GRAVITATIONAL FIELDS

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1473-1543

Largest objects in Solar System

Name	Orbits	Orbital radius (10^3 km)	Orbital Period	Diameter (km)	Mass(kg)
Sun				1,394,000	1.99×10^{30}
Jupiter	Sun	778,000	11.86 yrs	142,840	1.90×10^{27}
Saturn	Sun	1,429,000	29.46 yrs	120,540	5.69×10^{26}
Uranus	Sun	2,870,990	84 yrs	51,120	8.69×10^{25} *
Neptune	Sun	4,504,300	164.78 yrs	49,530	1.02×10^{26} *
Earth	Sun	149,600	1 yr	12,760	5.98×10^{24}
Venus	Sun	108,200	224.5 days	12,100	4.87×10^{24}
Mars	Sun	227,940	1.88 yrs	6,800	6.42×10^{23}
Ganymede	Jupiter	1,070	7.16 days	5,260	1.48×10^{23}
Titan	Saturn	1,222	15.95 days	5,150	1.35×10^{23}
Mercury	Sun	57,910	88 days	4,880	3.30×10^{23}
Callisto	Jupiter	1,883	16.69 days	4,800	1.08×10^{23}
Io	Jupiter	422	1.77 days	3,630	8.93×10^{22}
Moon	Earth	384	27.3 days	3,480	7.35×10^{22}
Europa	Jupiter	671	3,55 days	3,140	4.80×10^{22}
Triton	Neptune	355	5.88 days	2,710	2.15×10^{22}
Pluto	Sun	5,913,520	248.5 yrs	2,390	1.32×10^{22}
Titania	Uranus	436	8.71 days	1,610	3.5×10^{21}
Oberon	Uranus	583	13.46 days	1,550	3.0×10^{21}
Rhea	Saturn	527	4.52 days	1,530	2.3×10^{21}
Iapetus	Saturn	3,561	79.33 days	1,460	1.6×10^{21}
Charon	Pluto	19.6	6.39 days	1,168	1.9×10^{21}
Umbriel	Uranus	266	4,14 days	1,190	1.2×10^{21}
Ariel	Uranus	191	2.52 days	1,160	1.3×10^{21}
Dione	Saturn	377	2.74 days	1,120	1.1×10^{21}
Tethys	Saturn	295	1.89 days	1,060	0.6×10^{21}
Ceres	Sun	415,000	4.6 yrs	950	8.7×10^{20}

- **Note**: Neptune is slightly denser than Uranus.

For further information:
http://nssdc.gsfc.nasa.gov/planetary/planetfact.html

Solar system data obtained from NASA

Planet	Mass / kg	Radius / m	Density / kg m^{-3}	g / ms^{-2}	g/g_{Earth}
Mercury	3.30×10^{23}	2.44×10^{6}	5.42×10^{3}	3.70	0.38
Venus	4.87×10^{24}	6.05×10^{6}	5.25×10^{3}	8.87	0.90
Earth	5.97×10^{24}	6.37×10^{6}	5.51×10^{3}	9.81	1.00
Mars	6.42×10^{23}	3.39×10^{6}	3.93×10^{3}	3.73	0.38
Jupiter	1.90×10^{27}	6.99×10^{7}	1.33×10^{3}	25.94	2.64
Saturn	5.68×10^{26}	5.82×10^{7}	6.88×10^{2}	11.18	1.14
Uranus	8.68×10^{25}	2.54×10^{7}	1.26×10^{3}	8.97	0.91
Neptune	1.02×10^{26}	2.64×10^{7}	1.32×10^{3}	9.76	1.00
Pluto	1.25×10^{22}	1.14×10^{6}	2.03×10^{3}	0.64	0.07
Moon	7.35×10^{22}	1.74×10^{6}	3.33×10^{3}	1.62	0.17

Asteroid	Mass / kg	Radius / m	Density / kg m^{-3}	g / ms^{-2}	g/g_{Earth}
Ceres	8.70×10^{20}	4.73×10^{5}	1.96×10^{3}	0.26	0.026
Pallas	3.18×10^{20}	2.63×10^{5}	4.18×10^{3}	0.31	0.031
Juno	2.00×10^{19}	1.20×10^{5}	2.76×10^{3}	0.09	0.009
Vesta	3.00×10^{20}	2.65×10^{5}	3.85×10^{3}	0.28	0.029
Chiron	4.00×10^{18}	9.00×10^{4}	1.31×10^{3}	0.03	0.003

You may assume the following values for the introductory problems *unless* specified to the contrary in the question.

- Earth's mass, $M_E = 6.0 \times 10^{24}$ kg
- Earth's radius, $R_E = 6.4 \times 10^{6}$m
- Gravitational acceleration on the Earth's surface, g_E = 9.8 ms^{-2}
- Universal gravitational constant, $G = 6.7 \times 10^{-11}$ Nm^{2}kg^{-1}

Gravitational fields

1 Use values in table below to calculate the escape speed for an object launched from (a) Earth's surface, (b) Mars' surface, (c) Sun's surface, (d) inner solar system (say from the Earth's orbit)

Object	Mass/kg	Radius/m	Orbital radius/m
Sun	2.0×10^{30}	7.0×10^{8}	n.a.
Earth	6.0×10^{24}	6.4×10^{6}	1.50×10^{9}
Mars	6.6×10^{23}	3.4×10^{6}	2.28×10^{9}

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(b) ____

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(c) ____

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(d) ____

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2 (a) What is the Moon's escape speed as a whole number fraction of the Earth's? Take mass of Moon to be 1/81 of Earth's mass and Moon's radius to be 1/4 Earth's radius. Don't use a calculator, use ratios and cancel down.

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(b) Use this ratio, together with Earth's escape speed calculated in question 1 (a) to calculate a value for the Moon's escape speed in metres per second.

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3 How much work must be done to launch a rocket from the Moon's surface so that it escapes the Moon's gravity. Take the mass of the rocket to be 5000 kg, the Moon's mass to be 7.35×10^{22} kg, and its radius to be 1.74×10^{6} m.

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4 (a) Derive the relationship between the gravitational field strength, g, on the surface of a planet and its radius, R, and its density, p.

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(b) Suppose that you are able to jump vertically 0.5 m on Earth. What height might you be expected to jump on a planet which has a density that is 4/5 that of the Earth, but whose radius is one half that of the Earth? As a matter of interest, Mars is such a planet. Roll on the Mars Olympics!

Think of the jump as a transfer of energy from chemical energy in your muscles to g.p.e. : you wouldn't have any more energy available to you on the other planet.

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5 (a) Derive a relationship between the radius, *r*, of a spherical body, its density, *p*, and the escape velocity, *v*, from the body.

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(b) What is the radius of a body from which you could launch yourself into space by jumping vertically? You will have to assume, or (better) calculate, the speed at which you leave the ground when you make such a jump. The previous problem (number 4) may be of help. Take the density of the matter from which the body is made to be 2500 kgm^3.

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(c) What would be the minimum radius of a spherical body (made of the same stuff) if you could throw a ball from it at with a velocity equal to the escape velocity for the body? You will have to assume, or better, calculate the maximum speed with which you can throw a ball. Think in terms of how high you can throw it on Earth: how fast must the ball be travelling when it leaves your hand to reach this height?

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6 (a) With what minimum speed will a meteoroid enter the Earth's atmosphere?

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(b) What happens to its g.p.e. and k.e as it falls to Earth:

(i) when it is still above the atmosphere

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(ii) as it travels through the atmosphere?

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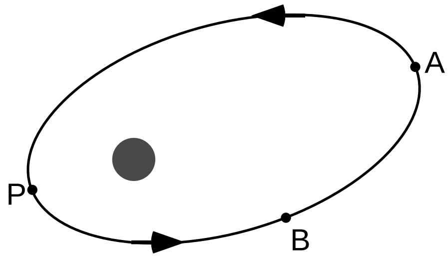
7 The diagram shows the orbit of a satellite about the Earth. P is *perigee* (point in orbit closest to Earth), A is *apogee* (point in orbit furthest from Earth) and B is a point on the orbit halfway between P and A.

(a) At which position in the does the satellite experience the greatest gravitational pull from the Earth?



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(b) At which position does it have (i) maximum, (ii) minimum potential energy and where does it have (iii) maximum, (iv) minimum kinetic energy?

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(c) What can be said about the speed of the satellite at B compared to that at P and at A?

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8 At what point in an elliptical orbit about the Earth should a rocket fire its engines to gain maximum k.e.: at apogee or perigee. Assume that the engine burn lasts the same time in either position.

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9 The first man-made device to escape the solar system was Pioneer 10. It was launched from Earth on March 2, 1972, from Cape Kennedy with a speed of only 15 km/s. At the time it was the fastest spacecraft ever to leave Earth. It was the first spacecraft to travel through the asteroid belt, the first to visit Jupiter, and the first to use a planet's gravity to change course and reach solar-system escape velocity.

Escape was accomplished by directing the probe into the path of oncoming Jupiter. It was whipped about by Jupiter's great gravitational field, picking up speed in the process—similar to the increase in the speed of a ball encountering an oncoming bat when it departs from the bat. Its speed of departure from Jupiter was increased enough to exceed the Sun's escape speed at the distance of Jupiter. Pioneer 10 passed beyond the orbit of Pluto in 1984.

(i) What is the escape speed from the Solar System for an object at a distance equal to Jupiter's orbit? Jupiter lies at 5.2 AU.

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On April 28th,2001 Pioneer was sent a radio signal from its base station here on Earth. When Pioneer received this it sent back a weak radio signal. The entire exchange, from the moment the signal was sent from Earth to the moment when the return signal from Pioneer was received back at the base took 22 hours.

(i) How far away from Earth was Pioneer at this time?

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(ii) Calculate the average speed at which Pioneer 10 has traveled in its journey.

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10 Which requires more fuel: a rocket travelling from Moon to Earth or one travelling from Earth to Moon? Explain your answer.

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11 (i) Prove that escape velocity for an object in orbit about a planet is always √2 greater than its orbital velocity.

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(ii) Is the Earth's orbital velocity √2 less than the escape velocity of the solar system from a point on the Earth's orbit? Support you answer with relevant calculations. Earth's orbital radius is 1 A.U. and its orbital period is 365.25 days.

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12 In what way is a contour map like a diagram of equipotential surfaces?

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13 (i) Plot a graph of gravitational potential energy, U, per unit mass for the Earth at intervals of R_E to a distance of $6R_E$. Use the following scale for your graph: R_E = 2cm and 10MJ/kg = 2 cm.

g.p.e. in the Earth's radial field given by $U = \frac{GM_E}{R_E}$ where $M_E = 6 \times 10^{24}$ kg, $R_E = 6.4 \times 10^6$ m.

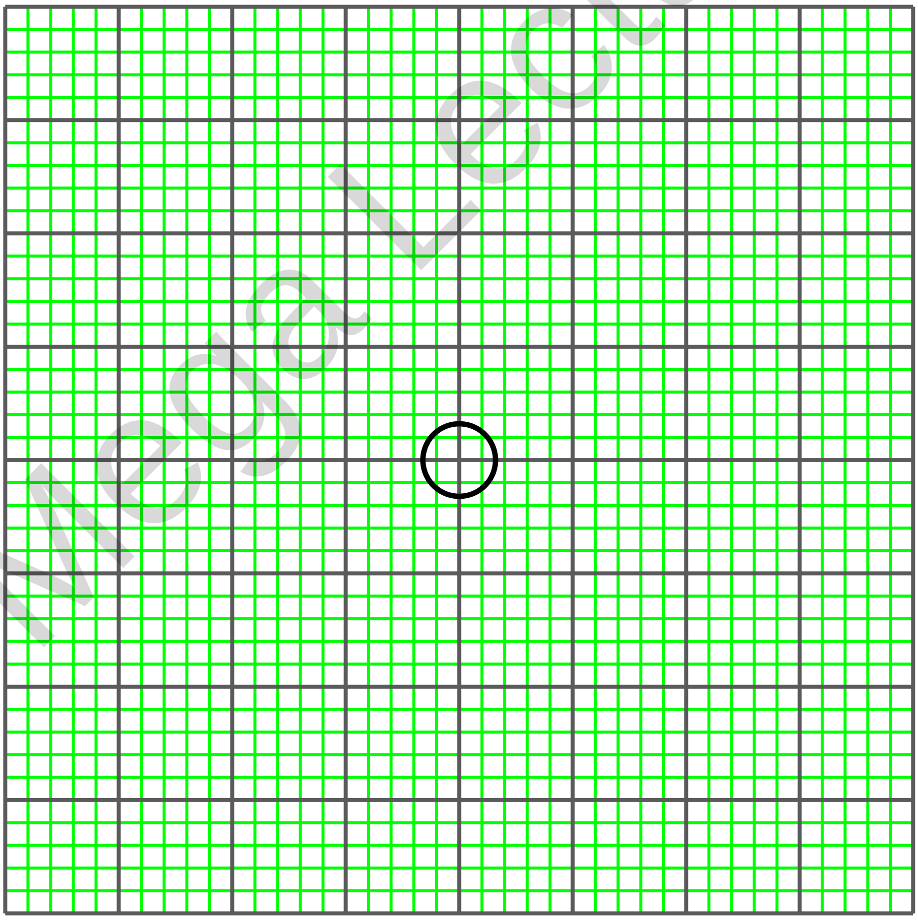
Calculate U and fill in the table below.

Distance from centre of earth / metres	G.P.E. / Joules
6.4×10^6	
12.8×10^6	
19.2×10^6	
25.6×10^6	
32.0×10^6	
38.4×10^6	

(ii) Use the graph to determine the distance of equipotential surfaces with energy intervals of 10 MJ/kg.

Energy intervals /MJ	Distance from centre of Earth / m
10	
20	
30	
40	
50	
60	

Now draw a scale diagram to show these equipotential surfaces around the Earth on the grid below. The Earth is represented by the circle in the centre. The diagram should be a series of concentric circles of different radii. Label each equipotential with its energy.



14 On Earth a fit man can jump vertically from rest a distance equal to one third of his height. In other words, he can raise his centre of gravity by this distance.

(i) How high could this man jump on the Moon?

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(ii) What is the largest radius of a planet or asteroid (assumed to be spherical) which this man can literally jump off? Take the density of the planet / asteroid to be the same as that of Earth.

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Answers to gravitational fields

1 (a) v_{Earth} = 11.2 km/s, (b) v_{Mars} = 5.1 km/s, (c) v_{Sun} = 617.5 km/s, (d) $v_{Solar\ System}$ = 42.2 km/s

2 (a) v_{Moon}/v_{Earth} = √(1/20), (b) 2500 m/s

3 1.41 x 10^{10} Joules

4 (a) $g = (4/3\ \pi\ G\ R\ p)$, (b) You can jump 1.25 m vertically on the smaller planet.

5 (a) $v = \sqrt{8/3G\pi R^2 p}$, (b) Use $mgh = 1/2\ mv^2$ to calculate the speed with which you leave the ground on Earth. This will be the same on the asteroid. Using this take-off speed as escape speed, the maximum diameter of the body that you can jump off is 2620 metres. (c) Assume that you can throw a ball 10 m vertically on Earth. This gives an initial speed of 14 m/s. The radius of the asteroid with an escape speed of 14 m/s is 11.85 km.

6 (a) minimum entry speed of a meteoroid is the same as the Earth's escape velocity, or 11.2 km/s, (b) (i) while still outside the Earth's atmosphere, the g.p.e. of the meteoroid decreases, and its k.e. increases, (ii) As it travels through the atmosphere, its g.p.e decreases and so does its k.e. (g.p.e becomes internal energy as meteoroid heats up due to adiabatic heating-not friction). A small meteoroid would reach the ground at terminal velocity, say 10 m/s