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Chapter 1 Probability

1 PROBABILITY

Objectives

After studying this chapter you should

• understand how the probability of an event happening is
  measured;

• recognise whether or not events are related in any way;

• be able to assess the likelihood of events occurring.

1.0   Introduction

'Sue is more likely than Jane to be head girl next year.'

'It will probably rain for the fete tomorrow.'

'A European football team has a better chance of winning the
next world cup than a South American one.'

'Reza is 'odds on' to beat Leif in the chess final.'

All these sentences express an opinion that one outcome is more
likely than another but in none of them is there any attempt to
say by how much. Yet if you want to take out insurance against
bad weather for the fete the insurance company you approach
must have a way of calculating the probability or likelihood of
rain to know how much to charge.

So how can you assess the chance that some event will
actually happen?

1.1   Theoretical probability: symmetry

Many intuitive ideas of chance and probability are based on the
idea of **symmetry**. Consider the following questions:

If you toss a coin repeatedly, how many times will it come
down heads?

If you roll a die how often will you get a four?

If you roll two dice several times, how often will you get two
sixes?

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For the second question, your answer should be about one in six times provided the die is a fair one. Another way of expressing this is to say that the probability of obtaining 4 is

$$\frac{1}{6} \quad \Rightarrow \quad p(4) = \frac{1}{6}.$$

The answer is dependent on the idea of symmetry. That is, every possible outcome (namely 1, 2, 3, 4, 5 and 6) is equally likely to occur. So the probability of any one score must be $\frac{1}{6}$.

Sometimes, though, you must be very careful to make sure that you have a complete list of **all** the possible outcomes of the event under consideration.
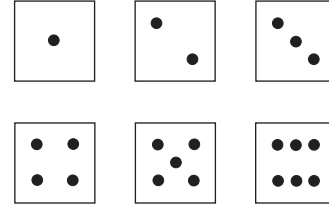
Activity 1   The three card game

Suppose you have three cards:

Card A   is white on both sides

Card B   is black on both sides

Card C   is black on side 1 and white on side 2.

You shuffle them and place them in a pile on the table so that you can see only the upper face of the top card, which is black.

If I were to say,

"I will pay you £5 if the reverse face of the top card is white   and you pay me £3 if it is black."

should you take the bet?
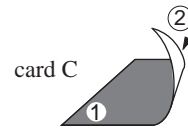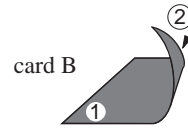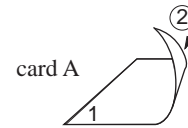
If you said that there are two possibilities - the lower face is either black or white - then this is certainly correct. However, if you have gone on to decide that you are just as likely to win as to lose then perhaps you have not listed all the possible cases.

With the three cards, if you can see a black face then the three possibilities are that you are looking at

   side 1 of Card C   side 1 of Card B   side 2 of Card B

and since two of these (side 1 and side 2 of Card B) have black on the reverse, the bet is not a good one for you.

In the long run, in three games you would win £5 once and lose £3 twice, so you can expect to lose £1 on average every three games or $33\frac{1}{3}$ p per go.

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Activity 2

Play the three card game a number of times with a friend. You can use either cards, as shown on the previous page, or a die with 1, 2, 6 painted black and 3, 4, 5 white. Remember to always bet on the same colour being underneath as is showing on top.

Listing all the equally likely outcomes can be very tedious so you may find it simpler and clearer to show them in a diagram.
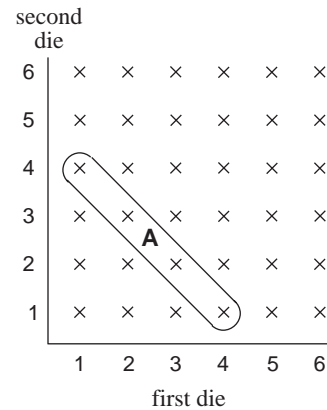
For example, when two dice are rolled there are thirty six possible outcomes which can be shown very neatly in a diagram (see opposite).

This is called the **sample spac**e. You can see by looking at the crosses in the area labelled **A** that, for example,

$$P(\text{total} = 5) = \frac{4}{36} = \frac{1}{9}.$$

This sort of diagram can be adapted to other problems so it is very useful.

What is $P(\text{total} = 7)$**?**

Example

Two of the five reserves for the school ski trip, Tamsin, John, Atanu, Robin and David can have places now that a couple of people have had to drop out. How likely is it that John and Tamsin will be chosen to go?

Solution

Only the two cases indicated out of the twenty in the diagram opposite are situations when John and Tamsin are chosen, so

$$P(\text{T and J}) = \frac{2}{20} = \frac{1}{10}.$$
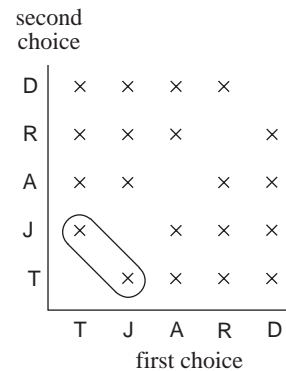
You know that with **one** die there are six different possible outcomes and the diagram for **two** dice showed that there are thirty six possible outcomes in this case.

**How many will there be if three dice are used?**

What sort of diagram could be drawn to show the different results?

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As one die needs a one-dimensional diagram which gives six possibilities and two dice need a two-dimensional diagram to show thirty six outcomes, a sensible idea to try for three dice would be a three-dimensional picture.

The diagram opposite shows six of the two-dimensional diagrams in layers on top of each other so there are $6 \times 36 = 216$ possibilities in this case or 6^3.

The • in the diagram represents 3 on the first die, 2 on the second and 6 on the third.

The number of dice used appears as a power in these examples so it should be possible to work out the total number of outcomes when more than three dice are used.
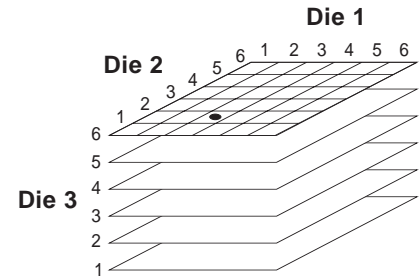
Example

What is the probability of getting five sixes when five dice are rolled?

Solution

Five dice produce $6^5 = 7776$ outcomes.

Only one outcome is all sixes, so

$$P(\text{five sixes}) \ = \ \frac{1}{7776}.$$

Example

What is the probability that there will be at least one head in five tosses of a fair coin?

Solution

Five coins produce $2^5 = 32$ outcomes.

Only T T T T T does not contain at least one head, so

$$P(\text{at least one H}) \ = \ \frac{31}{32}.$$

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Exercise 1A

1. What is the probability of choosing an even number from the set of numbers {1, 2, 3, 5, 6, 7, 8, 10}?

2. When two six-sided dice are rolled what is the probability that the product of their scores will be greater than six?

3. If you have three 10p coins and two 50p coins in your pocket and you take out two at random, what is the probability that they add up to 60p?

   (Draw a sample space.)

4. If two people are chosen at random what is the probability that they were born on the same day of the week?

5. List the ways in which one head and five tails may be obtained from six tosses of a coin. How many ways are there?

6. Two dice are rolled and the 'score' is the product of the two numbers showing uppermost. If the probability is $\frac{11}{36}$ that the score is at least N, what is the value of N?

7. Pierre and Julian each roll one die. If Pierre's shows the higher number then he wins 7p, otherwise he loses 5p. Explain why this is fair. If Pierre were to add three dots to convert the two on his die to a five, how will it affect his winning?

8. A card is chosen at random from a pack of fifty-two. It is replaced and a second card is selected. What is the probability that at least one is a picture card (Jack, Queen, King)? (Sketch a sample space but don't bother with all the crosses.)

9. Eight people are seated at random round a table. What is the probability that Sharif and Raijit will be next to each other?

1.2   Empirical probability: experiment

Mathematicians' early interest in the subject of probability in the seventeenth century came largely as a result of questions from gamblers in France. Since dice, cards, etc. were used, the situations involved had outcomes which were equally likely. All the arguments then could be based on symmetry. You must also be prepared, however, for other situations which do not have properties of symmetry.
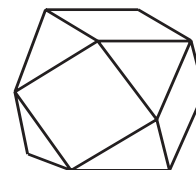
It was possible to answer the question about the die as there were six possible outcomes which were equally likely to occur as the cube is a simple regular solid. However, you might find questions about a **cuboctahedron** not as simple to answer.

This solid is formed by cutting equilateral triangles from the corners of a cube to produce six square and eight triangular faces.

Cuboctahedron

What is the probability of the solid ending with a square facing upwards when it is rolled?

Perhaps it depends on how many of the faces are squares. Or does considering the areas of the squares as a fraction of the total surface area seem more likely?

Without testing and evidence nobody will believe any answer

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you give to the question so you will need to experiment to find
the probability of a square facing upwards.

Activity 3

Find the answer for yourself by making the solid from a copy of
the net. Be prepared to roll it many times.
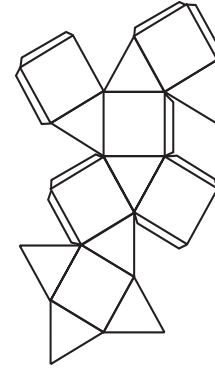
Net of a cuboctahedron

You can see the probability graphically by plotting the number
of rolls on the *x*-axis and the fraction of the times a square is
facing upwards on the *y*-axis.

You could, for example, see a square seven times in the first ten
goes and nine times in the next ten goes, so a table could start:

No. of squares in 10 goes	7	9	
Total no. of squares	7	16	
Total no. of rolls	10	20	30
Fraction (probability)	$\frac{7}{10} = 0.7$	$\frac{16}{20} = 0.8$	
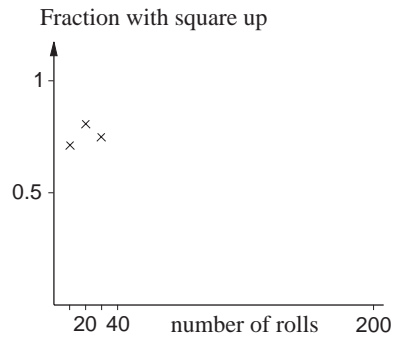
Of course, if several people are doing this experiment you could
put your results together to achieve a more reliable answer.

What you will often find from experiment is that the fraction
calculated will gradually cease to vary much and will become
closer to the value called the **probability.**

What is the probability of a square not appearing uppermost?

From this experiment you will have produced an **empirical
probability**; i.e. one based on experience rather than on a
logical argument.

The idea of experiment and observation then gives a probability
equal to

$$\frac{\text{the number of times a square was upwards}}{\text{the number of attempts}}$$

So if you saw it happen on 150 out of 200 times you will have
come to the conclusion that

$$P(\text{square}) = \frac{150}{200} = \frac{3}{4}.$$

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In reality, if the true probability was $\frac{3}{4}$, you would be unlikely to get exactly 150 out of 200 – but you should be somewhere near it.

Activity 4   Coin tossing

Toss an unbiased coin 100 times, and record the total fraction of heads after every 10 goes. Plot these on a graph of fraction of heads against number of goes. Does this indicate that the coin is a fair one?

1.3   Empirical probability: observation

How likely is it that the writer of this text is alive now?   It is hard to conduct an experiment on this but if I am forty now and writing this in 1991 then you can make use of observations on the life expectancy of forty-year-old males.

Male age	Average life expectancy beyond present age	Probability of surviving at least 5 years
35	38.1	0.993
36	37.1	
37	36.2	
38	35.2	
39	34.3	
40	33.3	0.988
41	32.4	
42	31.5	
43	30.5	
44	29.6	
45	28.7	0.979

Data like these are needed by insurance companies for their life policies. Some people will look at tables of figures for sunshine hours and rainfall when planning holidays.

Probability is of interest to people working in economics, genetics, astronomy and many other fields where it may be difficult to experiment but where data can be gathered by observation over a long period.

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Example

Jane travels to school on the train every weekday and often sees rabbits in a field by the track. In four weeks her observations were

Number of rabbits seen	0	1	2	3	4	5	6	7	8
Number of occasions	0	3	5	7	2	1	0	1	1

What is the probability that on her next journey she will see at least two rabbits?

Solution

$$P(\text{at least two rabbits}) = \frac{17}{20} = 0.85,$$

as on $5+7+2+1+0+1+1 = 17$ days out of the 20 she saw two or more rabbits.

Exercise 1B

1. Using the information from the example above, what is the probability that Jane sees:

   (a) 3 or 4 rabbits;   (b) 6 rabbits;

   (c) at least one rabbit?

2. The number of visitors to the UK from North America in 1988 is given below in categories to show mode of travel and purpose of visit.

	Air	Sea
Holiday	1269	336
Business	605	17
Friends and relatives	627	55
Miscellaneous	324	39

   (in 1000s)

   If you were to have met a visitor from North America in 1988 what would have been the probability that the visitor

   (a) was here on business;

   (b) came by sea;

   (c) came by air to visit friends or relatives;

   (d) was here on business if you know the visitor came by sea?

3. Draw a circle of radius 5 cm and add a square of side 10 cm so that the circle touches its four sides.

   Take random numbers from a table, four at a time, and interpret them as co-ordinates using the bottom left hand corner of the square as the origin. (For example, the numbers 4 6 2 0 give the point (4.6, 2.0) with measurements in cm.)

   Use a large number of points and see what fraction of them lie inside the circle.

   (The area of the square is 100 units and the area of the circle is $\pi 5^2 = 25\pi$. The fraction of the square taken up by the circle is

   $$\frac{25\pi}{100} = \frac{\pi}{4}$$

   so your result is an approximation to $\frac{\pi}{4}$ and can be used to estimate π.)

4. Take ten drawing pins and drop them onto a flat surface. Note how many finish point up. Repeat this several times and produce a table and graph like those you used with the cuboctahedron.

   What is the probability that a drawing pin accidentally dropped will fall into a point-up position?

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1.4   Combined events

Complement

In the probability experiment in Section 1.2 you will have obtained a value for probability by considering, for example, the number of times a square face finished uppermost as a fraction of the total number of rolls as

$$P(\text{square}) = \frac{\text{no. of times square finished upwards}}{\text{no. of trials}}$$

This is also called **relative frequency**.

The largest value this fraction can take is one, when a square face appears every trial, and the smallest it can be is zero, when a triangle is uppermost on each go, so

$$0 \leq \text{ probability} \leq 1.$$

Another result that may be obvious is that the number of times with a square facing up plus the number of times with a triangle facing up equals the number of trials.

Hence

$$\frac{\text{no. of times with square up}}{\text{no. of trials}} + \frac{\text{no. of times with triangle up}}{\text{no. of trials}} = 1$$

$$\Rightarrow \quad P(\text{square}) + P(\text{not square}) = 1$$

which is written in general as

$$\boxed{P(A) + P(A') = 1}$$

where A' means 'not A' or the '**complement** of A'.

You may well have used this idea earlier when you answered the question in Section 1.2 about how likely it is for a square not to appear on the top face when a cuboctahedron is rolled.

Intersection

Take a cube and mark on its different faces three black circles, one black cross and two red crosses.

When it is rolled, what are the probabilities of getting

red,   black,   circle   **and**   cross ?

What is the likelihood of getting a black symbol and a cross?

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You can see that just one of the cube's six faces is covered by this description, so

$$P(\text{black and cross}) = \frac{1}{6}.$$

This can be written as

$$P(\text{black} \cap \text{cross}) = \frac{1}{6}.$$

This is known as the intersection; so $P(A \cap B)$ means the probability of both events, A and B, happening.

Another way of showing all the possibilities is illustrated opposite.

These are called **Venn diagrams** after *John Venn*, an English mathematician and churchman, who studied logic and taught at Cambridge.

What is the value of $P(\text{red} \cap \text{cross})$**?**

You may have noticed that

$$P(\text{red} \cap \text{cross}) + P(\text{black} \cap \text{cross}) \;=\; P(\text{cross}).$$

If you were asked for the probability of a circle **and** a red symbol finishing uppermost from a single roll you should realise that

$$P(\text{red} \cap \text{circle}) = 0$$

as the two cannot happen at the same time. These are called **mutually exclusive** events as the occurrence of either excludes the possibility of the other one happening too.
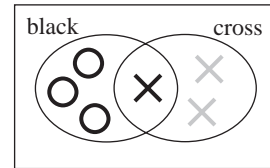
Union

Eight teams are entered for a knock-out netball tournament and two of these are the YWCA and the Zodiac youth club.

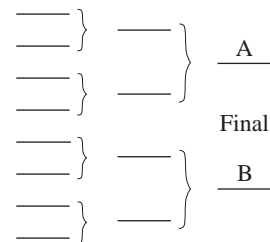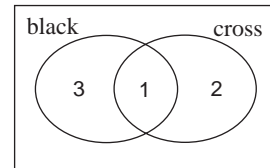**What is the probability that the YWCA or Zodiac will reach the final?**

('or' here means one or the other or both, more technically called the inclusive disjunction.)

How the competition will run is shown opposite but until the draw is made no names can be entered.

A diagram like the one you used earlier shows all the different possible ways in which the two final places A and B may be filled by the competing teams.
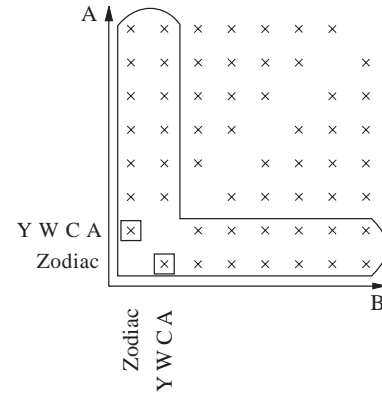
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From the figure opposite you can see that

(a) $P(\text{Zodiac in final}) = \dfrac{14}{56}$

(b) $P(\text{YWCA in final}) = \dfrac{14}{56}$

(c) $P(\text{Zodiac or YWCA}) = \dfrac{26}{56}$.

Note that $P(\text{Zodiac or YWCA}) \neq P(\text{Zodiac}) + P(\text{YWCA})$.

Why would you expect these not to be equal?

When the first two probabilities (a) and (b) were worked out, the two cases marked with squares in the diagram were included in each answer. When the probabilities are added together, these probabilities have been counted twice.

These correspond to the two ways of having both Zodiac and YWCA in the final. Their probability is given by

$$P(Z \cap Y) = \frac{2}{56}$$

and you can see that

$$P(Z \text{ or } Y) = P(Z) + P(Y) - P(Z \cap Y).$$

Taking off the $P(Z \cap Y)$ ensures that these two events are not counted twice.

Checking with the figures you get

$$\frac{14}{56} + \frac{14}{56} - \frac{2}{56} = \frac{26}{56}$$

which is true.

Now if you look back to the die marked with circles and crosses you will see that

$$P(\text{black}) = \frac{2}{3}, \quad P(\text{circle}) = \frac{1}{2}$$

so that if you tried to say that

$$P(\text{black or circle}) = P(\text{black}) + P(\text{circle})$$

you would get $P(B \cup C) = \frac{1}{2} + \frac{2}{3} = 1\frac{1}{6}$, where $B \cup C$ means B or C.

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This looks decidedly dubious as you know that probability is measured on a scale from zero to one! The problem is that once more you have counted some of the possibilities twice as they are in both categories. Again, if you try

$$P(B \cup C) = P(B) + P(C) - P(B \cap C)$$

then a true statement results:

$$\frac{2}{3} + \frac{1}{2} - \frac{1}{2} = \frac{2}{3}.$$

The $\frac{2}{3}$ on the left is correct as four of the six faces have a black colour or a circle or both.

Is it ever true that $P(A \cup B) = P(A) + P(B)$**?**

If it is, then $P(A \cap B)$ must be zero and this means that the events are **mutually exclusive**. A Venn diagram could be drawn and would look like the one here with no overlap. So if
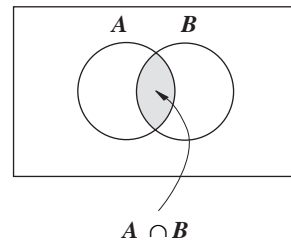
$$P(A \cap B) = 0$$

then $\qquad P(A \cup B) = P(A) + P(B).$

In general though,

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

and this can be illustrated by the Venn diagram opposite. The **intersection** of the two sets, $A \cap B$, is shown whilst the **union**, $A \cup B$, is given by everything inside A and B.
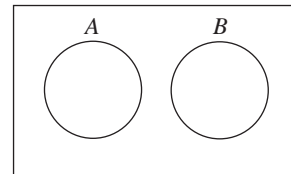
Exhaustive probabilities

The cube you looked at marked with crosses and circles had faces as shown opposite.

What is the value of $P(\text{black} \cup \text{cross})$**?**

Since each of the six symbols was black or a cross then

$$P(\text{black} \cup \text{cross}) = 1$$

and the events 'getting a black symbol 'and 'getting a cross' are said to form a pair of **exhaustive events**. Between them they exhaust all the possible outcomes and therefore all the probability, i.e. one.

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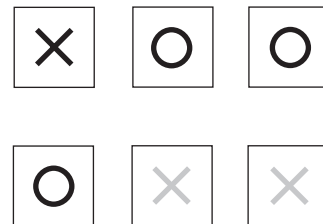
So, if A and B are exhaustive events

$$P(A \cup B) = 1$$

Exercise 1C

1. In a class at school $\frac{1}{2}$ of the pupils represent the school at a winter sport, $\frac{1}{3}$ represent the school at a summer sport and $\frac{1}{10}$ do both. If a pupil is chosen at random from this group what is the probability that someone who represents the school at sport will be selected?

2. If the probability that Andrea will receive the maths prize this year is $\frac{1}{3}$ and the probability that Philson will win it is $\frac{1}{4}$, what is the chance that one of them will get it?

3. In a certain road $\frac{1}{5}$ of the houses have no newspapers delivered. If $\frac{1}{2}$ have a national paper and $\frac{2}{3}$ have a local paper, what is the probability that a house chosen at random has both?

4. Consider the following possible events when two dice, one red and one green, are rolled:

    A : the total is 3

    B : the red is a multiple of 2

    C : the total is ≤ 9

    D : the red is a multiple of 3

    E : the total is ≥ 11

    F : the total is ≥ 10.

    Which of the following pairs are exhaustive or mutually exclusive?

    (a) A, D   (b) C, E   (c) A, B

    (d) C, F   (e) B, D   (f) A, E

1.5 Tree diagrams

Another approach to some of the problems examined earlier would be to use **'tree diagrams'**. These are sometimes called decision trees and may be used in other subjects such as business studies.

Example

While on holiday, staying with Rachel in Kent in the South East of England, Gabrielle saw a very large black bird. Rachel noticed that it was, in fact, not **all** black and they looked in a bird book to find what it might have been. The facts they discovered are shown in the tree diagram opposite.

By following along the branches from the left to the right can you decide what they actually saw?

Solution

As they were in Kent you should have moved from A to B. Since the bird was not all black, B to E is the correct choice next, and if the bird was very large then E to J tells you it was a rook.

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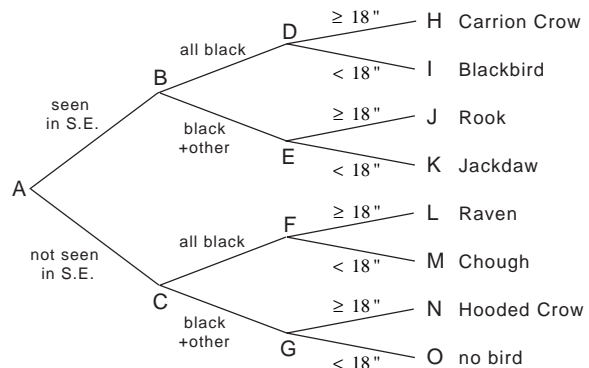
When you see people flocking from college to the local pub at lunchtime you might be able to identify the individuals by using the tree diagram opposite.

Now a shabbily dressed beer drinking worried person is a teacher. A happy, shabbily dressed beer drinker is a student, so keep smiling!

As a result of observation over a long period you might notice that 80% of those who come in are shabby. 90% of these and one third of the others are seen to drink beer. Three quarters of beer drinkers and half of those who prefer spirits look happy. If you put 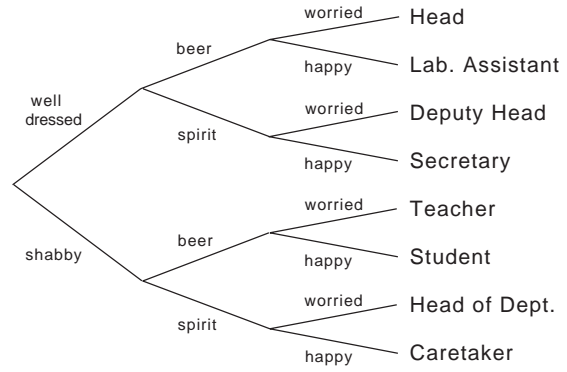these proportions on the branches as fractions you are in a position to work out how those who come in are divided up as students, teachers, etc.

Example

What proportion are teachers?

Solution

The 'teacher' branch is

shabby   –   beer   –   worried.

The proportion that are shabbily dressed and drink beer is

$$\frac{9}{10} \text{ of } \frac{4}{5} = \frac{9}{10} \times \frac{4}{5} = \frac{18}{25} \ .$$

Of those, $\frac{1}{4}$ are worried, giving the proportion of teachers as

$$\frac{1}{4} \text{ of } \frac{18}{25} = \frac{1}{4} \times \frac{18}{25} = \frac{9}{50} \ \ (= 0.18).$$

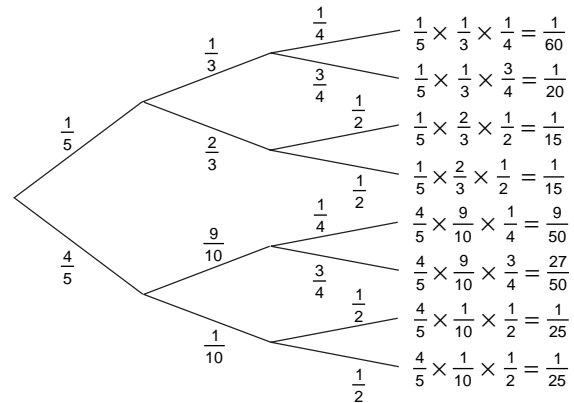**What fraction of customers from the college are secretaries?**

Example

What fraction of the customers from the college look worried?

Solution

The proportions in each category are shown on the tree diagram.

$$\text{So proportion worried } = \ \frac{1}{60} + \frac{1}{15} + \frac{9}{50} + \frac{1}{25} = \frac{91}{300} \ \ (\approx 0.3).$$

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Why is the sum of all the proportions in the tree diagram on the previous page equal to one?
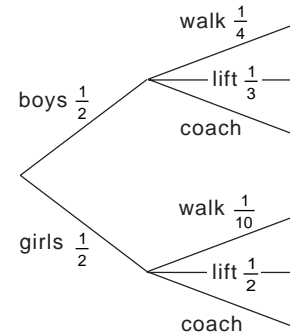
Example

If there are equal numbers of boys and girls in your school and you know that

$\frac{1}{4}$ of the boys and $\frac{1}{10}$ of the girls walk in every day,

$\frac{1}{3}$ of the boys and $\frac{1}{2}$ of the girls get a lift

and the rest come by coach, determine

(a) the proportion of the school population that are girls who go by coach;

(b) the proportion of the school population that go by coach.

Solution

The branches have missing entries but these can be calculated from the facts already known. Since

$$\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$$

of the boys have been accounted for, there remains $\frac{5}{12}$ who must use the coach.

Similarly, the proportion of girls going by coach is given by

$$1 - \left(\frac{1}{10} + \frac{1}{2} \right) = \frac{4}{10} = \frac{2}{5}.$$

All the values are entered on the diagram opposite, so that the answers to (a) and (b) are now easy to see.

(a) $\frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$

(b) $\frac{1}{5} + \frac{1}{2} \times \frac{5}{12} = \frac{49}{120}.$

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = P\,(\text{boy} \cap \text{walk})$

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} = P\,(\text{boy} \cap \text{lift})$

$\frac{1}{2} \times \frac{5}{12} = \frac{5}{24} = P\,(\text{boy} \cap \text{coach})$

$\frac{1}{2} \times \frac{1}{10} = \frac{1}{20} = P\,(\text{girl} \cap \text{walk})$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = P\,(\text{girl} \cap \text{lift})$

$\frac{1}{2} \times \frac{2}{5} = \frac{1}{5} = P\,(\text{girl} \cap \text{coach})$

Total = 1
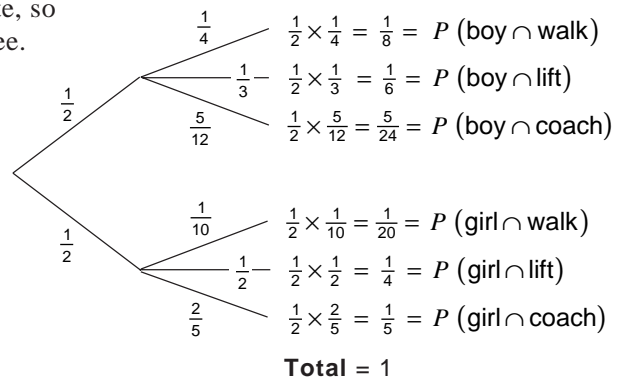
Example

When Sam and Jo play in the hockey team the probability that Sam scores is $\frac{1}{3}$ and that Jo scores is $\frac{1}{2}$, regardless of whether or not Sam does.

What is the probability that neither will score in the next game?

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Solution

The tree diagram opposite shows that the answer is $\frac{1}{3}$

since

$$P(S' \cap J') = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

Exercise 1D

1. The probability that a biased die falls showing a six is $\frac{1}{4}$. The biased die is thrown twice.

    (a) Draw a tree diagram to illustrate the probabilities of 'throwing a six' or 'not throwing a six'.

    (b) Find the probability that exactly one six is obtained.
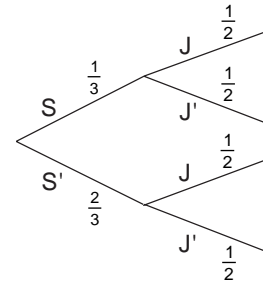
2. In each round of a certain game a player can score 1, 2, 3 only. Copy and complete the table which shows the scores and two of the respective probabilities of these being scored in a single round.

Score	1	2	3
Probability	$\frac{4}{7}$		$\frac{1}{7}$

3. A bag contains 7 black and 3 white marbles. Three marbles are chosen at random and in succession, each marble being replaced after it has been taken out of the bag.

    Draw a tree diagram to show all possible selections.

    From your diagram, or otherwise, calculate the probability of choosing:

    (a) three black marbles;

    (b) a white marble, a black marble and a white marble in that order;

    (c) two white marbles and a black marble in any order;

    (d) at least one black marble.

    State an event from this experiment which, together with the event described in (d), would be both exhaustive and mutually exclusive.

1.6   Conditional probability

Your assessment of how likely an event is to occur may well depend on some other event or variable. If you were asked, "What is the probability that it will rain next Monday?" your answer would depend on the time of year you were asked. If the question were in winter then $\frac{1}{2}$ might be a realistic assessment but in summer your reply might be $\frac{1}{10}$. This can be written

$$P(\text{rain next Monday} \mid \text{summer}) = \frac{1}{10},$$

that is, the probability of rain next Monday, given that it is summer, is one tenth.

This probability depends on a definitely known condition (that is, it is summer), hence the term '**conditional probability**'.

As another example, consider the following problem:

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If the probability of a school pupil wearing glasses is $\frac{1}{9}$, does it make any difference to how likely you think it is that the next one you see will wear glasses if you know that the pupil is female?

Is P (wearing glasses) **the same as** $P\big(\text{wearing glasses}\,\big|\,\text{female}\big)$?

It should be possible to find out by considering a large sample, perhaps when having lunch or at a main entrance.

There is nothing new in the idea of conditional probability and you may well have realised that you have used it already. Conditional probabilities appeared on branches of the tree diagrams to do with the pub's customers and pupils' transport in the last section. The fractions on the branches after the initial ones were conditional probabilities as they definitely depended on the previous ones. The transport tree could have been labelled

$$B = \text{boy}, \ G = \text{girl}, \ W = \text{walk}, \ \ L = \text{lift}, \ \ C = \text{coach}.$$

You can readily see that $P(B \cap W) = \frac{1}{8}$ since

$$P(B \cap W) \ = \ \frac{1}{2} \times \frac{1}{4} \ = \ \frac{1}{8}.$$

Now $\qquad P(B) = \frac{1}{2}, \ \text{and} \ P\big(W \,\big|\, B\big) = \frac{1}{4},$

leading to $\quad P(B \cap W) \ = \ P(B) \times P\big(W \,\big|\, B\big)$

or $\qquad \boxed{P\big(W \,\big|\, B\big) \ = \ \dfrac{P(W \cap B)}{P(B)}}$

$P\big(W \,\big|\, B\big) = \frac{1}{4} \longrightarrow P\,(B \cap W) = \frac{1}{8}$

$P\big(L \,\big|\, B\big) = \frac{1}{3} \longrightarrow P\,(B \cap L) \ = \frac{1}{6}$

$P(B) = \frac{1}{2}$

$P\big(C \,\big|\, B\big) = \frac{5}{12} \longrightarrow P\,(B \cap C) \ = \frac{5}{24}$

$P\big(W \,\big|\, G\big) = \frac{1}{10} \longrightarrow P\,(G \cap W) = \frac{1}{20}$

$P\big(L \,\big|\, G\big) = \frac{1}{2} \longrightarrow P\,(G \cap L) \ = \frac{1}{4}$

$P(G) = \frac{1}{2}$

$P\big(C \,\big|\, G\big) = \frac{2}{5} \longrightarrow P\,(G \cap C) \ = \frac{1}{5}$
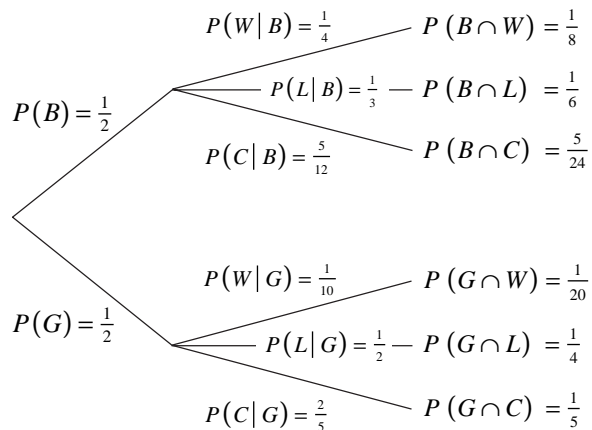
This is a very useful equation when working with conditional probability and holds in general. That is, if A and B are two events,

then $\qquad \boxed{P\big(A \,\big|\, B\big) \ = \ \dfrac{P(A \cap B)}{P(B)}}$

Example

Using the example from page 14, what is the probability that a worried person from the college is a teacher?

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Solution

$$P(\text{teacher}\,|\,\text{worried}) \;=\; \frac{P(\text{teacher}\cap\text{worried})}{P(\text{worried})}$$

$$=\; \frac{P(\text{teacher})}{P(\text{worried})}$$

$$=\; \left(\frac{4}{5}\times\frac{9}{10}\times\frac{1}{4}\right)\div\frac{91}{300}$$

$$=\; \frac{9}{50}\div\frac{91}{300}$$

$$=\; \frac{9}{50}\times\frac{300}{91}$$

$$=\; \frac{54}{91}.$$

So now you know what fraction of the worried people are teachers.

Conditional probabilities can also be found from sample space diagrams.
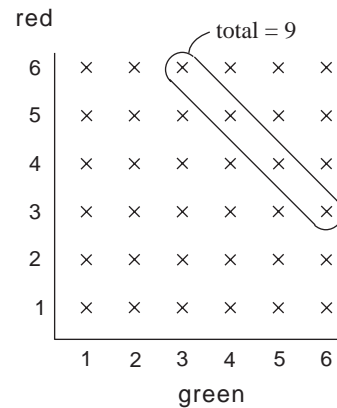
Example

If you roll two dice, one red and one green, what is the probability that the red one shows a six if the total on the two is 9?

Solution

Since you know that the total is 9 you need only look at the four crosses enclosed by the curve in the diagram opposite as they indicate all the possible ways of getting the 9 required. Now just considering these four, what is the chance that the red one shows 6?

$$P(r=6\,|\,r+g=9) \;=\; \frac{1}{4}$$

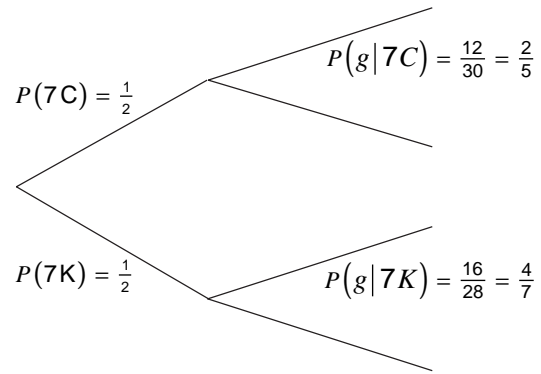as only one of the four crosses has a six on the red.

Example

Class 7C has 18 boys and 12 girls in it and 7K is made up of 12 boys and 16 girls. If you pick one of their registers and a pupil from it at random, what is the probability that you select

(a)   a girl   (b)   from 7C if the choice is a girl?

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Solution

(a) $P(\text{girl}) = \dfrac{1}{2} \times \dfrac{2}{5} + \dfrac{1}{2} \times \dfrac{4}{7}$

$\qquad\qquad = \dfrac{1}{5} + \dfrac{2}{7} = \dfrac{17}{35}.$

(b) $P(7C \mid \text{girl}) = \dfrac{P(7C \cap \text{girl})}{P(\text{girl})}$

$\qquad\qquad = \dfrac{\dfrac{1}{2} \times \dfrac{2}{5}}{\dfrac{17}{35}}$

$\qquad\qquad = \dfrac{1}{5} \div \dfrac{17}{35} = \dfrac{7}{17}.$

$P(7C) = \frac{1}{2}$ $P(g|7C) = \frac{12}{30} = \frac{2}{5}$

$P(7K) = \frac{1}{2}$ $P(g|7K) = \frac{16}{28} = \frac{4}{7}$

You might wonder why the answer to (a) was not

$$\frac{\text{no. of girls}}{\text{no. of pupils}} = \frac{12+16}{30+28} = \frac{28}{58} = \frac{14}{29}.$$

Why does this argument give the wrong answer?

The reason this method does not produce the correct answer here is that the pupils are not all equally likely to be chosen. Each pupil in 7C has a probability of

$$\frac{1}{2} \times \frac{1}{30} = \frac{1}{60}$$

of being selected, but for those in 7K it is

$$\frac{1}{2} \times \frac{1}{28} = \frac{1}{56}.$$

Exercise 1E

1. Two cards are drawn successively from an ordinary pack of 52 playing cards and kept out of the pack. Find the probability that:

    (a) both cards are hearts;

    (b) the first card is a heart and the second card is a spade;

    (c) the second card is a diamond, given that the first card is a club.

2. A bag contains four red counters and six black counters. A counter is picked at random from the bag and not replaced. A second counter is then picked. Find the probability that:

    (a) the second counter is red, given that the first counter is red;

    (b) both counters are red;

    (c) the counters are of different colours.

3. The two events A and B are such that
    $$P(A) = 0.6, \ P(B) = 0.2, \ P(A|B) = 0.1.$$
    Calculate the probabilities that:
    (a) both of the events occur;
    (b) at least one of the events occurs;
    (c) exactly one of the events occurs;
    (d) B occurs, given that A has occurred.

4. In a group of 100 people, 40 own a cat, 25 own a dog and 15 own a cat and a dog. Find the probability that a person chosen at random:
    (a) owns a dog or a cat;
    (b) owns a dog or a cat, but not both;
    (c) owns a dog, given that he owns a cat;
    (d) does not own a cat, given that he owns a dog.

1.7   Independence

In the previous section the answer to, "What is the probability that it will rain next Monday?" depended on the fact that you were told or knew about the season.

When two tetrahedral dice are rolled there are sixteen possible outcomes as shown in the diagram opposite.

What is $P(\text{total}) = 7$?

Now if I tell you that my cat has a broken leg, what is

$$P \ (\text{total} \ = 7 \mid \text{my cat has a broken leg})?$$

The answer is $\frac{2}{16}$ to both of these questions. The replies

are the same because the two things discussed, the chance of a total of 7 and my cat having a broken leg, are independent. Other examples may not be as immediately obvious.
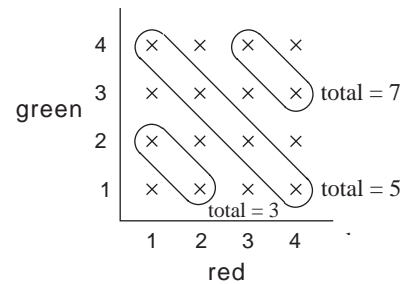
Example

What is the value of

(a) $P(\text{total} = 5)$ (b) $P(\text{total} = 5 \mid \text{red} = 2)$

(c) $P(\text{total} = 3)$ (d) $P(\text{total} = 3 \mid \text{red} = 2)?$

Solution

(a)   From the sample space diagram above

$$P(\text{total} = 5) \ = \ \frac{4}{16} \ = \ \frac{1}{4}.$$

(b)   Again $P(\text{total} = 5 \mid \text{red} = 2) \ = \ \frac{1}{4}$

since there is only one event (red $= 2$, green $= 3$) out of four possible events for red $= 2$.

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(c) $P(\text{total} = 3) = \dfrac{2}{16} = \dfrac{1}{8}.$

(d) $P(\text{total} = 3 \mid \text{red} = 2) = \dfrac{1}{4}.$

The answers to (a) and (b) are both $\frac{1}{4}$, so the answer to ,"How likely is a total of 5?", is independent of (not affected by) the fact that you were told in (b) that the red score was 2.

(c) and (d) have different answers, however, $\frac{1}{8}$ and $\frac{1}{4}$ respectively, so your assessment of how likely a total of 3 is depends on the fact given in (d).

If two events, *A* and *B,* are such that

$$P(A \mid B) = P(A)$$

then they are said to be **independen**t. Otherwise they are **dependent.**

In Section 1.5 there were examples of both cases. The tree diagram showing how pupils travelled to school included

$$P(\text{walk} \mid \text{boy}) = \frac{1}{4}$$

and $P(\text{walk} \mid \text{girl}) = \dfrac{1}{10},$

so how likely you think a pupil is to walk would depend on their sex.

On the other hand, in another example in Section 1.5, the chance of Jo scoring was not related to how likely Sam was to score so these events were independent. (In a tree diagram to show two events the branches are duplicated after each initial one if the second event is independent of the first.)

Example

In one year at school, 25 out of 154 failed the end of term maths exam. One class was particularly badly behaved and 7 out of 31 of them failed. Does bad behaviour in class affect how likely a pupil is to fail the test?

Solution

$$P(\text{fail}) = \frac{25}{154} = 0.162 \text{ (to 3 d.p.)}$$

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$$P\left(\text{fail} \mid \text{badly behaved class}\right) = \frac{7}{31} = 0.226 \text{ (to 3 d.p.).}$$

Since these are certainly different the events are dependent, so the answer is 'Yes'.

Example

A family has three children. What is the probability that all three are the same sex? If you know at least two of them are girls what is the probability that they are all the same sex? Has this piece of information been of any help?

Solution

The possible combinations are shown below.

At least two girls	G	G	G	all same sex
At least two girls	G	G	B	
At least two girls	G	B	G	
	G	B	B	
At least two girls	B	G	G	
	B	G	B	
	B	B	G	
	B	B	B	all same sex

$$P\left(\text{all same sex}\right) = \frac{2}{8} = \frac{1}{4}$$

$$P\left(\text{all same sex} \mid \text{at least 2 girls}\right) = \frac{1}{4}.$$

So the events are independent, and the answer is 'No'.

Starting from the definition of independence,

$$P(A) = P\left(A \mid B\right)$$

$$= \frac{P\left(A \cap B\right)}{P(B)}$$

$$\Rightarrow \quad P(A)\, P(B) = P\left(A \cap B\right).$$

Testing to see whether or not $P(A) \times P(B)$ is, in fact, equal to $P\left(A \cap B\right)$ can also be used as a test for independence. So in our last example,

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$$P(\text{at least two girls}) = \frac{4}{8} = \frac{1}{2}$$

$$P(\text{all three the same sex}) = \frac{2}{8} = \frac{1}{4}$$

$$P(\text{at least two girls} \cap \text{all three the same sex}) = \frac{1}{8}.$$

Since $\dfrac{1}{2} \times \dfrac{1}{4} = \dfrac{1}{8}$ you can see that these events are independent.

If A and B are independent then the occurrence of B does not affect the likelihood of A happening and similarly it seems very likely that the non-occurrence of B should have no effect.
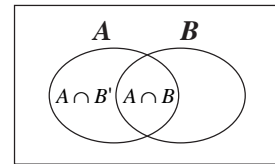
If A and B are independent, then

$$P(A \cap B) = P(A)\,P(B)$$

$$P(A) - P(A \cap B) = P(A) - P(A)\,P(B)$$

$$\Rightarrow \quad P(A \cap B') = P(A) - P(A)\,P(B)$$

$$= P(A)\left[1 - P(B)\right]$$

$$\Rightarrow \quad P(A \cap B') = P(A)\,P(B').$$

So A and B' are also independent.

Exercise 1F

1. A card is selected at random from an ordinary pack of 52. If

 A = the card is an ace

 D = the card is a diamond

 P = the card is a picture (Jack, Queen or King)

 R = the card is from a red suit

 X = the card is not the three of diamonds or the two of clubs,

 what are the values of the following:

 (a) $P(A)$ (b) $P(A|D)$ (c) $P(D)$

 (d) $P(D|P)$ (e) $P(D|R)$ (f) $P(P)$

 (g) $P(P|A)$ (h) $P(P|A')$ (i) $P(A|X)$

 (j) $P(D|X')$ (k) $P(X|D)$ (l) $P(R|X)$?

2. Which of the following pairs of events from Question 1 are independent:

 (a) A, D (b) D, P (c) P, A

 (d) R, X (e) D, R (f) D, R'?

3. Work out the six probabilities on the branches labelled a to f and also the value of g.

 - $\frac{1}{2}$
 - $\frac{1}{6}$
 - c — $\frac{1}{30}$
 - a
 - d — $\frac{2}{15}$
 - b
 - e — $\frac{1}{6}$
 - f — g
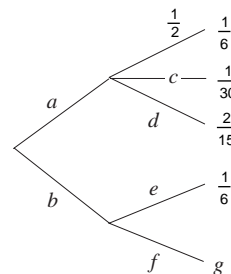
4. Two boxes, A and B, each contain a mixture of black discs and white discs. A contains 8 black and 7 white while B has 5 black and 7 white. A box is selected at random and a disc taken from it.

 Draw a tree diagram and calculate the probability that:

 (a) the disc is white;

 (b) the disc came from B if it is white.

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5. A box contains 60 balls each of which is either red, blue or white. If the numbers of these are in the ratio 6:3:1 how many are there of each?

    By drawing a tree diagram, find the probability that when two balls are drawn at random together:

    (a) they are the same colour;

    (b) no red ball is drawn;

    (c) they are both white if you are told they are the same colour.

6. In a quiz competition the first question is worth one point and answered correctly with probability $\frac{5}{6}$. After any question is answered correctly the contestant receives one worth a point more than the previous one. After a wrong answer the contestant receives a one point question and two consecutive wrong answers eliminates the competitor.

    If the probabilities of correctly answering 2, 3 and 4 point questions are $\frac{4}{5}$, $\frac{3}{4}$ and $\frac{2}{3}$ respectively, calculate the probability that after four rounds the contestant has:

    (a) been eliminated;

    (b) scored at least six points.

7. In May, three mornings out of every five on average are fine.

    When the weather is fine Sarita walks to work with probability $\frac{1}{2}$, goes on the bus with probability $\frac{1}{3}$ and drives when she does not walk or use the bus. She never walks when the weather is wet and drives four times as often as she uses the bus. The probability of her arriving late when she walks in is $\frac{1}{2}$ and when she uses the bus it is $\frac{1}{4}$. She is always on time when she drives.

    On a particular May morning what is the probability that Sorita:

    (a) arrives on time;

    (b) travels by bus and is late?

    If she arrives late what is the probability that it is a fine morning?

1.8   Miscellaneous Exercises

1. One die has the numbers 1, 2, 3, 4, 5, 6 on its faces and another has 1, 1, 2, 2, 3, 3 on its faces. When the two are rolled together what is the probability that one of the scores will divide exactly into the other?

2. There are prizes for the first two runners home in a race with six competitors. What is the probability that:

    (a) both Dave and Raj will win prizes;

    (b) neither Dave nor Raj will win a prize?

3. A two digit number is written down at random. What is the probability that it:

    (a) is divisible by 5;   (b) is divisible by 3;

    (c) is greater than 50;   (d) is a square number?

4. When four coins are tossed together, what is the probability of at least three heads?

5. A counter starts at the point (0, 0). A coin is tossed and when a tail results it moves one unit to the right. When a head is seen it moves one unit upwards.

    What is the probability that after three goes it is still on the *x*-axis?

6. **Buffon's Needle** Take some ordinary pins and draw a set of straight lines across a sheet of paper so that they are the same distance apart as the length of a pin. Drop ten pins onto the lined paper several times and record your results in the same way as in Section 1.2, noting how many lie across a line. Draw a graph and estimate the probability of a pin crossing a line when dropped.

7. **Off-centre spinner** Make a hexagonal spinner and put a cocktail stick or something similar through a point to divide AB in the ratio 2 : 3.
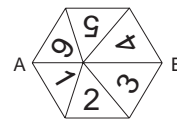
    What are the probabilities of the various scores?

8. Four unbiased coins are tossed together. For the events *A* to *D* below, say whether statements (a) to (d) are true or false and give a reason for each answer.

    A = no heads B = at least one head

    C = no tails D = at least two tails

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(a) *A* and *B* are mutually exclusive;

(b) *A* and *B* are exhaustive;

(c) *B* and *D* are exhaustive;

(d) *A'* and *C'* are mutually exclusive.

9. In a class of 30 pupils, 17 have a dog, 19 have a cat and 5 have neither. If a member of the form is selected at random what is the probability that this pupil has both a cat and a dog?

10. The probability that Suleiman will be chosen to play in goal for the next match is $\frac{1}{4}$ and the probability that Paul will be selected for that position is $\frac{2}{5}$. Find the probability that:

(a) Suleiman or Paul will be selected to play in goal;

(b) neither of them will be asked to play in goal.

11. A number is to be formed by arranging the digits 1, 4, 7 and 8 in some order.

If   *A* = the number is odd

and   *B* = the number is greater than 4000,

find the value of:

(a) $P(A)$ (b) $P(B)$ (c) $P(B|A)$

(d) $P(A \cap B)$ (e) $P(A'|B)$.

12. John does $\frac{3}{5}$ of the jobs that come into the workshop and Dave does the rest. If 35% of John's work and 55% of Dave's work is perfect, find the probability that a job done in the work-shop will be done:

(a) perfectly;

(b) by Dave if it was not done perfectly.

13. A warehouse receives 60% of its supplies from factory *A*, 30% from *B* and the rest from *C*.

   A sends large, medium and small items in the ratio 1 : 3 : 2.

   B's supplies are $\frac{1}{3}$ large size and no small size.

   C provides three times as many medium as small items and no large ones.

If an item is selected at random from the warehouse, what is the probability that it is:

(a) medium;

(b) from *B* and large;

(c) from *C* if it is found to be medium?

14. A box contains 8 discs of which 5 are red and 3 are blue. One is selected at random and its colour noted. It is returned to the box together with an extra one of the other colour. This process is repeated twice more.

What is the probability that:

(a) the third disc selected is red;

(b) more reds are selected than blues;

(c) the third disc is red if there are more blues shown than reds?

15. At a fete, one of the games consists of throwing a 2p coin onto a large board of coloured squares, each 2 inches by 2 inches. If a coin lies completely within a single square it is returned to a player with another 2p, otherwise it goes to the organiser. A 2p coin has a diameter of 1 inch. By considering where the centre of the coin must land for a win, work out the player's probability of success.

How much money should the organiser expect to take in one hundred goes?

To make more profit, you could draw up a board to use with 10p coins. What size should the square be if the player is to have a probability of $\frac{1}{3}$ of winning? (Answer to the nearest mm.)

16. A circular spinner has three sections numbered 1, 2 and 3. If these numbers came up twenty-five, thirty and forty-five times in an experiment, what do you consider the likely values for the angles of the sectors?

17. Twenty discs numbered 1 to 20 are put at random into four boxes with five in each.

What is the probability that numbers 15 and 19 will be in the same box?

Would the answer be different if the discs had been split into five groups of four?

18. A forgetful teacher leaves his mark book in a room where he has had a lesson once in every three occasions on average. If he teaches three lessons in different rooms in the morning, what is the probability that:

(a) he will arrive at the lunch break having lost his mark book;

(b) he left it in the second room if he finished the morning without it?

19. Three bags, *A, B* and *C*, each contain three 5p coins and two 2p coins. A coin is selected at random from *A* and placed in *B*. A coin is drawn from *B* and put in *C*. Finally, a coin is drawn from bag *C*. Find the probability that:

(a) the coin selected from *C* is a 2p;

(b) the coin selected from *A* was a 5p if the one from *C* was a 2p.

Explain why the answer to (a) might have been expected. Repeat (a) for *x* 2p coins and *y* 5p coins.

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Chapter 1 Probability

20. Four girls each try to catch a ball and the probability that each will succeed is independently $\frac{2}{3}$.

What is the probability that it will:

(a) not be caught;

(b) be caught?

21. Three students, Dave, Jane and Mary, share a house. Each of the girls is twice as likely as Dave to receive a telephone call in the evening. The probabilities that each will be out on any evening are independently $\frac{1}{2}$, $\frac{2}{5}$ and $\frac{3}{5}$ respectively. If the telephone rings one evening find the probability that the call is:

(a) for Jean who is in;

(b) for someone who is out;

(c) for Dave given that it is for someone who is out.

22. Two gamblers play a game with two coins. The first tosses them and pays the second £1 for each head showing. Then the second has a turn and pays £1 for each tail showing. After each has had one go what is the probability that the first player has made a profit?

23. A school has three minibuses and the probability that each is free after school is independently $\frac{2}{5}$.

Find the probability that after school on a particular day:

(a) at least one minibus is free;

(b) all the minibuses are free if at least one is free.

24. A set of dominoes consists of twenty eight pieces, each of which shows two sets of spots from zero to six, and no two dominoes are the same. A single domino is selected at random. Show the 28 possibilities on a diagram.

What is the probability that:

(a) the smaller number is 2;

(b) it is a double;

(c) it contains neither a 4 nor a 5?

25. Three coins are tossed. Event X is that at least one head and at least one tail result. Event Y is that at most one head shows. Are events X and Y independent?

26. Vehicles approaching a crossroad must go in one of three directions - left, right or straight on. Observations by traffic engineers showed that of vehicles approaching from the north, 45% turn left, 20% turn right and 35% go straight on. Assuming that the driver of each vehicle chooses direction independently, what is the probability that of the next three vehicles approaching from the north

(a) (i)   all go straight on;

   (ii)   all go in the same direction;

   (iii)   two turn left and one turns right;

   (iv)   all go in different directions;

   (v)   exactly two turn left?

(b) Given that three consecutive vehicles all go in the same direction, what is the probability that they all turn left?   (AEB)

27. The results of a traffic survey of the colour and type of car are given in the following table.

	Saloon	Estate
White	68	62
Green	26	32
Black	6	6

One car is selected at random from this group.

Find the probability that the selected car is

(i)   a green estate car,

(ii)   a saloon car,

(iii)   a white car given that it is not a saloon car.

Let W and G denote the events that the selected car is white and green respectively and let S be the event that the car is a saloon.

Show that the event $W \cup G$ is independent of the event S.   Show, however, that colour and type of car are not independent.   (AEB)

28. The staff employed by a college are classified as academic, administrative or support. The following table shows the numbers employed in these categories and their sex.

	Male	Female
Academic	42	28
Administrative	7	13
Support	26	9

A member of staff is selected at random.

A is the event that the person selected is female.

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B is the event that the person selected is academic staff.

C is the event that the person selected is administrative staff.

($\overline{A}$ is the event not *A*, $\overline{B}$ is the event not *B*, $\overline{C}$ is the event not *C*)

(a) Write down the values of

  (i) $P(A)$,

  (ii) $P(A \cap B)$

  (iii) $P(A \cup \overline{C})$

  (iv) $P(\overline{A}|C)$

(b) Write down one of the events which is

  (i)   not independent of *A*,

  (ii)   independent of *A*,

  (iii)   mutually exclusive of *A*.

  In each case, justify your answer.

(c) Given that 90% of academic staff own cars, as do 80% of administrative staff and 30% of support staff,

  (i)   what is the probability that a staff member selected at random owns a car?

  (ii)   A staff member is selected at random and found to own a car. What is the probability that this person is a member of the support staff?

(AEB)

29. A vehicle insurance company classifies drivers as A, B or C according to whether or not they are a good risk, a medium risk or a poor risk with regard to having an accident. The company estimates that A constitutes 30% of drivers who are insured and B constitutes 50%. The probability that a class A driver will have one or more accidents in any 12 month period is 0.01, the corresponding values for B and C being 0.03 and 0.06 respectively.

(a) Find the probability that a motorist, chosen at random, is assessed as a class C risk and will have one or more accidents in a 12 month period.

(b) Find the probability that a motorist, chosen at random, will have one or more accidents in a 12 month period.

(c) The company sells a policy to a customer and within 12 months the customer has an accident. Find the probability that the customer is a class C risk.

(d) If a policy holder goes 10 years without an accident and accidents in each year are independent of those in other years, show that the probabilities that the policy holder belongs to each of the classes can be expressed, to 2 decimal places, in the ratio 2.71 : 3.69 : 1.08.

(AEB)

30. A hospital buys strawberry jam in standard sized tins from suppliers A, B and C. (The table on the next page gives information about the contents.)

Find the probability of a tin selected at random being

(a) from supplier A,

(b) underweight.

What is the probability of

(c) a tin from B being both underweight and poor quality,

(d) an underweight tin from A containing poor quality jam,

(e) a tin from C being both underweight and poor quality,

(f) a tin from C which contains poor quality jam being underweight,

(g) a tin selected at random being both underweight and poor quality.

(h) a tin being from A given that it is both underweight and of poor quality?

(AEB)

Supplier	% of hospital requirements supplied	% of tins with underweight contents	% of tins containing poor quality jam	Other information
A	55	3	7	1% are both underweight and poor quality
B	35	5	12	probability of poor quality is independent of probability of being underweight
C	10	6	20	40% of underweight tins contain poor quality jam