

This is a method for proving that a certain result is true for all positive integers n (ie, for $n \in \mathbb{N}$). There are two steps:

- ① We show that the result is true for $n=1$
- ② We ASSUME the result is true for $n=k$, and show that it follows that it is true for $n=k+1$

Examples

- ① Prove that $7^n + 2^{2n} + 1$ is divisible by 6 for all $n \in \mathbb{N}$.

Try $n=1$: $7^1 + 2^2 + 1 = 12 = 2 \times 6$

Assume this is true for $n=k$ ($k \geq 1$)
 ie, that $7^k + 2^{2k} + 1 = 6a$ ($a \in \mathbb{N}$)

Now consider $n=k+1$

$$\begin{aligned}
 7^{k+1} + 2^{2k+2} + 1 &= 7(7^k) + 4(2^{2k}) + 1 \\
 &= 7^k + 2^{2k} + 1 + 6(7^k) + 3(2^{2k}) \\
 &= 6a + 6(7^k) + 6(2^{2k-1}) \\
 &= 6[a + 7^k + 2^{2k-1}] \\
 &= 6b \quad (b \in \mathbb{N})
 \end{aligned}$$

Hence by induction the result is true for all $n \in \mathbb{N}$.

- ② Prove that $1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$ for $n \in \mathbb{N}$.

Try $n=1$: $1^3 = 1 = \frac{1^2 \times 2^2}{4}$ which is true.

Now assume result is true for $n=k$

ie, $1^3 + 2^3 + \dots + k^3 = \frac{k^2(k+1)^2}{4}$

Consider $n = k+1$ www.youtube.com/megalecture

$$\begin{aligned}
 1^3 + 2^3 + \dots + k^3 + (k+1)^3 &= \frac{k^2(k+1)^2}{4} + (k+1)^3 \\
 &= \frac{1}{4}(k+1)^2 [k^2 + 4(k+1)] \\
 &= \frac{1}{4}(k+1)^2 (k^2 + 4k + 4) \\
 &= \frac{(k+1)^2(k+2)^2}{4}
 \end{aligned}$$

which is the formula with $n = k+1$

Hence by induction the result is true for all $n \in \mathbb{N}$.

③ A sequence is defined by:
 $u_1 = 1$, $u_2 = 5$, $u_{n+2} = 5u_{n+1} - 6u_n$ ($n \geq 1$)

[ie/ 1, 5, 19, 65,]

Prove by induction that $u_n = 3^n - 2^n$

Try $n=1$ $3^1 - 2^1 = 1 = u_1$
 and $n=2$ $3^2 - 2^2 = 5 = u_2$

Assume the result is true for $n=k$ and $n=k+1$

ie, that $u_k = 3^k - 2^k$
 and $u_{k+1} = 3^{k+1} - 2^{k+1}$

Then, from the given definition,

$$\begin{aligned}
 u_{k+2} &= 5(3^{k+1} - 2^{k+1}) - 6(3^k - 2^k) \\
 &= 3^k(15 - 6) + 2^k(6 - 10) \\
 &= 9(3^k) - 4(2^k) \\
 &= 3^2 \times 3^k - 2^2 \times 2^k \\
 &= 3^{k+2} - 2^{k+2}
 \end{aligned}$$

which is the result for $n = k+2$

Hence by induction the result is true for all $n \in \mathbb{N}$.

PLS8 Ex 7.1 Q 2ac, 3, 4, 5, 6