Reference Guide & Formula Sheet for Physics

Dr. Hoselton & Mr. Price

#3 Components of a Vector

if $V = 34$ m/sec $\angle 48°$

then

$V_i = 34$ m/sec•(cos 48°); **and** $V_J = 34$ m/sec•(sin 48°)

#4 Weight = m•g

g = 9.81 m/sec² near the surface of the Earth
= 9.795 m/sec² in Fort Worth, TX

Density = mass / volume

$$\rho = \frac{m}{V}\left(unit : kg/m^3\right)$$

#7 Ave speed = distance / time = v = d/t

Ave velocity = displacement / time = v = d/t

Ave acceleration = change in velocity / time

#8 Friction Force

$F_F = \mu \cdot F_N$

If the object is not moving, you are dealing with static friction and it can have any value from zero up to $\mu_s F_N$

If the object is sliding, then you are dealing with kinetic friction and it will be constant and equal to $\mu_K F_N$

#9 Torque

$\tau = F \cdot L \cdot \sin\theta$

Where θ is the angle between F and L; unit: Nm

#11 Newton's Second Law

$F_{net} = \Sigma F_{Ext} = m \cdot a$

#12 Work = F•D•cos θ

Where D is the distance moved and θ is the angle between **F** and the direction of motion, unit : J

#16 Power = rate of work done

$$Power = \frac{Work}{time} \quad \text{unit : watt}$$

Efficiency = Work$_{out}$ / Energy$_{in}$

Mechanical Advantage = force out / force in

M.A. = F_{out} / F_{in}

#19 Constant-Acceleration Linear Motion

$v = v_o + a \cdot t$	x
$(x-x_o) = v_o \cdot t + \frac{1}{2} \cdot a \cdot t^2$	v
$v^2 = v_o^2 + 2 \cdot a \cdot (x - x_o)$	t
$(x-x_o) = \frac{1}{2} \cdot (v_o + v) \cdot t$	a
$(x-x_o) = v \cdot t - \frac{1}{2} \cdot a \cdot t^2$	v_o

#20 Heating a Solid, Liquid or Gas

$Q = m \cdot c \cdot \Delta T$ (no phase changes!)

Q = the heat added
c = specific heat.
ΔT = temperature change, K

#21 Linear Momentum

momentum = **p** = m•v = mass • velocity
momentum is conserved in collisions

#23 Center of Mass – point masses on a line

$x_{cm} = \Sigma(mx) / M_{total}$

#25 Angular Speed vs. Linear Speed

Linear speed = v = r•ω = r • angular speed

#26 Pressure under Water

$P = \rho \cdot g \cdot h$

h = depth of water
ρ = density of water

#28 Universal Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

G = 6.67 E-11 N m² / kg²

#29 Mechanical Energy

$PE_{Grav} = P = m \cdot g \cdot h$
$KE_{Linear} = K = \frac{1}{2} \cdot m \cdot v^2$

#30 Impulse = Change in Momentum

$F \cdot \Delta t = \Delta(m \cdot v)$

#31 Snell's Law

$n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$

Index of Refraction

n = c / v
c = speed of light = 3 E+8 m/s

#32 Ideal Gas Law

$P \cdot V = n \cdot R \cdot T$

n = # of moles of gas
R = gas law constant
= 8.31 J / K mole.

#34 Periodic Waves

$v = f \cdot \lambda$
$f = 1 / T$ T = period of wave

#35 Constant-Acceleration Circular Motion

$\omega = \omega_o + \alpha \cdot t$	θ
$\theta - \theta_o = \omega_o \cdot t + \frac{1}{2} \cdot \alpha \cdot t^2$	ω
$\omega^2 = \omega_o^2 + 2 \cdot \alpha \cdot (\theta - \theta_o)$	t
$\theta - \theta_o = \frac{1}{2} \cdot (\omega_o + \omega) \cdot t$	α
$\theta - \theta_o = \omega \cdot t - \frac{1}{2} \cdot \alpha \cdot t^2$	ω_o

#36 Buoyant Force - Buoyancy

$F_B = \rho \bullet V \bullet g = m_{Displaced\ fluid} \bullet g = weight_{Displaced\ fluid}$

ρ = density of the fluid
V = volume of fluid displaced

#37 Ohm's Law

$V = I \bullet R$

V = voltage applied
I = current
R = resistance

Resistance of a Wire

$R = \rho \bullet L / A_x$

ρ = resistivity of wire material
L = length of the wire
A_x = cross-sectional area of the wire

#39 Heat of a Phase Change

$Q = m \bullet L$

L = Latent Heat of phase change

#41 Hooke's Law

$F = k \bullet x$

Potential Energy of a spring

$W = \frac{1}{2} \bullet k \bullet x^2$ = Work done on spring

#42 Electric Power

$P = I^2 \bullet R = V^2 / R = I \bullet V$

#44 Speed of a Wave on a String

$$T = \frac{mv^2}{L}$$

T = tension in string
m = mass of string
L = length of string

#45 Projectile Motion

Horizontal: $x - x_o = v_o \bullet t + 0$

Vertical: $y - y_o = v_o \bullet t + \frac{1}{2} \bullet a \bullet t^2$

#46 Centripetal Force

$$F = \frac{mv^2}{r} = m\omega^2 r$$

#47 Kirchhoff's Laws

Loop Rule: $\Sigma_{Around\ any\ loop}\ \Delta V_i = 0$

Node Rule: $\Sigma_{at\ any\ node}\ I_i = 0$

#51 Minimum Speed at the top of a Vertical Circular Loop

$$v = \sqrt{rg}$$

#53 Resistor Combinations

SERIES

$R_{eq} = R_1 + R_2 + R_3 + \ldots$

PARALLEL

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}$$

#54 Newton's Second Law and Rotational Inertia

τ = torque = $I \bullet \alpha$

I = moment of inertia = $m \bullet r^2$ (for a point mass)

(See table in Lesson 58 for I of 3D shapes.)

#55 Circular Unbanked Tracks

$$\frac{mv^2}{r} = \mu mg$$

#56 Continuity of Fluid Flow

$A_{in} \bullet v_{in} = A_{out} \bullet v_{out}$

A = Area
v = velocity

#58 Moment of Inertia - I

Moment of Inertia	I
cylindrical hoop	$m \bullet r^2$
solid cylinder or disk	$\frac{1}{2}\ m \bullet r^2$
solid sphere	$\frac{2}{5}\ m \bullet r^2$
hollow sphere	$\frac{2}{3}\ m \bullet r^2$
thin rod (center)	$\frac{1}{12}\ m \bullet L^2$
thin rod (end)	$\frac{1}{3}\ m \bullet L^2$

#59 Capacitors $Q = C \bullet V$

Q = charge on the capacitor
C = capacitance of the capacitor
V = voltage applied to the capacitor

RC Circuits (Discharging)

$V_c = V_o \bullet e^{-t/RC}$

$V_c - I \bullet R = 0$

#60 Thermal Expansion

Linear: $\Delta L = L_o \bullet \alpha \bullet \Delta T$

Volume: $\Delta V = V_o \bullet \beta \bullet \Delta T$

#61 Bernoulli's Equation

$P + \rho \bullet g \bullet h + \frac{1}{2} \bullet \rho \bullet v^2$ = constant

$Q_{Volume\ Flow\ Rate} = A_1 \bullet v_1 = A_2 \bullet v_2$ = constant

#62 Rotational Kinetic Energy (See LEM, pg 8)

$KE_{rotational} = \frac{1}{2} \bullet I \bullet \omega^2 = \frac{1}{2} \bullet I \bullet (v / r)^2$

$KE_{rolling\ w/o\ slipping} = \frac{1}{2} \bullet m \bullet v^2 + \frac{1}{2} \bullet I \bullet \omega^2$

Angular Momentum = L = $I \bullet \omega = m \bullet v \bullet r \bullet \sin\theta$

Angular Impulse equals CHANGE IN Angular Momentum

$\Delta L = \tau_{orque} \bullet \Delta t = \Delta(I \bullet \omega)$

#63 Period of Simple Harmonic Motion

$$T = 2\pi\sqrt{\frac{m}{k}}$$ where k = spring constant

$f = 1 / T = 1$ / period

#64 Banked Circular Tracks

$v^2 = r \cdot g \cdot \tan\theta$

#66 First Law of Thermodynamics

$\Delta U = Q_{Net} + W_{Net}$

Change in Internal Energy of a system =
+Net Heat added to the system
+Net Work done on the system

Flow of Heat through a Solid

$\Delta Q / \Delta t = k \cdot A \cdot \Delta T / L$

k = thermal conductivity
A = area of solid
L = thickness of solid

#68 Potential Energy stored in a Capacitor

$P = \frac{1}{2} \cdot C \cdot V^2$

RC Circuit formula (Charging)

$V_c = V_{cell} \cdot (1 - e^{-t/RC})$

$R \cdot C = \tau$ = time constant

$V_{cell} - V_{capacitor} - I \cdot R = 0$

#71 Simple Pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$ and $f = 1/T$

#72 Sinusoidal motion

$x = A \cdot \cos(\omega \cdot t) = A \cdot \cos(2 \cdot \pi \cdot f \cdot t)$

ω = angular frequency
f = frequency

#73 Doppler Effect

$$f' = f\,\frac{343 \pm_{Away}^{Toward} v_o}{343 \mp_{Away}^{Toward} v_s}$$

v_o = velocity of observer: v_s = velocity of source

#74 2nd Law of Thermodynamics

The change in internal energy of a system is
$\Delta U = Q_{Added} + W_{Done\ On} - Q_{lost} - W_{Done\ By}$

Maximum Efficiency of a Heat Engine (Carnot Cycle) (Temperatures in Kelvin)

$$\%Eff = \left(1 - \frac{T_c}{T_h}\right) \cdot 100\%$$

#75 Thin Lens Equation

$$\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{o} + \frac{1}{i}$$

f = focal length
i = image distance
o = object distance

Magnification

$M = -D_i / D_o = -i / o = H_i / H_o$

Helpful reminders for mirrors and lenses

Focal Length of:	positive	negative
mirror	concave	convex
lens	converging	diverging
Object distance = o	all objects	
Object height = H_o	all objects	
Image distance = i	real	virtual
Image height = H_i	virtual, upright	real, inverted
Magnification	virtual, upright	real, inverted

#76 Coulomb's Law

$$F = k\frac{q_1 q_2}{r^2}$$

$$k = \frac{1}{4\pi\varepsilon_o} = 9E9\frac{N \cdot m^2}{C^2}$$

#77 Capacitor Combinations

PARALLEL

$C_{eq} = C_1 + C_2 + C_3 + \ldots$

SERIES

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n} = \sum_{i=1}^{n}\frac{1}{C_i}$$

#78 Work done on a gas or by a gas

$W = P \cdot \Delta V$

#80 Electric Field around a point charge

$$E = k\frac{q}{r^2}$$

$$k = \frac{1}{4\pi\varepsilon_o} = 9E9\frac{N \cdot m^2}{C^2}$$

#82 Magnetic Field around a wire

$$B = \frac{\mu_o I}{2\pi r}$$

Magnetic Flux

$\Phi = B \cdot A \cdot \cos\theta$

Force caused by a magnetic field on a moving charge

$F = q \cdot v \cdot B \cdot \sin\theta$

#83 Entropy change at constant T

$\Delta S = Q / T$

(Phase changes only: melting, boiling, freezing, etc)

#84 Capacitance of a Capacitor

$C = \kappa \cdot \varepsilon_o \cdot A / d$

κ = dielectric constant
A = area of plates
d = distance between plates
ε_o = 8.85 E(-12) F/m

#85 Induced Voltage N = # of loops

$$Emf = N\frac{\Delta\Phi}{\Delta t}$$

Lenz's Law – induced current flows to create a B-field opposing the change in magnetic flux.

#86 Inductors during an increase in current

$V_L = V_{cell} \cdot e^{-t/(L/R)}$

$I = (V_{cell}/R) \cdot [1 - e^{-t/(L/R)}]$

$L / R = \tau$ = time constant

#88 Transformers

$N_1 / N_2 = V_1 / V_2$

$I_1 \cdot V_1 = I_2 \cdot V_2$

#89 Decibel Scale

B (Decibel level of sound) = 10 log (I / I_o)

I = intensity of sound
I_o = intensity of softest audible sound

#92 Poiseuille's Law

$\Delta P = 8 \cdot \eta \cdot L \cdot Q/(\pi \cdot r^4)$

η = coefficient of viscosity
L = length of pipe
r = radius of pipe
Q = flow rate of fluid

Stress and Strain

Y or S or **B = stress / strain**

stress = F/A

Three kinds of strain: unit-less ratios

I. Linear: strain = $\Delta L / L$
II. Shear: strain = $\Delta x / L$
III. Volume: strain = $\Delta V / V$

#93 Postulates of Special Relativity

1. Absolute, uniform motion cannot be detected.
2. No energy or mass transfer can occur at speeds faster than the speed of light.

#94 Lorentz Transformation Factor

$$\beta = \sqrt{1 - \frac{v^2}{c^2}}$$

#95 Relativistic Time Dilation

$\Delta t = \Delta t_o / \beta$

#96 Relativistic Length Contraction

$\Delta x = \beta \cdot \Delta x_o$

Relativistic Mass Increase

$m = m_o / \beta$

#97 Energy of a Photon or a Particle

$E = h \cdot f = m \cdot c^2$

h = Planck's constant = 6.63 E(-34) J sec
f = frequency of the photon

#98 Radioactive Decay Rate Law

$A = A_o \cdot e^{-kt} = (1/2^n) \cdot A_0$ (after n half-lives)

Where k = (ln 2) / half-life

#99 Blackbody Radiation and the Photoelectric Effect

$E = n \cdot h \cdot f$ where h = Planck's constant

#100 Early Quantum Physics

Rutherford-Bohr Hydrogen-like Atoms

$$\frac{1}{\lambda} = R \cdot \left(\frac{1}{n_s^2} - \frac{1}{n^2}\right) meters^{-1}$$

or

$$f = \frac{c}{\lambda} = cR\left(\frac{1}{n_s^2} - \frac{1}{n^2}\right) Hz$$

R = Rydberg's Constant
= 1.097373143 E7 m^{-1}
n_s = series integer (2 = Balmer)
n = an integer > n_s

Mass-Energy Equivalence

$m_v = m_o / \beta$

Total Energy = KE + $m_o c^2 = m_o c^2 / \beta$

Usually written simply as $E = m c^2$

de Broglie Matter Waves

For light: $E_p = h \cdot f = h \cdot c / \lambda = p \cdot c$

Therefore, momentum: $p = h / \lambda$

Similarly for particles, $p = m \cdot v = h / \lambda$, so the matter wave's wavelength must be

$\lambda = h / m v$

Energy Released by Nuclear Fission or Fusion Reaction

$E = \Delta m_o \cdot c^2$

MISCELLANEOUS FORMULAS

Quadratic Formula

if $a x^2 + b x + c = 0$

then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric Definitions

sin θ = opposite / hypotenuse
cos θ = adjacent / hypotenuse
tan θ = opposite / adjacent

sec θ = 1 / cos θ = hyp / adj
csc θ = 1 / sin θ = hyp / opp
cot θ = 1 / tan θ = adj / opp

Inverse Trigonometric Definitions

$\theta = \sin^{-1}$ (opp / hyp)
$\theta = \cos^{-1}$ (adj / hyp)
$\theta = \tan^{-1}$ (opp / adj)

Law of Sines

a / sin A = b / sin B = c / sin C
or
sin A / a = sin B / b = sin C / c

Law of Cosines

$a^2 = b^2 + c^2 - 2\,b\,c\cos A$
$b^2 = c^2 + a^2 - 2\,c\,a\cos B$
$c^2 = a^2 + b^2 - 2\,a\,b\cos C$

T-Pots

For the functional form

$$\frac{1}{A} = \frac{1}{B} + \frac{1}{C}$$

You may use "The **P**roduct **o**ver **t**he **S**um" rule.

$$A = \frac{B \cdot C}{B + C}$$

For the Alternate Functional form

$$\frac{1}{A} = \frac{1}{B} - \frac{1}{C}$$

You may substitute T-Pot-d

$$A = \frac{B \cdot C}{C - B} = -\frac{B \cdot C}{B - C}$$

Fundamental SI Units

Unit	Base Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Luminous Intensity	candela	cd
Quantity of Substance	moles	mol
Plane Angle	radian	rad
Solid Angle	steradian	sr or str

Some Derived SI Units

Symbol/Unit	Quantity	Base Units
C coulomb	Electric Charge	A•s
F farad	Capacitance	A^2•s4/(kg•m^2)
H henry	Inductance	kg•m^2/(A^2•s^2)
Hz hertz	Frequency	s^{-1}
J joule	Energy & Work	kg•m^2/s^2 = N•m
N newton	Force	kg•m/s^2
Ω ohm	Elec Resistance	kg•m^2/(A^2•s^2)
Pa pascal	Pressure	kg/(m•s^2)
T tesla	Magnetic Field	kg/(A•s^2)
V volt	Elec Potential	kg•m^2/(A•s^3)
W watt	Power	kg•m^2/s^3

Non-SI Units

°C	degrees Celsius	Temperature
eV	electron-volt	Energy, Work

Aa acceleration, Area, A_x=Cross-sectional Area, Amperes, Amplitude of a Wave, Angle,
Bb Magnetic Field, Decibel Level of Sound, Angle,
Cc specific heat, speed of light, Capacitance, Angle, Coulombs, ^{o}Celsius, Celsius Degrees, candela,
Dd displacement, differential change in a variable, Distance, Distance Moved, distance,
Ee base of the natural logarithms, charge on the electron, Energy,
Ff Force, *frequency of a wave or periodic motion*, Farads,
Gg Universal Gravitational Constant, acceleration due to gravity, Gauss, grams, Giga-,
Hh depth of a fluid, height, vertical distance, Henrys, Hz=Hertz,
Ii Current, Moment of Inertia, image distance, Intensity of Sound,
Jj Joules,
Kk K or KE = Kinetic Energy, force constant of a spring, thermal conductivity, coulomb's law constant, kg=kilograms, Kelvins, kilo-, rate constant for Radioactive decay $=1/\tau=\ln 2$ / half-life,
Ll Length, Length of a wire, Latent Heat of Fusion or Vaporization, Angular Momentum, Thickness, Inductance,
Mm mass, Total Mass, meters, milli-, Mega-, m_o=rest mass, mol=moles,
Nn index of refraction, moles of a gas, Newtons, Number of Loops, nano-,
Oo
Pp Power, Pressure of a Gas or Fluid, Potential Energy, momentum, Power, Pa=Pascal,
Qq Heat gained or lost, Maximum Charge on a Capacitor, object distance, Flow Rate,
Rr radius, Ideal Gas Law Constant, Resistance, magnitude or length of a vector, rad=radians
Ss speed, seconds, Entropy, length along an arc,
Tt time, Temperature, Period of a Wave, Tension, Teslas, $t_{1/2}$=half-life,
Uu Potential Energy, Internal Energy,
Vv velocity, Velocity, Volume of a Gas, velocity of wave, Volume of Fluid Displaced, Voltage, Volts,
Ww weight, Work, Watts, Wb=Weber,
Xx distance, horizontal distance, x-coordinate east-and-west coordinate,
Yy vertical distance, y-coordinate, north-and-south coordinate,
Zz z-coordinate, up-and-down coordinate,

Αα **Alpha** angular acceleration, coefficient of linear expansion,
Ββ **Beta** coefficient of volume expansion, Lorentz transformation factor,
Χχ **Chi**

Δδ **Delta** Δ=change in a variable,

Εε **Epsilon** ε_o = permittivity of free space,

Φϕ **Phi** Magnetic Flux, angle,

Γγ **Gamma** surface tension = F / L, 1 / γ = Lorentz transformation factor,
Ηη **Eta**

Ιι **Iota**

ϑφ **Theta** and **Phi** lower case alternates.
Κκ **Kappa** dielectric constant,

Λλ **Lambda** wavelength of a wave, rate constant for Radioactive decay $=1/\tau=\ln 2$/half-life,

Μμ **Mu** friction, μ_o = permeability of free space, micro-,
Νν **Nu** alternate symbol for frequency,

Οο **Omicron**
Ππ **Pi** 3.1425926536…,

Θθ **Theta** angle between two vectors,

Ρρ **Rho** density of a solid or liquid, resistivity,

Σσ **Sigma** Summation, standard deviation,
Ττ **Tau** torque, time constant for a exponential processes; eg τ=RC or τ=L/R or τ=1/k=1/λ,
Υυ **Upsilon**
ςϖ **Zeta** and **Omega** lower case alternates
Ωω **Omega** angular speed or angular velocity, Ohms
Ξξ **Xi**

Ψψ **Psi**

Ζζ **Zeta**

Values of Trigonometric Functions for 1st Quadrant Angles

(simple mostly-rational approximations)

θ	sin θ	cos θ	tan θ
0°	**0**	**1**	**0**
10°	*1/6*	*65/66*	*11/65*
15°	*1/4*	*28/29*	*29/108*
20°	*1/3*	*16/17*	*17/47*
29°	*$15^{1/2}/8$*	*7/8*	*$15^{1/2}/7$*
30°	**1/2**	**$3^{1/2}/2$**	**$1/3^{1/2}$**
37°	**3/5**	**4/5**	**3/4**
42°	*2/3*	*3/4*	*8/9*
45°	**$2^{1/2}/2$**	**$2^{1/2}/2$**	**1**
49°	*3/4*	*2/3*	*9/8*
53°	**4/5**	**3/5**	**4/3**
60	**$3^{1/2}/2$**	**1/2**	**$3^{1/2}$**
61°	*7/8*	*$15^{1/2}/8$*	*$7/15^{1/2}$*
70°	*16/17*	*1/3*	*47/17*
75°	*28/29*	*1/4*	*108/29*
80°	*65/66*	*1/6*	*65/11*
90°	**1**	**0**	**∞**

(Memorize the **Bold** *rows for future reference.)*

Derivatives of Polynomials

For polynomials, with individual terms of the form Ax^n, we define the derivative of each term as

$$\frac{d}{dx}\left(Ax^n\right) = nAx^{n-1}$$

To find the derivative of the polynomial, simply add the derivatives for the individual terms:

$$\frac{d}{dx}\left(3x^2 + 6x - 3\right) = 6x + 6$$

Integrals of Polynomials

For polynomials, with individual terms of the form Ax^n, we define the indefinite integral of each term as

$$\int \left(Ax^n\right)dx = \frac{1}{n+1}Ax^{n+1}$$

To find the indefinite integral of the polynomial, simply add the integrals for the individual terms and the constant of integration, C.

$$\int (6x+6)dx = \left[3x^2 + 6x + C\right]$$

Prefixes

Factor	Prefix	Symbol	Example
10^{18}	exa-	E	**38 Es** (Age of the Universe in Seconds)
10^{15}	peta-	P	
10^{12}	tera-	T	**0.3 TW** (Peak power of a 1 ps pulse from a typical Nd-glass laser)
10^{9}	giga-	G	**22 G$** (Size of Bill & Melissa Gates' Trust)
10^{6}	mega-	M	**6.37 Mm** (The radius of the Earth)
10^{3}	kilo-	k	**1 kg** (SI unit of mass)
10^{-1}	deci-	d	**10 cm**
10^{-2}	centi-	c	**2.54 cm** (=1 in)
10^{-3}	milli-	m	**1 mm** (The smallest division on a meter stick)
10^{-6}	micro-	μ	
10^{-9}	nano-	n	**510 nm** (Wave-length of green light)
10^{-12}	pico-	p	**1 pg** (Typical mass of a DNA sample used in genome studies)
10^{-15}	femto-	f	
10^{-18}	atto-	a	**600 as** (Time duration of the shortest laser pulses)

Linear Equivalent Mass

Rotating systems can be handled using the linear forms of the equations of motion. To do so, however, you must use a mass equivalent to the mass of a non-rotating object. We call this the Linear Equivalent Mass (LEM). *(See Example I)*

For objects that are both rotating and moving linearly, you must include them twice; once as a linearly moving object (using m) and once more as a rotating object (using LEM). *(See Example II)*

The LEM of a rotating mass is easily defined in terms of its moment of inertia, I.

$$LEM = I/r^2$$

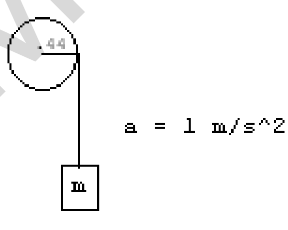
For example, using a standard table of Moments of Inertia, we can calculate the LEM of simple objects rotating on axes through their centers of mass:

	I	LEM
Cylindrical hoop	mr^2	m
Solid disk	$\frac{1}{2}mr^2$	$\frac{1}{2}m$
Hollow sphere	$\frac{2}{5}mr^2$	$\frac{2}{5}m$
Solid sphere	$\frac{2}{3}mr^2$	$\frac{2}{3}m$

Example I

A flywheel, M = 4.80 kg and r = 0.44 m, is wrapped with a string. A hanging mass, m, is attached to the end of the string.

When the hanging mass is released, it accelerates downward at 1.00 m/s^2. Find the hanging mass.



To handle this problem using the linear form of Newton's Second Law of Motion, all we have to do is use the LEM of the flywheel. We will assume, here, that it can be treated as a uniform solid disk.

The only external force on this system is the weight of the hanging mass. The mass of the system consists of the hanging mass plus the linear equivalent mass of the fly-wheel. From Newton's 2nd Law we have

F = ma, therefore, $mg = [m + (LEM = \frac{1}{2}M)]a$

$mg = [m + \frac{1}{2}M]\, a$

$(mg - ma) = \frac{1}{2}M\, a$

$m(g - a) = \frac{1}{2}Ma$

$m = \frac{1}{2} \bullet M \bullet a / (g - a)$

$m = \frac{1}{2} \bullet 4.8 \bullet 1.00 / (9.81 - 1)$

$m = 0.27$ kg

If $a = g/2 = 4.905$ m/s^2, $m = 2.4$ kg

If $a = \frac{3}{4}g = 7.3575$ m/s^2, $m = 7.2$ kg

Note, too, that we do not need to know the radius unless the angular acceleration of the fly-wheel is requested. If you need α, and you have r, then $\alpha = a/r$.

Example II

Find the kinetic energy of a disk, m = 6.7 kg, that is moving at 3.2 m/s while rolling without slipping along a flat, horizontal surface. ($I_{DISK} = \frac{1}{2}mr^2$; LEM = $\frac{1}{2}$m)

The total kinetic energy consists of the linear kinetic energy, $K_L = \frac{1}{2}mv^2$, plus the rotational kinetic energy, $K_R = \frac{1}{2}(I)(\omega)^2 = \frac{1}{2}(I)(v/r)^2 = \frac{1}{2}(I/r^2)v^2 = \frac{1}{2}(LEM)v^2$.

$KE = \frac{1}{2}mv^2 + \frac{1}{2} \bullet (LEM = \frac{1}{2}m) \bullet v^2$

$KE = \frac{1}{2} \bullet 6.7 \bullet 3.2^2 + \frac{1}{2} \bullet (\frac{1}{2} \bullet 6.7) \bullet 3.2^2$

$KE = 34.304 + 17.152 = 51$ J

Final Note:

This method of incorporating rotating objects into the linear equations of motion works in every situation I've tried; even very complex problems. Work your problem the classic way and this way to compare the two. Once you've verified that the LEM method works for a particular type of problem, you can confidently use it for solving any other problem of the same type.