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10 TRIGONOMETRY

Objectives

After studying this chapter you should

- know that radians are a unit for measuring angles;
- be able to convert from degrees to radians and vice versa;
- know and be able to use formulae for arc length and sector area in terms of radians;
- be familiar with basic properties of sine, cosine and tangent functions;
- be able to solve simple trigonometric equations.

10.0 Introduction

You are familiar with using degrees to measure angles, but this is not the only way to do it. In fact, as you see in some areas of mathematics, it is not even the most convenient way.

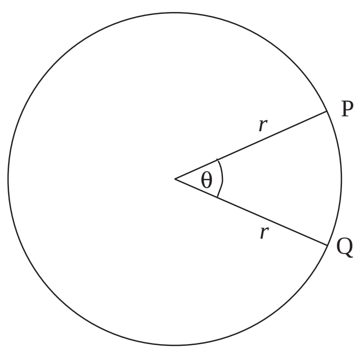
Activity 1 History of circular measure

(a) Your calculator will have settings for 'degrees', 'radians' and 'gradians'. Look up gradians in an encyclopaedia, and find out when these were first introduced. When did they last get used in Europe? How many gradians are there in a full turn, or in a right angle?

(b) Find out why 360 degrees make a full turn, and which civilizations invented them. What were the reasons for adopting 360? Why are 360 degrees still used?

Activity 2

The figure opposite shows a circle of radius r and arc PQ which is subtended by an angle of θ degrees.



(a) What is the length of the complete circumference?

(b) What is the arc length PQ in terms of θ and r?

(c) If $PQ = r$, what is the value of θ?

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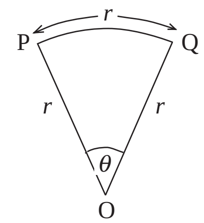
10.1 Radian measure



The second activity gives the clue as to how radians are defined. One **radian** corresponds to the angle which gives the same arc length as the radius. So if θ is the angle POQ in degrees.

$$\frac{\theta}{360} = \frac{r}{2\pi r} = \frac{1}{2\pi}$$

Thus $\theta = \frac{360}{2\pi} = \frac{180}{\pi}$ showing that

$$1 \text{ radian} \equiv \frac{180}{\pi} \text{ degrees}$$

This is not a very convenient definition, so it is usual to note that

$$\boxed{\pi \text{ radians} \equiv 180 \text{ degrees}}$$

Example

Convert the following angles in degrees to radians.

(a) 90°   (b) 360°   (c) 720°   (d) 60°

Solution

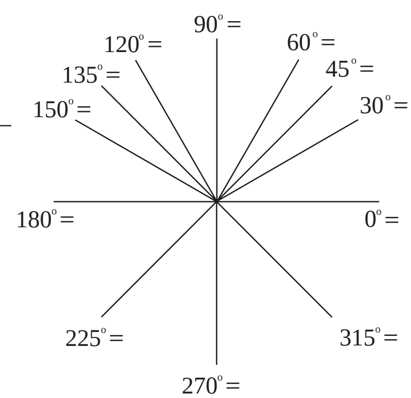
Since $180° = \pi$ radians,

(a) $90° = \frac{\pi}{2}$ radians   (b) $360° = 2\pi$ radians

(c) $720° = 4\pi$ radians   (d) $60° = \frac{\pi}{3}$ radians

Activity 3 Conversion between degrees and radians

Copy the diagram opposite, putting in the equivalent radian measure for each angle given in degrees.



As you have seen, there is a one to one correspondence between degrees and radians so that

$$\theta° \equiv \theta \times \frac{\pi}{180} \text{ radians}$$

or

$$\theta \text{ radians} \equiv \left(\theta \times \frac{180}{\pi}\right)^{\circ}$$

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Example

Use the formulae above to convert

(a) to radians (i) $45°$ (ii) $30°$ (iii) $150°$

(b) to degrees (i) $\frac{2\pi}{3}$ radians (ii) $\frac{\pi}{12}$ radians.

Solution

(a) (i) $45° = 45 \times \frac{\pi}{180}$ radians $= \frac{\pi}{4}$ radians

(ii) $30° = 30 \times \frac{\pi}{180}$ radians $= \frac{\pi}{6}$ radians

(iii) $150° = 150 \times \frac{\pi}{180}$ radians $= \frac{5\pi}{6}$ radians.

(b) (i) $\frac{2\pi}{3}$ radians $= \left(\frac{2\pi}{3} \times \frac{180}{\pi}\right)^{\circ} = 120°$

(ii) $\frac{\pi}{12}$ radians $= \left(\frac{\pi}{12} \times \frac{180}{\pi}\right)^{\circ} = 15°$.

Exercise 10A

1. Convert these angles in radians to degrees:

   (a) $\frac{7\pi}{6}$ radians (b) 3π radians

   (c) 2 radians (d) $\frac{11\pi}{12}$ radians.

2. Convert these angles in degrees to radians:

   (a) $12\frac{1}{2}^{\circ}$ (b) $72\frac{1}{2}^{\circ}$

   (c) $210°$ (d) $20°$.

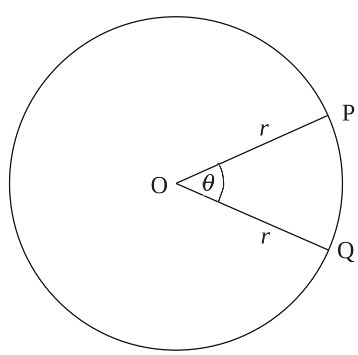
10.2 Arc length and sector area

You may already have seen how to find the arc length. If θ is measured in degrees, then

$$\text{arc length PQ} = \frac{\theta}{360} \times 2\pi r = \frac{\pi r \theta}{180},$$

but if θ is measured in radians

$$\text{arc length PQ} = \frac{\theta}{2\pi} \times 2\pi r = r\theta.$$



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There is a simple formula for the area of a sector of a circle subtended by an angle.

Activity 4 Sector area

(a) What is the area of a circle, of radius r?

(b) The sector subtends an angle of θ radians at the centre of a circle. What proportion is the area of the sector to the area of the whole circle?

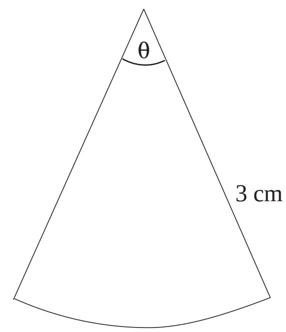
(c) Deduce the formula for the sector area?

So, using radian measure, the formula for both arc length and sector area take a simple form, namely

$$\text{arc length} = r\theta$$

$$\text{sector area} = \frac{1}{2}r^2\theta$$

You will see how these formulae are used in the following example.



Example

(a) A silver pendant is made in the form of a sector of a circle as shown opposite. If the radius is 3 cm, what is the angle, θ, in radians so that the area is 6 cm^2 ?

(b) Another pendant has the same total perimeter, but with radius 2.5 cm. What is the required angle, θ, in radians?

Solution

(a) Sector area $= 6 = \frac{1}{2} \times 3^2 \times \theta$

$$\Rightarrow \quad \theta = \frac{4}{3} \text{ radians.}$$

(b) Total perimeter length $= 3 + 3 + 3 \times \frac{4}{3}$

$$= 10 \text{ cm.}$$

Hence, for the second pendant

$$10 = 2.5 + 2.5 + 2.5 \times \theta$$

$$\Rightarrow \quad \theta = 2 \text{ radians.}$$

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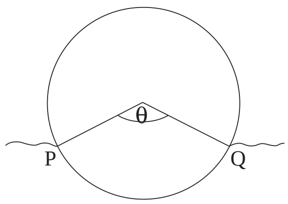
Exercise 10B

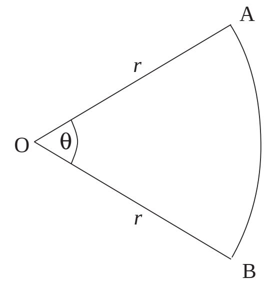
1. An oil drum of diameter 60 cm is floating as shown in the diagram below :



   (a) Given that the arc length PQ is 50 cm, find θ in radians.

   (b) The drum is 1 metre long, find the volume of the drum which lies below the surface level.

2. The length of the arc AB is 20 cm, and the area of the sector AOB is 100cm^2.



   Form two equations involving r and θ when θ is measured in radians. Solve these equations simultaneously to find r and θ.

3. A stone, swung round on the end of a 200 cm string, completes an arc subtending an angle at the centre of 2 radians every second. Find the speed at which the stone is moving.

10.3 Sine, cosine and tangent functions

You are probably familiar with the uses of sine, cosine and tangents in right angled triangles, but the following activities below will further revise the concepts.

Activity 5

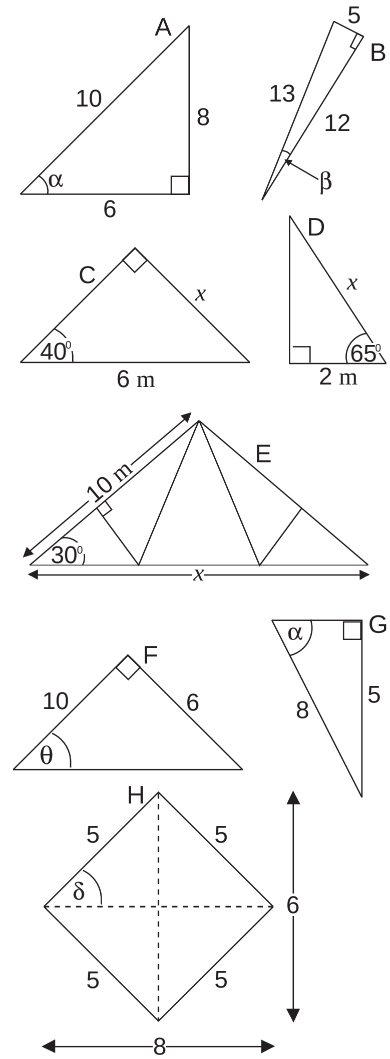
(a) Write down the sine, cosine and tangent of the angles marked in the triangles A and B.

(b) Find the lengths marked x to 2 d.p. in these triangles C and D.

(c) The sketch labelled E shows the frame of a roof. Find the width x.

(d) Find the angle marked, to 1 d.p., in each of the triangles, F and G.

(e) Find the angle δ in the rhombus shown opposite, as H.



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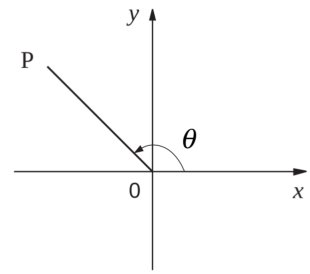
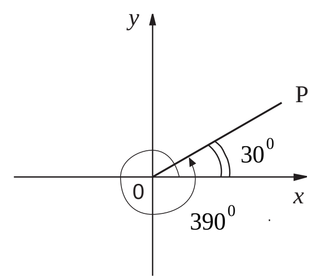
You have seen how sine, cosine and tangent are defined for angles between $0°$ and $90°$ but this can be extended to other angles.



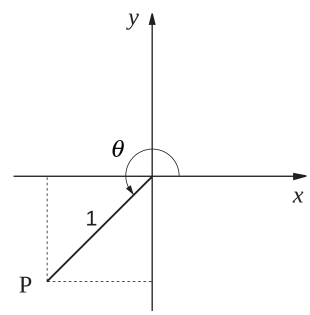
Angles of more than $90°$ can be defined as the angle θ made between a rotating 'arm' OP and the positive x axis, as shown opposite. It is possible to define angles of more than 360^0 in this way, or even negative angles, as shown opposite.



If the length of OP is 1 unit, then the sine, cosine and tangent of any angle is defined in terms of the x and y co-ordinates of the point P as follows;

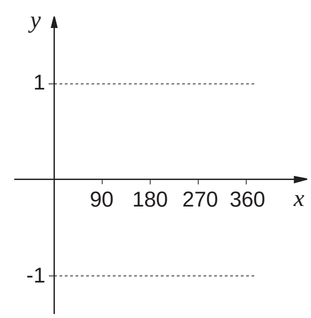
$$\sin\theta = y \qquad \cos\theta = x \qquad \tan\theta = \frac{y}{x}$$

Note that $\sin\theta$, $\cos\theta$ and $\tan\theta$ may be negative for certain values of θ. For instance in the third figure opposite, y and x are negative, whilst $\tan\theta$ is positive.



Scientific calculators give sine, cosine and tangent of any angle.

Activity 6



Using graph paper, plot the function

$$y = \sin x$$

for every $10°$ interval between $0°$ and $360°$. Draw a smooth curve through the points. What are the maximum and minimum values of the function y?

Repeat the process for the function

$$y = \cos x.$$

Can you explain why both functions, $\sin x$ and $\cos x$, lie between ± 1?

Whilst you have only plotted the functions for $0 \le x \le 360°$, it is easy to see how it can be extended.

What is the period of each function?

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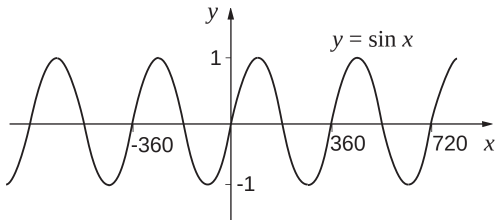
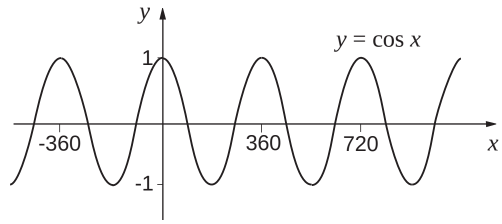
For the range $360° \le x \le 720°$, the pattern will repeat itself, and similarly for negative values. The graphs are illustrated below





You can see from these graphs that $y = \sin x$ is an **odd** function $(y(-x) = -y(x))$, whilst $y = \cos x$ is an **even** function $(y(-x) = y(x))$.

You can always use your calculator to find values of sine and cosine of an angle, although sometimes it is easier to use the properties of the functions.

Example

Show that $\sin 30° = \frac{1}{2}$, and find the following without using tables:

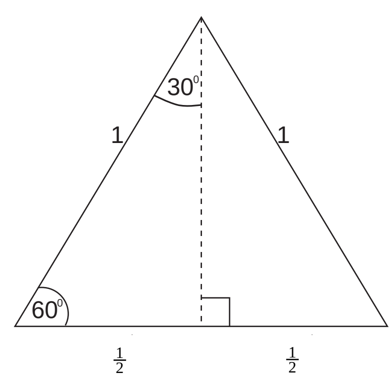
(a) $\sin 150°$ (b) $\sin(-30°)$ (c) $\sin 210°$

(d) $\sin 390°$ (e) $\cos 60°$ (f) $\cos 120°$

(g) $\cos(-60°)$ (h) $\cos 240°$

Solution

From the sketch opposite of an equilateral triangle,

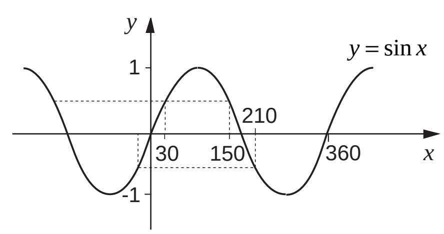


$$\sin 30° = \frac{1/2}{1} = \frac{1}{2}.$$

(a) $\sin 150^\circ = \sin\left(180^\circ - 30^\circ\right) = \sin 30^\circ = \frac{1}{2}$

(b) $\sin(-30°) = -\sin 30°$ (function is odd)

$= -\frac{1}{2}$



(c) $\sin 210° = -\frac{1}{2}$

(d) $\sin 390° = \sin 30° = \frac{1}{2}$

(e) Since $\cos\theta = \sin(90 - \theta°)$

$\cos 60° = \sin(90 - 60°)$

$= \sin 30°$

$= \frac{1}{2}$

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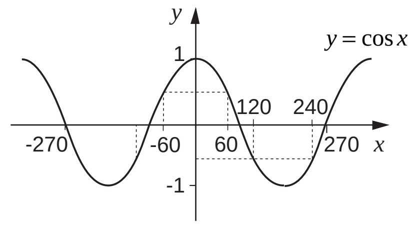
(f) $\cos 120° = -\cos 60°$ (function is odd about $90°$)

$= -\frac{1}{2}$

(g) $\cos(-60°) = \cos 60 = \frac{1}{2}$ (function is even)

(h) $\cos 240° = \cos 120° = -\frac{1}{2}$



In the above example, the relationship

$$\cos\theta = \sin(90 - \theta°)$$

was used.

Why is this result true?

Similarly, it should be noted that

$$\sin\theta = \cos(90 - \theta°)$$

The last function to consider here is $\tan x$, which has rather different properties from $\sin x$ and $\cos x$.

Activity 7 tan x

Using a graphic calculator or computer, sketch the function

$$y = \tan x$$

for x in the range $-360°$ to $360°$. What is the period of the function?

Unlike the sine and cosine functions, $\tan x$ is not bounded by ± 1. In fact, as x increases to $90°$, $\tan x$ increases without limit.

Example

Using the values

$$\tan 45° = 1 \quad \text{and} \quad \tan 30° = \frac{1}{\sqrt{3}}$$

find, without using a calculator,

(a) $\tan 135°$ (b) $\tan(-45°)$ (c) $\tan 315°$

(d) $\tan(-30°)$ (e) $\tan 150°$ (f) $\tan 60°$.

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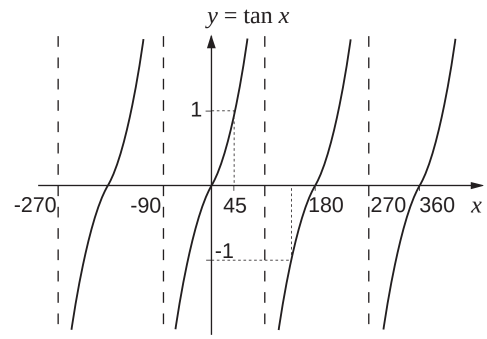
Solution



(a) $\tan 135° = \tan(180° - 45°) = -\tan 45° = -1.$

(b) $\tan(-45°) = -\tan 45°$

$= -1.$

(c) $\tan 315° = -\tan 45°$

$= -1.$

(d) $\tan(-30) = -\tan 30° = -\dfrac{1}{\sqrt{3}}.$

(e) $\tan 150° = -\tan 30° = -\dfrac{1}{\sqrt{3}}.$

(f) $\tan 60° = \dfrac{\sin 60°}{\cos 60°} = \dfrac{\cos 30°}{\sin 30°} = \dfrac{1}{\tan 30°}$

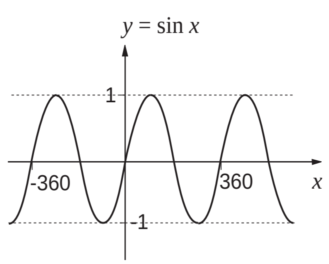
$= \dfrac{1}{\left(1/\sqrt{3}\right)} = \sqrt{3}.$

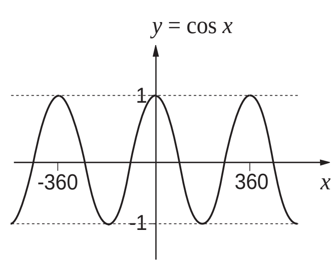
In (f), the result is equivalent to using

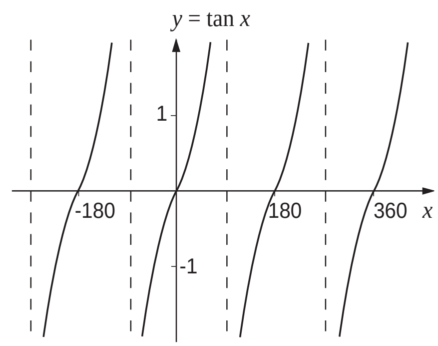
$$\tan\theta = \frac{1}{\tan(90 - \theta)}$$

which is set in the exercise below.

The full results are summarised below.



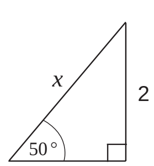




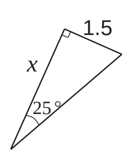
Exercise 10C

1. Find the length marked x to 2 d.p. for each of the figures below.

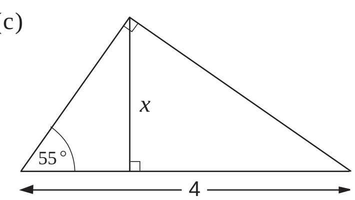
(a)



(b)



(c)



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2. Given that $\sin 45° = \frac{1}{\sqrt{2}}$, find, without using tables

   (a) $\sin 135°$ (b) $\sin(-45°)$ (c) $\sin 315°$

   (d) $\cos 45°$ (e) $\cos(225°)$ (f) $\tan 45°$

   (g) $\tan 135°$ (h) $\tan(-45°)$

3. Prove that $\sin\theta = \cos(90 - \theta)$.

4. Prove that .

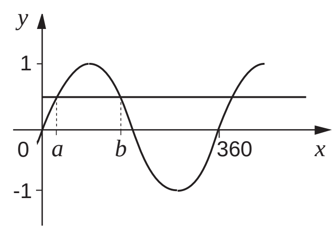
10.4 Solving trigonometrical equations

Trigonometric functions can be used to model physical phenomena but applying these functions to problems in the real world will often result in a trigonometric equation to solve. In this section, you will consider some simple equations to solve. These will highlight the difficulties that can occur.

Example

Solve $\sin x = \frac{1}{2}$ for $0° \le x \le 360°$

Solution



In solving equations of this sort it is **vital** to be aware that there may be **more than one** possible solution in the allowable domain - this possibility results from the periodic nature of this function. It is usually helpful to make a sketch of the relevant function, and this will help to identify the number of possible solutions.

In this case, the functions to plot are

$$y = \tfrac{1}{2} \text{ and } y = \sin x.$$

The points of intersection in the range $0° \le x \le 360°$ are solutions of the equation.

The figure shows that there are just **two** solutions, denoted by *a* and *b*, in the range. You can find the value of *a* by entering 0.5 in your calculators and using the 'inverse' sin or 'arc sin' buttons.

This gives $a = 30°$ (or $\frac{\pi}{6} \approx 0.524$ radians, if you are using radians).

Your calculator will only give you this single value, but it is easy to find *b* provided you have used a sketch. This shows that the function $y = \sin x$ is **symmetrical** about $x = 90°$ and so $b = 90° + 60° = 150°$ (≈ 2.618 radians). So the equation has two solutions, namely $30°$ or $150°$.

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Note: Even though your calculator cannot directly give you both solutions, it can be usefully used to check the answers. To do this enter $30°$ and press the sin button; similarly for $150°$.

If in the example above, the range had been $-360°$ to $360°$, how many solutions would there be?

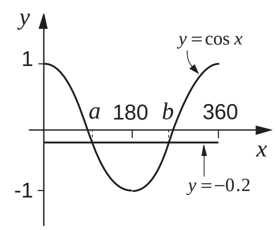
Example

Solve $3\cos x = -0.6$ for $0° \le x \le 360°$, giving your answer to 1 d.p.

Solution

Rearranging the equation

$$\cos x = -\frac{0.6}{3} = -0.2.$$



As before, you plot the functions

$$y = \cos x \text{ and } y = -0.2.$$

There are two solutions, a and b, with a between $90°$ and $180°$, and b between $180°$ and $270°$. Enter -0.2 into your calculator and use the 'inverse' cosine button - this will give you the answer

$$a = 101.5°.$$

From the graph the second solution b is given by

$$b - 180° = 180° - a \approx 78.5°$$

$$\Rightarrow \quad b = 258.5°.$$

and to 1 d.p. the solutions are $101.5°$ or $258.5°$.

(Remember to check the solutions on your calculator)

What would be the solutions to the previous problem if the range had been given as $-360° \le x \le 0°$?

Example

Solve the equation $\tan x = -2$ where x is measured in radians and $-2\pi \le x \le 2\pi$. Give your answers to 2 d.p. (Set your calculator to radian mode).

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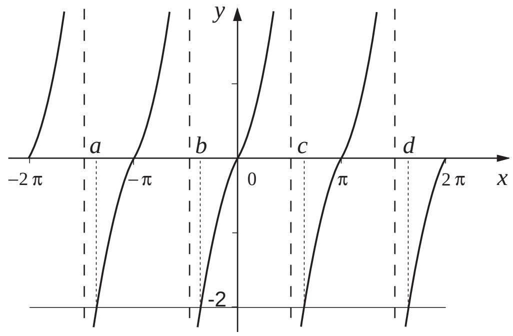
Solution

Note that x is given in radians rather than degrees here and remember that π radians $\equiv 180°$.

The graph of $y = \tan x$ and $y = -2$ are shown opposite.



You can see that there are four solutions denoted by a, b, c, and d.

To find one of these solutions, enter -2 in your calculator and use the 'inverse tan' (or 'arctan')button. This should give -1.107 radians, so that $b = -1.107$. Since the function $\tan x$ has period π (or $180°$).

$$a = b - \pi = -4.249$$

$$c = b + \pi = 2.035$$

$$d = b + 2\pi = 5.177$$

To 2 d.p.'s, the solutions in radians are

$$-4.25, \; -1.11, \; 2.04 \text{ and } 5.18$$

Exercise 10D

Give all answers to 3 significant figures

1. Solve $\cos x = 0.5, \; 0° \le x \le 360°$
2. Solve $\tan x = 1, 0° \le x \le 360°$
3. Solve $\sin x = \frac{1}{4}, 0° \le x \le 180°$
4. Solve $\sin x = -0.5, 0° \le x \le 360°$
5. Solve $4 \tan x = 1, 0° \le x \le 720°$
6. Solve $\sin x = -\frac{1}{3}, \; -\pi \le x \le \pi$
7. Solve $3 \cos x = 1 - \pi \le x \le \pi$
8. Which of these equations have no solutions?

   (a) $\sin x = 1, \; 0° \le x \le 360°$

   (b) $\cos x = -\frac{1}{4}, \; 0° \le x \le 90°$

   (c) $\cos x = 2, \; 0° \le x \le 360°$

   (d) $\tan x = 2, 0° \le x \le 90°$

   (e) $4 \sin x = -5, \; 0° \le x \le 360°$

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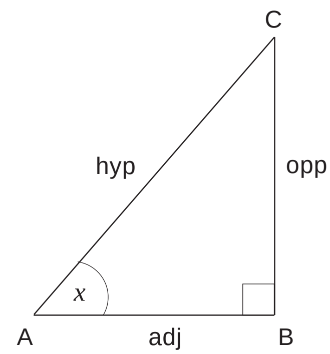
10.5 Properties of trig functions

There are a number of important properties, which you have already been using. These are

	Degrees	Radians
1.	$\cos(x) = \sin(90° - x)$	$\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$
2.	$\sin(x) = \cos(90° - x)$	$\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$
3.	$\cos(180° - x) = -\cos(x)$	$\cos(\pi - x) = -\cos(x)$
4.	$\sin(180° - x) = +\sin(x)$	$\sin(\pi - x) = \sin(x)$
5.	$\tan x = \frac{\sin x}{\cos x}$	
6.	$\sin^2 x + \cos^2 x = 1$	

The last result is a very important one and is proved below for $0 < x < 90°$.

Theorem $\sin^2 x + \cos^2 x = 1$



Proof

Using Pythagoras' Theorem,

$$BC^2 + AB^2 = AC^2.$$

Dividing by AC^2,

$$\left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2 = 1.$$

But $$\sin x = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AC}$$

$$\cos x = \frac{\text{adj}}{\text{hyp}} = \frac{AB}{AC}$$

giving $$\sin^2 x + \cos^2 x = 1.$$

Note $\sin^2 x$ is used for $(\sin x)^2$ so that there is no confusion between $(\sin x)^2$ and $\sin(x^2)$.

These properties can be used to solve more complex trig equations.

Example

Find all solutions between $0°$ and $360°$ of the equation

$$\sin x = 2\cos x \quad \text{to 2 d.p.}$$

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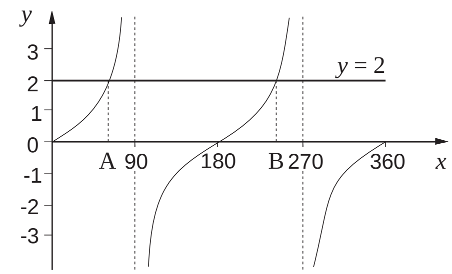
Solution

Assuming $\cos x \neq 0$, you can divide both sides of the equation by $\cos x$ to give

$$\tan x = 2$$

(since $\frac{\sin x}{\cos x} = \tan x$).



The graph of $y = \tan x$ and $y = 2$ is shown opposite. This shows that there are 2 solutions in the range $0°$ to $360°$.

Using a calculator, the inverse tangent of 2 is $63.4°$ (to 1d.p.). This is marked A on the graph. Since the tangent graph has a period of $180°$, B is given by

$$63.4° + 180° = 243.4°.$$

Note that at the start of the solution, $\cos x = 0$ was excluded.

When $\cos x = 0$, this gives $\sin x = 0$. These two equations are not both true for any value of x.

Example

Find all solutions between $0°$ and $360°$ of the equation

$$\cos x \sin x = 3\cos x$$

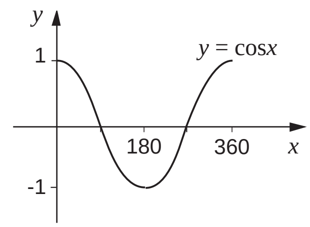
giving answers to 1 d.p.

Solution

The equation can be rewritten as

$$\cos x \sin x - 3\cos x = 0$$

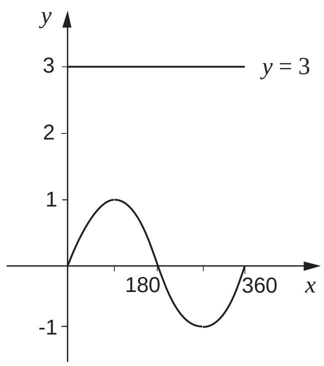
or $$\cos x(\sin x - 3) = 0.$$



So either $\cos x = 0$ or $\sin x - 3 = 0 \Rightarrow \sin x = 3$.

The graph of $y = \cos x$ is shown opposite for $0° \leq x \leq 360°$. The solutions are given by $\cos x = 0$, which gives

$$x = 90° \text{ or } 270°.$$



The other possibility is $\sin x = 3$. The graph of $y = \sin x$ and $y = 3$ are shown opposite. They do not intersect so there are no solutions.

The only solutions of the equation are therefore $x = 90°$ or $270°$.

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The example above showed that algebraic techniques like factorising can be applied to equations involving trigonometric functions. The next example shows that **quadratics** in sines or cosines can also be solved.

Example

Find all solutions between $0°$ and $360°$ to the equation

$$3\cos x = 2\sin^2 x.$$

Solution

This equation contains a mixture of 'cos' and 'sin' terms, but using the identity

$$\cos^2 x + \sin^2 x = 1$$

will make it possible to express the equation in 'cos' terms only.

Now $\sin^2 x = 1 - \cos^2 x$

so that the equation becomes

$$3\cos x = 2\left(1 - \cos^2 x\right)$$

$$= 2 - 2\cos^2 x$$

Hence $2\cos^2 x + 3\cos x - 2 = 0$.

(This is a quadratic equation in $\cos x$)

This can be factorised to give

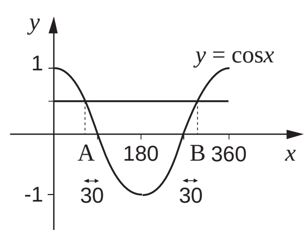
$$(2\cos x - 1)(\cos x + 2) = 0$$

and either $2\cos x - 1 = 0$ or $\cos x + 2 = 0$.

The first equation gives

$$\cos x = \tfrac{1}{2}$$

and the graphs of $y = \cos x$ and $y = \tfrac{1}{2}$ are shown in the figure opposite.



Using the inverse cosine function on a calculator gives $x = 60^0$, and this corresponds to A on the figure. Using symmetry about 180^0, gives B as

$$270° + 30° = 300°.$$

The second equation, $\cos x = -2$, has no solutions. The only possible solutions of the original equation are 60° and 300°.

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Exercise 10E

1. Find all the solutions to the equation $\sin x \cos x + \sin x = 0$ for $0° \le x \le 360°$
2. Solve completely the equation $1 - \sin x = 2\cos^2 x$, $0° \le x \le 360°$. (Hint: there are three solutions).
3. Find all solutions between $0°$ and $360°$, to 1 d.p., of the equation $5 - 2\cos x = 8\sin^2 x$.
4. Find all the solutions between 0 and 2π radians of the equation $\sin x = -\cos x$, leaving π in your answers.
5. Solve $3\sin x = \cos x$ for $0° \le x \le 360°$

10.6 Miscellaneous Exercises

1. Find in radians the angle subtended at the centre of a circle of circumference 36 cm by an arc of length 7.5 cm.
2. Find the length of the arc of a circle of radius 2 cm which subtends an angle of $\frac{\pi}{6}$ at the centre.
3. Find the radius of a circle if a chord of length 10 cm subtends an angle of $85°$ at the centre.
4. Change the following angles into radians.

   (a) $45°$ (b) $73.78°$ (c) $178.83°$
5. Find the area of a sector of a circle of radius 2 cm which subtends an angle of $\frac{\pi}{4}$ at the centre.
6. Determine the angle in radians subtended by the sector of a circle of radius 5 cm such that the area of the sector is 10 cm^2.
7. Solve the following equations completely for the values of x shown, to 1 d.p.if necessary.

   (a) $\tan(x + 90°) = 1,\ 0° \le x \le 360°$

   (b) $\cos 2x = \frac{1}{2},\ 0 \le x \le 2\pi$

   (c) $3\cos\left(\frac{1}{2}x + 45°\right) = -1,\ 0° \le x \le 360°$

   (d) $\sin 3x = -\frac{1}{2},\ 0 \le x \le 2\pi$
8. Solve these equations completely giving answers to 1 d.p. where necessary.

   (a) $\sin x = \cos x,\ 0° \le x \le 360°$

   (b) $3\sin x + \cos x = 0,\ 0 \le x \le 2\pi$

   (c) $3\sin^2 x - 4\sin x + 1 = 0,\ 0° \le x \le 360°$

   (d) $2\sin^2 x + 5\cos x + 1 = 0,\ 0° \le x \le 360°$
9. Solve these equations completely

   (a) $3\sin 2x = \cos 2x,\ 0° \le x \le 180°$

   (b) $\cos^2 x - 1 = 0,\ 0° \le x \le 360°$

   (c) $\cos^2 3x + 4\sin 3x = 1,\ 0° \le x \le 360°$

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