnitude of (i) 1% and (ii) a factor of 10?

(i) ____

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____ [2]

(ii) ____

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____ [2]

15 (a) Calculate the percentage difference in g_E between the equator and the pole taking into account the fact that the pole is 40 km nearer to the centre of the Earth than the equator.

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(b) What difference, if any, does the fact that Earth is spinning make to g_E?

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____ [2]

16 If a large piece of Earth were somehow transferred to the Moon by how much, if at all, would the gravitational force between them change?

(i) Calculate the present gravitational force between them in terms of the Moon's mass (M_m)

[Take mass of Earth: mass of Moon as 81:1 & the distance between them as 60R_E]

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____ [2]

(ii) Calculate the gravitational attraction between them (in terms of the Moon's mass, M_m) if a piece of Earth equal to the mass of the Moon were transferred to the Moon.

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(iii) If transferring some of Earth's mass to the Moon does change the gravitational attraction between Earth and Moon, how much of Earth must be transferred to the Moon for the attractive force to reach its greatest value?

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(iv) Do bits of one planet ever get transferred to other planets? How? Give examples.

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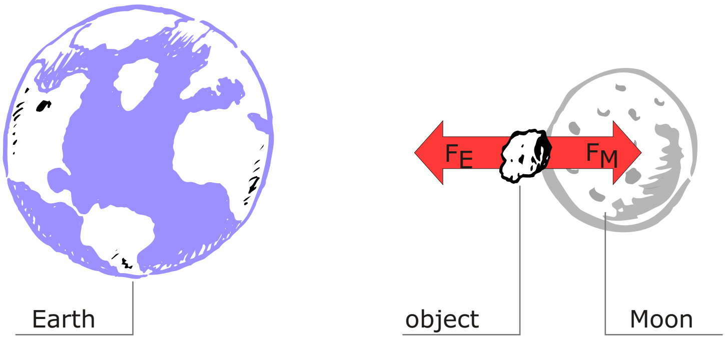
17 The Roche limit is defined as the smallest distance from a primary body (e.g. a planet or a star) at which gravitational forces alone can hold together an orbiting satellite of the same average density as the primary body. If a body is closer than this, tidal forces between itself and the primary body would break up the secondary body.

This break-up will occur when the gravitational attraction of a planet on a fragment of its satellite is greater than the gravitational attraction of the satellite on that fragment.



If the Moon's near side were at the Roche limit, an object on its surface is pulled towards the Earth with the same force as it is pulled towards the Moon.

(a) If the Moon were at the Roche limit, an object on Moon's near side surface would be on the point of flying off towards the Earth. Use this fact to calculate the Roche limit for the Earth. Use data for Earth & Moon on page 3.

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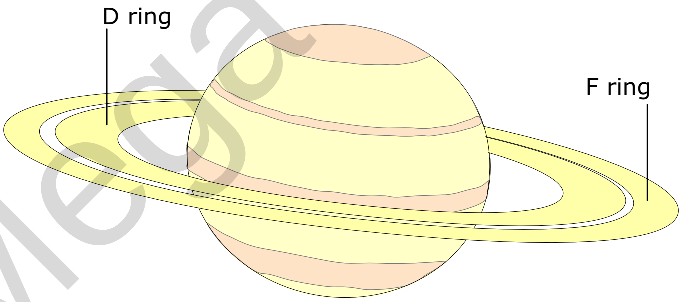
____ [3]

(b) Saturn is rightly famous for its dazzling system of rings. In fact all the giant planets in the Solar System have ring systems, though only Saturn's ring system is visible from Earth.

Do the rings of Saturn lie within the Roche limit for Saturn?



Is the outer edge of Saturn's ring system at the Roche limit?

For this calculation consider the gravitational forces acting on an object on the surface of a natural satellite having the same density as Saturn in orbit about Saturn.

Take the diameter and density of Saturn as 60,300 km and 6.9 × 10^2 kgm^{-3} respectively. The inner ring, called the 'D' ring is at a distance of 67,000 km from the centre of Saturn, and the outer ring, the 'F' ring is at 140,000 km.

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____ [4]

Answers to Gravitational attraction

1 Gravitational force between spheres: 1.67×10^{-7} N

2 Gravitational force between Earth and Moon: 1.92×10^{20} N

3 Gravitational force between Earth and Sun: 3.56×10^{22} N

4 (i) Gravitational force between the Earth and you: for a mass of 70 kg, the force is 684 N.
(ii) Free body diagram:

5 (a) Gravitational force between spheres: 1.85×10^{-9} N
(b) Gravity is such a weak force that the gravitational force between objects small enough to be handled in a laboratory is tiny, which makes it very difficult to measure accurately. This introduces a large uncertainty in the calculated value of *G*.

6 (a) (i) Force at perigee = 2.31×10^{20} N, (ii) Force at apogee = 1.78×10^{20} N
(b) Force increases by 22.9 %
(c) The force exerted by the Moon on the Earth is always equal to the force exerted by the Earth on the Moon (Newton's 3rd law of motion.)

7 (a) (i) $M_e = g_e R_e^2 / G$, (ii) 5.99×10^{24} kg, (iii) 5455 kgm^{-3}, (iv) Earth has a large dense core , (b) (i) new $g = 4 \times 9.8$ Nkg^{-1}, (e) new $g = 1/2 \times 9.8$ Nkg^{-1}

8 Choose the lump on the Moon. The mass of a body having a weight of 1 N is 6 times greater on the Moon. Its mass will be six times greater than the 1 N lump on Earth because the Moon's gravitational attraction is 6 times less than the Earth's.

9 Although the Earth's mass is greater that that of the Moon, which on its own would cause a gravitational force 81 times greater than that due to the Moon, the Earth's surface is R_e/R_m (= 3.7) times further from its centre, which reduces the gravitational force at its surface by 3.7^2 = 13 times. Hence overall value of Earth's surface gravity = 81/13.4 = 6 times more than Moon's surface gravity.

10 (i) $g_M = 16/100 \times g_E$ (ii) $g_M = 1.57$ ms^{-2}

11 (a) (i) $F_E = 9.8$ N, (ii) $F_M = 3.3 \times 10^{-5}$N
(b) (i) $F_E = 1.05 \times 10^{-2}$ N , (ii) $F_M = 1.3\ 10^{-4}$ N,
(c) (i) $F_E = 2.6 \times 10^{-3}$ N, (ii) $F_M = 1.9$ N
(d) (i) Verne thought that weightlessness would occur only at the point in the journey when the pull of the Moon's gravity on the travellers was exactly equal to the pull of the Earth's gravity on the travellers.
(ii) Pull of Moon = pull of earth at a distance of 351×10^6 km from Earth (= 9/10 of the earth Moon distance) (iii) In fact astronauts are weightless for the whole journey because the spacecraft is in orbital motion and so they they are in free fall.

12 (a) (i) $g_E = GM_E/R_E^2$, (ii) $D_E = 3M_E/4\pi R_E^3$, (iii) $g_E = (4/3)\pi G D_E R_E$ (b) (i) $g_P = 2g_E$, (ii) $D_P = 4\ D_E$,.(c) (i) $g_P = g_E/3.5$, (ii) Moon's average density is about half that of Earth's.

13 (i) Density of neutron star = 478×10^{15} kg m^{-3}, (ii) Gravitational field strength = 1.34×10^{12} Nkg^{-1}

14

15 (a) percentage difference = 1.25%

16 (i) Force between Earth & Moon, (ii) $F_1 = 81\ GM_M^2/R^2$, $F_2 = 160\ GM_M^2/R^2$, (iii) Maximum gravitational force between them when half of Earth's mass has been transferred to Moon i.e. $F_{Max} = 41^2\ GM_M^2/R^2$, (iv) Small amounts are sometimes knocked off a planet when it is struck by a large meteoroid. Small fragments of both the Moon and Mars (i.e. meteorites) have been found on Earth

17 (a) Roche limit for Earth Moon system = 15.6×10^3 km.