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* Polynomials

→ Remainder theorem:

- when $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$
- when $f(x)$ is divided by $(ax-b)$, the remainder is $f(b/a)$

→ Factor theorem:

- when $f(x)$ is divided by a factor, remainder is zero.

* Modulus function

→ The modulus of a real number is the magnitude of that numb

The modulus $f(x) = |x|$ is defined as:

$|x| = x$ for $x \geq 0$

$|x| = -x$ for $x < 0$

→ The curve $y = |f(x)|$ is obtained by reflecting the negative part of the curve in the x-axis.
The positive parts remain unchanged.

- $|ab| = |a| \times |b|$; same for division.
- $|x^2| = |x|^2 = x^2$
- $\sqrt{x^2} = \pm x = |x|$
- $|x| = |a| \Rightarrow x^2 = a^2$; same for inequalities.

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* Partial Fractions

- $$\frac{ax+b}{(px+q)(rx+s)} \equiv \frac{A}{(px+q)} + \frac{B}{(rx+s)}$$

- $$\frac{ax^2+bx+c}{(px+q)(rx^2+s)} \equiv \frac{A}{px+q} + \frac{Bx+C}{(rx^2+s)}$$

- $$\frac{ax^2+bx+c}{(px+q)(rx+s)^2} \equiv \frac{A}{(px+q)} + \frac{B}{(rx+s)} + \frac{C}{(rx+s)^2}$$

$\therefore$ $ax^2+bx+c \equiv A(rx+s)^2 + B(px+q)(rx+s) + C(px+q)$

→ when degree of numerator is equal to denominator:

- $$\frac{ax^2+bx+c}{(px+q)(rx+s)} \equiv A + \frac{B}{(px+q)} + \frac{C}{(rx+s)}$$

$\therefore$ $ax^2+bx+c \equiv A(px+q)(rx+s) + B(rx+s) + C(px+q)$

→ put $x = -s/r$ and $x = -q/p$ to find B and C

then → put $x =$ any value (eg: 0) to find A.

* Binomial expansions.

→ We can use partial fractions to simplify the expansions of complex expressions.

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* Logs and exponentials.

- A function $f(x) = ar^t$ describes:
  - -> exponential growth when $r > 1$
  - -> exponential decay when $r < 1$

- $a^b = c \qquad \log_a c = b$

The logarithmic function is the inverse of the exponential function to the same base.

PROPERTIES OF LOGS:

- $\log ab = \log a + \log b$
- $\log a/b = \log a - \log b$
- $\log a^b = b \log a$
- $\log_a a = 1$
- $\log_a 1 = 0$; at any base except zero.
- $\log_{b^c} a = 1/c \log_b a$
- $\log_b a = \dfrac{\log a}{\log b}$

Bases of logs are same on both sides.

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* Trigonometry

The following are positive in the respective quadrants:

sin, cosec	All
tan, cot	cos, sec

- $\sec\theta = \dfrac{1}{\cos\theta}$
- $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$
- $\cot\theta = \dfrac{1}{\tan\theta}$

→ IDENTITIES

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$

- $\cos^2\theta + \sin^2\theta \equiv 1$
- $\sec^2\theta - \tan^2\theta \equiv 1$
- $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$

$$\left.\begin{array}{l} \sin(90-\theta) = \cos\theta \\ \tan(90-\theta) = \cot\theta \\ \sec(90-\theta) = \operatorname{cosec}\theta \end{array}\right\} \text{and vice versa.}$$

→ Addition Formulae

- $\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$
- $\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$
- $\tan(A \pm B) \equiv \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

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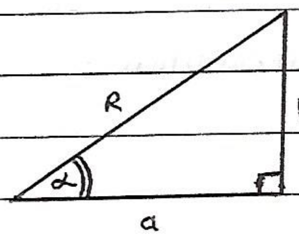
→ Double angle formulae.

- $\sin 2A \equiv 2\sin A\cos A$
- $\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 1 - 2\sin^2 A \equiv 2\cos^2 A - 1$
- $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

*



$R = \sqrt{a^2+b^2}$

$\alpha = \tan^{-1}(b/a)$ → $R\cos\alpha = a$

→ $R\sin\alpha = b$

- $a\sin x \pm b\cos x \equiv R\sin(x \pm \alpha)$
- $a\cos x \pm b\sin x \equiv R\cos(x \mp \alpha)$

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* Differentiation

- if $y = x^n$ then $\frac{dy}{dx} = n \cdot x^{n-1}$

- $\frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$; multiply by differential of the power.

- $\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)}$; multiply by differential

- $\frac{d}{dx} \log_{10} x = \frac{1}{x \ln 10}$

$f(x)$		$f'(x)$
$\sin(x)$	=	$\cos x$
$\cos(x)$	=	$-\sin x$
$\tan(x)$	=	$\sec^2 x$
$\sec(x)$	=	$\sec x \tan x$
$\operatorname{cosec}(x)$	=	$-\cot x \cdot \operatorname{cosec} x$
$\cot(x)$	=	$-\operatorname{cosec}^2 x$

  → multiply by differential of inside bracket (x)

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* Integration

- if $\frac{dy}{dx} = x^n$ then $y = \frac{x^{n+1}}{n+1}$

- $\int e^{f(x)} dx = \frac{1}{f'(x)} \cdot e^{f(x)} + c$; divide by differential of power.

- $\int \frac{1}{f(x)} = \frac{1}{f'(x)} \cdot \ln|f(x)| + c$; divide by differential

integrals are given in formula sheet

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* Numerical solutions of equations.

* Sign change rule.

if $f(a)$ is negative and $f(b)$ is positive, then the root lies between a and b.

* Decimal search

use sign-change-rule and increase the number of decimal places.

* Interval bisection

keep on finding mid-point of intervals.

* Iterative

$$x_{n+1} = f(x_n)$$

use an initial value then answers from previous values to obtain x_{n+1}

when the answer is almost constant, this will be the solution

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* Complex Numbers

$$|z| = \sqrt{[Re(z)]^2 + [Im(z)]^2}$$

→ If $F(z) = 0$, then complex roots occur in conjugate pairs (only if the coefficients of quadratic are real). *

■ $\tan\theta = \frac{Im(z)}{Re(z)}$ and $\arg(z) = \theta$

■ if $z = x + iy$ then $z^* = x - iy$

→ $z = r(\cos\theta + i\sin\theta) = re^{i\theta}$

where $r = |z|$

$\theta = \arg z$

■ If $z_1 = (r_1, \theta_1)$ and $z_2 = (r_2, \theta_2)$

then:

$$|z_1 z_2| = (r_1 r_2, \theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \left(\frac{r_1}{r_2}, \theta_1 - \theta_2\right)$$

* Sum of roots $= -b/a$.

* Product of roots $= c/a$.

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VECTORS

* **Vector equation of a line:**

$r = \vec{OA} + \mu(\vec{AB})$

* **Point on the line.**

$$\underset{\text{line}}{r} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \mu \begin{pmatrix} i \\ j \\ k \end{pmatrix} = \begin{pmatrix} d \\ e \\ f \end{pmatrix} \rightarrow \text{position vector of point.}$$

form 3 simultaneous equations to find μ.
if μ is satisfied by all 3 equations, the point is on the line.

* **Intersection of two lines.**

$$\rightarrow l_1 = \begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} + \mu \begin{pmatrix} i_1 \\ j_1 \\ k_1 \end{pmatrix} \qquad l_2 = \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix} + \lambda \begin{pmatrix} i_2 \\ j_2 \\ k_2 \end{pmatrix}$$

- if the direction vectors are scalar multiples the lines are parallel

- solve using simultaneous equations to find μ and λ.
  if μ and λ are satisfied by all 3 equations, the lines intersect.
  Point of intersection = substitute value of μ in l_1

... are skew.

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- For the lines to be perpendicular, the scalar product of direction vectors is zero

i.e $(i_1 \times i_2) + (j_1 \times j_2) + (k_1 \times k_2) = 0$

* Angle between 2 lines

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$\text{i.e } \cos\theta = \frac{(i_1 \times i_2) + (j_1 \times j_2) + (k_1 \times k_2)}{\sqrt{i_1^2 + j_1^2 + k_1^2} \times \sqrt{i_2^2 + j_2^2 + k_2^2}}$$

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* Equation of a plane:

$$r \cdot n = d$$

OR $ax + by + cz = d$

- n and (a, b, c) are normal of the plane.
- r and (x, y, z) is a point through which plane passes

* Equation of a plane when given:
  - → 2 lines in the plane
  - → point through which plane passes.

① Finding normal of plane:

- Equate direction vectors of 2 lines × n to zero

i.e: $\begin{pmatrix} i_1 \\ j_1 \\ k_1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$ and $\begin{pmatrix} i_2 \\ j_2 \\ k_2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$

$\therefore$ $i_1 a + j_2 b + k_2 c = 0$ and $i_2 a + j_2 b + k_2 c = 0$

- Find a ratio between $a : b : c$

This will give the normal.

② Finding $ax + by + cz = d$.

- Substitute the point in plane for (x, y, z) to find d.

Now you have the full vector equation.

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* Intersection of line and plane

Line $= \begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} + \mu \begin{pmatrix} i_1 \\ j_1 \\ k_1 \end{pmatrix}$ and plane $= ax + by + cz = d$

- If scalar product is zero

i.e $\begin{pmatrix} i_1 \\ j_1 \\ k_1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$

then the line is perpendicular to normal of plane
∴ the line is in the plane or parallel to plane.

■ substitute line into (x, y, z) of plane.

ie : $a(a_1 + \mu i_1) + b(b_1 + \mu j_1) + c(c_1 + \mu k_1) = d$

→ Solve for μ & substitute into line for point of intersection.

→ If there are no solutions, the line is parallel to plane and doesn't lie in the plane.

* Angle between line and plane

$$\cos\theta = \frac{(a_1 \cdot a) + (b_1 \cdot b) + (c_1 \cdot c)}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a^2 + b^2 + c^2}}$$

→ using direction vector of line and normal of plane.
This is angle between normal and plane

∴ angle line and plane $= \underline{\underline{90° - \theta}}$

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* 2 Planes

- Intersect if they have a line, hence infinite number of points in common.

- Are coincident if they are the same plane
  ↳ the equations of the planes are scalar multiples

- Are parallel if they have no points in common
  ↳ The normals are multiples of each other.

* Equation of line joining 2 planes

① Finding direction vector of line:

- Let $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ be direction of line (i)

- Form simultaneous equations using $i \cdot n = 0$ for both planes.
- Find ratios for $a:b:c$; this will give direction of line

② Finding position vector of line:

- Assign any value for z in both planes
- Solve to find x and y
- This will give the position vector.

∴ Equation of line = position vector + μ(direction vector)

* Angle between 2 planes
  ↳ it's the angle between their normals.

$$\cos\theta = \frac{n_1 \cdot n_2}{|n_1||n_2|}$$

* Distance between point (P) and plane

① Find unit vector of plane

$$\hat{n} = \frac{1}{|n|} \cdot n = \frac{1}{\sqrt{a^2+b^2+c^2}} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot r$$

where (a,b,c) is normal of plane.

② Find a point (Q) in the plane
  ↳ Find any set of values for (x, y, z) which satisfy the plane equation.

③ Find vector from this point in plane to the point given.
$\vec{PQ} = \vec{OQ} - \vec{OP}$

↳ Substitute this point into r to get.

$$d = \frac{1}{|n|} \cdot n \cdot r \quad \text{i.e} \quad \frac{(ar_1)+(br_2)+(cr_3)}{\sqrt{a^2+b^2+c^2}}$$

ignore negative sign in answer

OR ① distance $= \dfrac{a(i)+b(j)+c(k)-d}{\sqrt{a^2+b^2+c^2}}$ → (i, j, k) is the point
→ $ax + by + cz = d$
...e.

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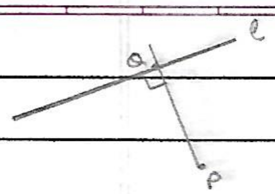
* Distance between point and line



① Find a point Q on the line for which $\vec{PQ}$ is perpendicular to line l.

$\vec{PQ} = \vec{OQ} - \vec{OP}$ → $\vec{OP}$ is known
→ $\vec{OQ}$ is in terms of μ

$$\vec{OQ} = \begin{pmatrix} a + \mu i \\ b + \mu j \\ c + \mu k \end{pmatrix}$$

∴ Find $\vec{PQ}$ in terms of μ

② $\vec{PQ}$ × direction vector of line l = 0
↳ solve to find μ

③ Substitute value of μ into $\vec{PQ}$
Find $|PQ|$
↳ This is distance from point P to line l.

* Distance between 2 planes

① Find distance of each plane from origin :

$$\frac{d}{\sqrt{a^2+b^2+c^2}}$$

② Subtract the 2 distances to find distance between planes.

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